

# 1 Estimation of CH<sub>4</sub> emission based on advanced 4D-LETKF assimilation system

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8

## 9 Abstract

10 Methane (CH<sub>4</sub>) is the second major greenhouse gas after carbon dioxide (CO<sub>2</sub>) which has  
11 substantially increased during last decades in the atmosphere, raising serious sustainability and  
12 climate change issues. Here, we develop a data assimilation system for in situ and column averaged  
13 concentrations using Local ensemble transform Kalman filter (LETKF) to estimate surface emissions  
14 of CH<sub>4</sub>. The data assimilation performance is tested and optimized based on idealized settings using  
15 Observation System Simulation Experiments (OSSEs) where a known surface emission distribution  
16 (the truth) is retrieved from synthetic observations. We tested three covariance inflation methods to  
17 avoid covariance underestimation in the emission estimates, namely; fixed multiplicative (FM),  
18 relaxation to prior spread (RTPS) and adaptive multiplicative. First, we assimilate the synthetic  
19 observations at every grid point at the surface level. In such a case of dense observational data, the  
20 normalized Root Mean Square Error (RMSE) in the analyses over global land regions are smaller by  
21 10-15% in case of RTPS covariance inflation method compared to FM. We have shown that  
22 integrated estimated flux seasonal cycles over 15 regions using RTPS inflation are in reasonable  
23 agreement between true and estimated flux with 0.04 global normalized annual mean bias. We have  
24 then assimilated the column averaged CH<sub>4</sub> concentration by sampling the model simulations at  
25 GOSAT observation locations and time for another OSSE experiment. Similar to the case of dense  
26 observational data, RTPS covariance inflation method performs better than FM for GOSAT synthetic  
27 observation in terms of normalized RMSE (2-3%) and integrated flux estimation comparison with the  
28 true flux. The annual mean averaged normalized RMSE (normalized mean bias) in LETKF CH<sub>4</sub> flux  
29 estimation in case of RTPS and FM covariance inflation is found to be 0.59 (0.18) and 0.61 (0.23)  
30 respectively. The chi-square test performed for GOSAT synthetic observations assimilation suggests  
31 high underestimation of background error covariance in both RTPS and FM covariance inflation

32 methods, however, the underestimation is much higher (>100% always) for FM compared to RTPS  
33 covariance inflation method.

## 34 **1. Introduction**

35 Methane (CH<sub>4</sub>) is the second major greenhouse gas, after carbon dioxide (CO<sub>2</sub>), that has  
36 anthropogenic sources. According to the contemporary record of the global CH<sub>4</sub> budget, the total of  
37 all CH<sub>4</sub> sources ranges 538–593 Tg yr<sup>-1</sup> during 2008–2017 (Saunio et al., 2020). The primary natural  
38 sources are from wetlands (~40%). The main anthropogenic CH<sub>4</sub> emissions are from microbial  
39 emissions associated with ruminant (livestock and waste), rice cultivation, fugitive emissions (oil and  
40 gas production and use), and incomplete combustion of bio and fossil fuels. The major fraction of  
41 atmospheric CH<sub>4</sub> sinks (range: 474 - 532 Tg yr<sup>-1</sup>) occurs in the troposphere by oxidation via reaction  
42 with hydroxyl (OH) radicals (Patra, et al., 2011; Saunio et al., 2020); other loss processes include  
43 oxidation by soil, and reactions with O<sup>1</sup>D and Cl. The lifetime of CH<sub>4</sub> in the atmosphere is estimated  
44 to be 9.1 ± 0.9 years (Szopa et al. 2021).

45 Regional CH<sub>4</sub> emissions can be estimated from CH<sub>4</sub> concentration fields and chemistry transport  
46 models using Bayesian synthesis approaches based on inverse modeling techniques (e.g., Enting,  
47 2002). In such approach, emissions are optimized on a coarse resolution (e.g., for a limited number of  
48 pre-defined regions) mostly using surface-based observations. CH<sub>4</sub> concentrations are provided by the  
49 NOAA cooperative air sampling network sites (Dlugokencky et al., 2020) and other networks by the  
50 World Data Centre for Greenhouse Gases (WDCGG) website, hosted by the Japan Meteorological  
51 Agency. In the recent years, satellite measurements are made from the Greenhouse Gases Observing  
52 Satellite (GOSAT) or the TROPOspheric Monitoring Instrument (TROPOMI) (Lorente et al., 2021),  
53 covering the globe with fine spatio-temporal scales. GOSAT provide an extensive global observations  
54 of column CH<sub>4</sub> concentrations since 2009 (Yoshida et al., 2013). Some of the inverse modeling  
55 studies utilize the satellite observations for CH<sub>4</sub> flux estimation (Zhang et al., 2021; Maasakkers et al.,  
56 2016), but, it requires enormous computational resources while dealing with more flux regions and  
57 more observations.

58 Grid-based CH<sub>4</sub> flux optimization is also performed using adjoint technique (4-D Var data  
59 assimilation) and Ensemble Kalman Filter (EnKF), but was limited to small sets of observations  
60 (Houweling et al., 1999; Meirink et al., 2008; Bruhwiler et al., 2014). Bruhwiler et al. (2014) followed  
61 the EnKF method of Peters et al. (2005) to estimate the CH<sub>4</sub> surface fluxes that utilizes an off-line  
62 ACTM framework. Techniques such as 4-D Var and EnKF are important to estimate CH<sub>4</sub> fluxes since  
63 they can assimilate a large number of observations, manage high-resolution fluxes. In the EnKF  
64 system, a flow-dependent forecast error covariance structure is provided by ensemble model forecasts,  
65 while it does not need an adjoint model that makes it simple but powerful tool for flux estimation.

66 One of the limitations in EnKF method is the dependence of the resolution of state vector on  
67 ensemble size, which can give spurious results if the number of ensemble members is much smaller  
68 than the rank of the error covariance matrix (Houtekamer and Zhang, 2016).

69 LETKF is a type of square-root EnKF that performs analysis locally in space without perturbing the  
70 observations (Ott et al., 2002, 2004; Hunt et al., 2007). LETKF is computationally efficient since the  
71 observations are assimilated simultaneously not serially, it is simple to account for observation error  
72 correlation. Miyazaki et al. (2011) and Kang et al. (2012) demonstrated the implementation of  
73 LETKF data assimilation system by coupling an ACTM for carbon-cycle research using atmospheric  
74 CO<sub>2</sub> observations. It is also extensively applied for the emission estimation of short-lived species  
75 using satellite data (Skachko et al., 2016; Miyazaki et al., 2019; Sekiya et al., 2021). In this work, we  
76 will estimate the CH<sub>4</sub> fluxes using a LETKF data assimilation system. Assimilation windows ranging  
77 from 6 hours (Kang et al., 2012) to several months (Bruhwiler et al., 2014) have been used, depending  
78 on the desired time resolution of the estimated emissions, which is often limited by the observational  
79 data density. The time frame over which the system behaves linearly, and in what time frame the  
80 observations respond to the control variables such as, atmospheric transport, as well as observation  
81 abundance, must also be taken into consideration. Within an assimilation window, where and when  
82 the fluxes would be constrained by specific observations is to be ascertained by the correlation  
83 between ensemble prior fluxes and the ensemble CH<sub>4</sub> concentrations simulation from a forward  
84 model (Liu et al., 2016).

85 main objective of this work is to develop an advanced 4-D data assimilation system based on LETKF  
86 that simultaneously estimates atmospheric distributions and surface fluxes of CH<sub>4</sub>. OSSEs are  
87 conducted to assess the performance of LETKF since it is important to test the system against the  
88 known emissions or the truth. The OSSE LETKF set-up of top-down CH<sub>4</sub> flux estimation using online  
89 ACTM is an essential step before implementing on real in situ and satellite observation.

## 90 **2. Formulation of LETKF system**

91 We briefly describe the LETKF in the application of CH<sub>4</sub> flux estimation, while detailed derivation of  
92 equations and code implementation are given elsewhere (Hunt et al., 2007; Miyazaki et al., 2011;  
93 Miyoshi et al., 2010). The notation used here for LETKF formulation is adopted from Kotsuki et al.  
94 (2017). In the LETKF, the background ensemble (columns of matrix  $x^b$ ) in a local region evolved  
95 from a set of perturbed initial conditions. The background ensemble mean,  $\bar{x}^b$ , and its perturbation,  
96  $X^b$ , are estimated from the ensemble forecast such as:

$$\bar{x}^b = \frac{1}{m} \sum_{i=1}^m x_i^b; \quad X_i^b = x_i^b - \bar{x}^b \quad (1)$$

97 Where ‘m’ indicates the ensemble size. The background error covariance matrix  $P^b$  in the m-  
 98 dimensional ensemble is defined as:

$$P^b = \frac{1}{m-1} X^b [X^b]^T \quad (2)$$

99 The analysis ensemble mean  $\bar{x}^a$  is derived using background ensemble mean  $\bar{x}^b$  and ensemble  
 100 perturbations  $X^b$  such as:

$$\bar{x}^a = \bar{x}^b + X^b \tilde{P}^a (Y^b)^T R^{-1} (y^o - H\bar{x}^b) = \bar{x}^b + X^b w^a \quad (3)$$

101 where H, Y, R, and  $\tilde{P}^a$  denote the linear observation operator, ensemble perturbation matrix in the  
 102 observation space ( $Y \equiv Hx$ ), observation error covariance matrix, and analysis error covariance matrix  
 103 in the ensemble space, respectively. The superscripts ‘o’, ‘b’ and ‘a’ denote the observations,  
 104 background (prior), and analysis (posterior), respectively.  $w^a$  defines the analysis increment (or  
 105 analysis weight) in observation space and is derived using the information about observational  
 106 increment  $y^o - H\bar{x}^b$ . The analysis error covariance matrix ( $\tilde{P}^a$ ) in the m-dimensional ensemble space  
 107 is spanned by ensemble perturbation (Hunt et al., 2007) and defined as:

$$\tilde{P}^a = \{(m-1)I + (HX^b)^T R^{-1} HX^b\}^{-1} \quad (4)$$

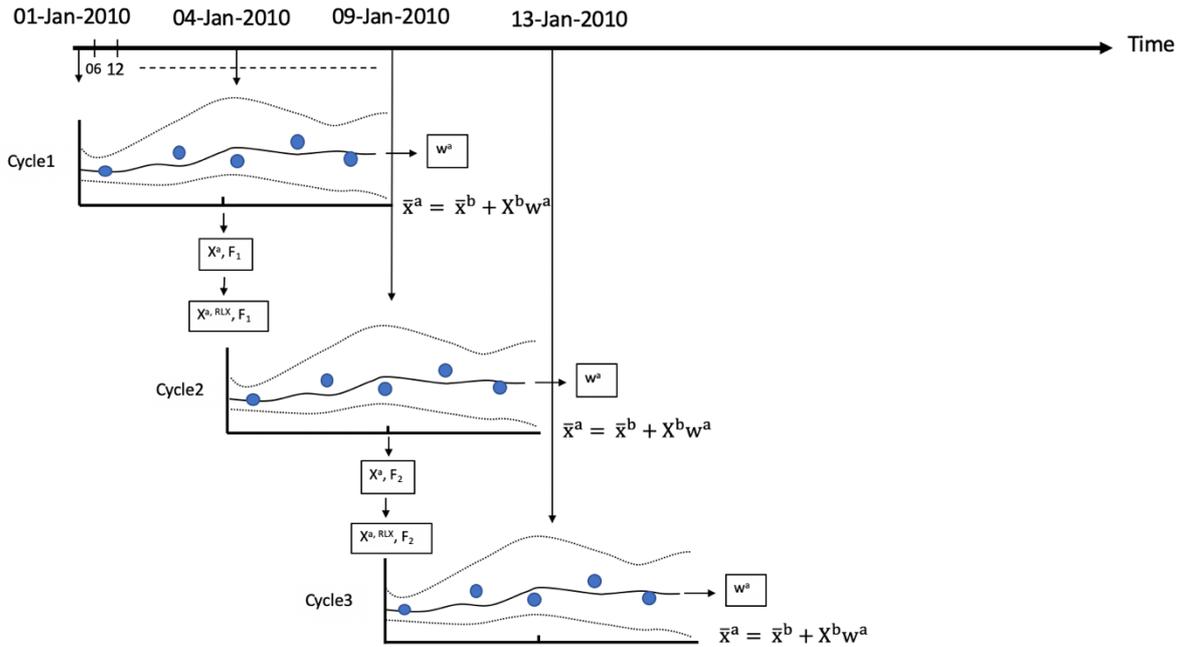
108 Finally, the analysis ensemble perturbations  $X^a$  at the central grid point are derived such as:

$$X^a = X^b \{(m-1)\tilde{P}^a\}^{1/2} \quad (5)$$

109 Where,  $\{(m-1)\tilde{P}^a\}^{1/2}$  is a multiple of the symmetric square root of the local analysis error  
 110 covariance matrix in ensemble space and could be computed by singular vector decomposition  
 111 method. The LETKF solves the analysis update equations 3 and 5 at every model grid point  
 112 independently by assimilating local observations within the localization cut-off radius.

113 We have applied a gross error check as a quality control to exclude observations that are far from the  
 114 first guess, the appropriate degrees of the gross error check are also examined. Figure 1 shows the  
 115 schematic diagram of our LETKF set-up with two ensemble members for 3 consecutive assimilation  
 116 cycles with 8 days assimilation window. The analysis is obtained at mid-point time of the assimilation  
 117 window (Figure 1). The analyzed (updated) surface flux is used for next data assimilation cycle  
 118 starting from the mid-point time of the previous data assimilation window. The state vector  
 119 augmentation approach is used to estimate the atmospheric CH<sub>4</sub> surface flux (Kang et al., 2012;  
 120 Miyazaki et al., 2011).

121 Assimilation window size and ensemble members are chosen based on computational efficiency and  
 122 estimation accuracy. A larger assimilation window means fluxes are constrained by more  
 123 observations, however, it requires handling of large matrix optimization which is difficult in cases of  
 124 dense observation and introduces sampling errors related to transport errors. In this study, few  
 125 sensitivity experiments performed to demonstrate the choice of assimilation window length and  
 126 ensemble size when GOSAT synthetic observation are assimilated in Section 4.2.



127

128 **Figure 1:** Schematic represents the temporal evolution of LETKF cycle. In the first assimilation  
 129 window (Cycle1), the dotted lines show the ensemble forecast of CH<sub>4</sub> concentrations (with 2  
 130 ensemble members), the solid line shows the linear combination of the forecasts, the filled circles  
 131 show the observations of CH<sub>4</sub> concentration. The data assimilation finds the linear combination of the  
 132 ensemble forecast by estimating the weight ( $w^a$ ) that best fits the observations throughout the  
 133 assimilation window. The analysis weight is applied to obtain optimal surface fluxes ( $F$ ) and the  
 134 concentration of CH<sub>4</sub> at the intermediate time of the data assimilation window. The updated analyzed  
 135 concentration ensembles are used as initial condition after relaxation ( $X^{a, RLX}$ ) (Eq. 8) for the next  
 136 ensemble forecast. The spread of the ensemble members represents the forecast error. The schematic  
 137 is adapted from Kalnay and Yang (2010) and Miyazaki et al. (2011).

### 138 2.1 Covariance inflation

139 The LETKF data assimilation needs variance inflation to mitigate the under dispersive ensemble. We  
 140 tested three methods; fixed multiplicative (FM), relaxation-to-prior spread (RTPS), and adaptive  
 141 multiplicative covariance inflation.

142 The fixed multiplicative (FM) inflation method (Anderson and Anderson, 1999) inflates the prior  
 143 ensemble by inflating the background error covariance matrix  $P^b$  defined in equation (Eq. 2) such as:

$$P_{\text{inf}}^b = \gamma P_{\text{tmp}}^b \quad (6)$$

144 where  $P_{\text{tmp}}^b$  represents the temporary background error covariance matrix which is inflated by a factor  
 145  $\gamma$ .

146 The other inflation methods used to prevent the reduction of ensemble spread are relaxation-to-prior  
 147 perturbation (RTPP) (Zhang et al., 2004) and relaxation-to-prior spread (RTPS) (Whitaker and  
 148 Hamill, 2012). The RTPP methods relax the reduction of the ensemble spread after updating the  
 149 ensemble perturbations which blends the background and analysis ensemble perturbations as:

$$X_{\text{inf}}^a = \alpha_{\text{RTPP}} X^b + (1 - \alpha_{\text{RTPP}}) X_{\text{tmp}}^a \quad (7)$$

150 where  $\alpha_{\text{RTPP}}$  denotes the relaxation parameter of the RTPP.

151 The RTPS inflation method relaxes the reduction of ensemble spread by relaxing the analysis spread  
 152 to prior spread such as:

$$X_{\text{RLX}}^a = \left( \frac{\alpha_{\text{RTPS}} \sigma^b + (1 - \alpha_{\text{RTPS}}) \sigma^a}{\sigma^a} \right) X_{\text{tmp}}^a \quad (8)$$

153 where  $\sigma$  and  $\alpha_{\text{RTPS}}$  denote the ensemble spread, and relaxation parameter of the RTPS, respectively.  
 154 The range of parameter  $\alpha_{\text{RTPS}}$  is bounded by [0, 1]. This study focuses mainly on the FM and RTPS  
 155 covariance inflation methods.

156 In addition, Miyoshi (2011) applied adaptive inflation by determining the multiplicative inflation  
 157 factors at every grid point at every analysis step using the observation-space statistics derived by  
 158 Daley (1992) and Desroziers et al. (2005).

$$\langle dd^T \rangle = HP_{\text{inf}}^b H^T + R \quad (9)$$

159 Where the operator ' $\langle \bullet \rangle$ ' denotes the statistical expectation and  $d = y^o - H\bar{x}^b$  (observation-minus-  
 160 first-guess), and  $R$  is the error observation covariance matrix.

161 The impact of using the adaptive multiplication inflation method is discussed in the GOSAT synthetic  
 162 observation assimilation experiments in Section 4.2.

## 163 2.2 MIROC4-ACTM

164 Model for Interdisciplinary Research on Climate, version 4.0 (MIROC4) based ACTM (hereafter  
 165 referred to as MIROC4-ACTM) (Patra et al., 2018; Bisht et al., 2021) is used here for  $\text{CH}_4$

166 concentration simulations. The model simulations have been performed at horizontal grid resolution  
167 of approximately  $2.8 \times 2.8^\circ$  latitude-longitude grid (T42 spectral truncations) and hybrid vertical  
168 coordinate of 67 levels (Earth's surface to 0.0128 hPa, Watanabe et al., 2008). Bisht et al., 2021  
169 performed the multi-tracer analysis and demonstrated the importance of very well-resolved  
170 stratosphere in the MIROC4-ACTM that illustrates better extratropical stratospheric variabilities, and  
171 simulated tropospheric dynamical fields. The meteorological fields in MIROC4-ACTM are nudged to  
172 the JMA Re-analysis (JRA-55) data (Kobayashi et al., 2015).

### 173 **3. Experimental set-up**

#### 174 **3.1 Construction of known surface emissions (truth)**

175 Present OSSEs intend to develop basic tuning strategies before the actual data to be assimilated which  
176 is useful to accelerate the operational use of real observations. The OSSE has been discussed here by  
177 exploiting the known "truth". The synthetic observations to be assimilated in the OSSE are generated  
178 from nature runs which uses bottom-up surface emission (true) data to simulate global 3-D CH<sub>4</sub>  
179 concentrations. The true surface CH<sub>4</sub> emissions are prepared on the monthly scale using  
180 anthropogenic and natural sectors, minus the surface sinks due to bacterial consumption in the soil  
181 (Chandra et al., 2021). The anthropogenic emissions were obtained from the Emission Database for  
182 Global Atmospheric Research, version 4.3.2 inventory (EDGARv4.3.2) (Maenhout et al., 2019) that  
183 includes the emissions from the major sectors such as; fugitive, enteric fermentation and manure  
184 management, solid waste and wastewater handling. The biomass burning emissions are taken from the  
185 Global Fire Database (GFEDv4s) (van der Werf et al., 2017) and Goddard Institute for Space Studies  
186 emissions (Fung et al., 1991). The wetland and rice emissions are taken from the process-based model  
187 of the terrestrial biogeochemical cycle, Vegetation Integrated Simulator of Trace gases (VISIT) (Ito,  
188 2019) that is based on Cao et al. (1996). The other natural emission such as, ocean, termites, mud  
189 volcano are taken from TransCom-CH<sub>4</sub> inter-comparison experiment (Patra et al., 2011). The total  
190 emissions are taken as the truth for the OSSEs and the concentration simulated by MIROC4-ACTM  
191 will be referred to as synthetic observations.

#### 192 **3.2 Prior flux preparation and LETKF setting**

193 Based on our understanding of CH<sub>4</sub> inverse modelling, the uncertainty in regional flux estimation is  
194 found to be 30% or lower (Chandra et al., 2021). Therefore, we attempted to reproduce the true flux  
195 by starting with a prior flux that is lower by 30% of the true flux (prior flux has same seasonal cycles  
196 as true flux). The MIROC4-ACTM is initialized with the spin-up of 3 years (2007 – 2009) with prior  
197 flux distribution. The initial CH<sub>4</sub> distribution on 01 January 2007 was taken from an earlier simulation  
198 of 27 years. An initial perturbation with standard deviation of approximately 6-8% spread is applied

199 to the a priori flux as the initial ensemble spread, whereas no ensemble perturbation was applied to the  
200 initial CH<sub>4</sub> concentration. The sensitivity of the initial ensemble spread to CH<sub>4</sub> flux estimation is  
201 discussed in Section 4.2. The uncertainty to perturb prior fluxes is generated based on random  
202 positive values with normal distribution. The monthly scale prior emission is linearly interpolated at 6  
203 hourly intervals to be used in the MIROC4-ACTM simulation for data assimilation. This study  
204 performs two LETKF data assimilation experiments. In these experiments, we provided initial  
205 perturbation on regional basis over land (53 different land regions; Chandra et al., 2021) and at every  
206 grid over ocean, no spatial error correlation between grid points is considered among ensemble  
207 members. However, in Section 4.2.5, we also discussed the sensitivity of CH<sub>4</sub> data assimilation by  
208 providing initial ensemble spread at every grid by considering horizontal spatial error correlation  
209 between grid points among ensemble members, with a global mean correlation of 20%.

### 210 **3.3 Experiment 1: Synthetic dense observation formulation**

211 The OSSE setting with very accurate and dense observation surface data is an attempt to demonstrate  
212 that data assimilation system works reasonably in the estimation of the true surface flux. Errors in the  
213 estimated flux could arise due to the insufficient ensemble size and also the implemented inflation  
214 methods to overcome the under-sampling, along with simplified forecast process of emissions. In real  
215 data assimilation, there are additional sources of potential errors, such as, atmospheric transports, and  
216 inappropriate prior or observation uncertainties. In our OSSEs, CH<sub>4</sub> fluxes as mentioned in Section  
217 3.2 are used as “true” fluxes in generating synthetic observations (CH<sub>4</sub> concentrations). In the  
218 experiment 1, the simulated surface layer CH<sub>4</sub> concentrations at each grid for the entire globe were  
219 used as synthetic assimilated observations. We added a constant measurement uncertainty of 5 ppb,  
220 which is typically achieved by the present-day measurement systems (e.g., Dlugokencky et. al, 2020).

221 In this study, the CH<sub>4</sub> observations are assimilated by applying the observation error covariance  
222 localization (Kotsuki et al., 2020) to reduce the spurious spatial correlation due to smaller ensemble  
223 size than the degrees of freedom of the system ( $R \leftarrow R \times \exp\left(-\frac{1}{2}\{(d_h/\sigma_h)^2 + (d_v/\sigma_v)^2\}\right)$ ). Where  
224  $d_h$  and  $d_v$  denote the horizontal distance (km) and vertical difference (log[Pa]) between the analysis  
225 model grid point and observation location. The tunable parameters  $\sigma_h$  and  $\sigma_v$  are the horizontal  
226 localization scale (km) and vertical localization scale (log[Pa]), respectively. Using the spatial  
227 localization technique, we have estimated the CH<sub>4</sub> flux for each grid by choosing the CH<sub>4</sub>  
228 observations that influence the grid point using optimal cutoff radius ( $\approx 3.65\sigma_{h,v}$ ; Miyoshi et al.,  
229 2007) with horizontal covariance localization ( $\sigma_h$ ) of 2200 km and vertical covariance localization  
230 ( $\sigma_v$ ) of 0.3 in the natural logarithmic pressure (log[Pa]) coordinate. The localization is performed to  
231 improve the signal to noise ratio of ensemble-based covariance. Numerous sensitivity experiments  
232 have been performed by varying the horizontal and vertical localization length in order to obtain the  
233 optimized CH<sub>4</sub> flux that best compare with the truth. The LETKF assimilates the observations within

234 the specified radius to solve the analysis state at each grid point independently (Liu et al., 2016;  
 235 Kotsuki et al., 2020). State vector of the analysis includes the atmospheric CH<sub>4</sub> concentration, which  
 236 is the prognostic variable of forecast model and the state vector is further augmented by surface CH<sub>4</sub>  
 237 flux, which is not a model prognostic variable. This augmentation enables the LETKF to directly  
 238 estimate the parameter through the background error covariance with observed variables (Baek et al.,  
 239 2006). The state vector augmentation is implemented similar to that used by Miyazaki et al. (2011).  
 240 This approach analyses CH<sub>4</sub> flux during the analysis step. The purpose of the simultaneous CH<sub>4</sub>  
 241 emission and concentration optimization is to reduce the uncertainty of the initial CH<sub>4</sub> concentrations  
 242 on the CH<sub>4</sub> evolution during the assimilation window and to maximize the observations potential  
 243 (Tian et al., 2014).

244 The atmospheric CH<sub>4</sub> concentration is changed during both the analysis and forecast steps. A  
 245 challenge of this scheme is that, the analysis increment is added to the model state at each analysis  
 246 step, without considering the global total CH<sub>4</sub> mass conservation in the model, but consistent with the  
 247 observed local CH<sub>4</sub> abundance.

248 In this case, surface flux at every model grid point is analyzed with 8-days assimilation window  
 249 during the year 2010 with the 100 ensemble members. The ensemble size and assimilation window  
 250 are chosen based on the CH<sub>4</sub> flux estimation accuracy calculated by performing sensitivity experiment  
 251 for ensemble size (60, 80, and 100) and assimilation window (3-days and 8-days), respectively (not  
 252 shown).

### 253 **3.4 Experiment2: synthetic satellite observation formulation**

254 One way to address the real-world CH<sub>4</sub> flux estimation problem is to first make the OSSE dataset like  
 255 real observations. In this OSSE experiment, we have assimilated synthetic column average CH<sub>4</sub>  
 256 concentrations with a coverage mimicking GOSAT satellite observations. We prepared a model  
 257 simulated column averaged CH<sub>4</sub> concentrations (XCH<sub>4</sub>) dataset that is spatiotemporally sampled with  
 258 GOSAT-observations as follows:

$$XCH_4 = XCH_{4(a\ priori)} + \sum_j h_j a_j (CH_{4(CTM)} - CH_{4(a\ priori)})_j \quad (10)$$

259 Where, XCH<sub>4</sub> is the column-averaged model simulated CH<sub>4</sub> concentration. XCH<sub>4(a priori)</sub> is a priori  
 260 column-averaged concentration. CH<sub>4(CTM)</sub> and CH<sub>4(a priori)</sub> are the CH<sub>4</sub> profile from CTM and a  
 261 priori, respectively.  $h_j$  is the pressure weighting function ( $j$  is the vertical layer index), and  $a_j$   
 262 represents averaging kernel matrix for the column retrieval which is the sensitivity of the retrieved  
 263 total column at the various ( $j$ ) atmospheric levels. In the next step, we added the same retrieval

264 (XCH<sub>4</sub>) error as GOSAT to the XCH<sub>4</sub> (ACTM simulated) to make the OSSE more realistic and then  
265 attempt to estimate the true fluxes.

266 In this case, the CH<sub>4</sub> flux has been estimated for each grid by choosing the CH<sub>4</sub> observation with  
267 cutoff radius ( $\approx 3.65 \sigma_{h,v}$ ) with horizontal covariance localization ( $\sigma_h$ ) of 5000 km and vertical  
268 covariance localization ( $\sigma_v$ ) of 0.35 in the natural logarithmic pressure (log[Pa]) coordinate. The  
269 optimal horizontal and vertical covariance localization values are chosen based on trial and error  
270 method (those best fits to estimate CH<sub>4</sub> flux when compared with truth). A long cutoff radius has been  
271 chosen due to sparse observational coverage of GOSAT. Covariance localization is necessary to  
272 remove long-range erroneous correlations and for mitigating sampling errors in the ensemble-based  
273 error covariance with a limited ensemble size (Miyoshi et al., 2007; Greybush et al., 2011; Kotsuki et  
274 al., 2020). The surface flux is analyzed at every model grid point with 8-days assimilation window  
275 and 100 ensemble members those are chosen based on sensitivity experiments discussed in Section  
276 4.2.

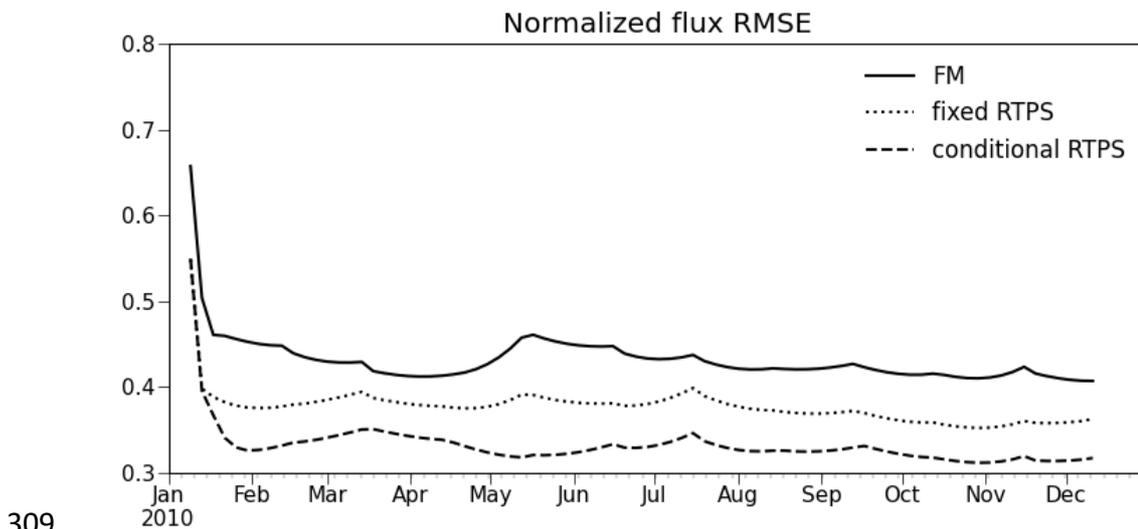
## 277 4. Results and Discussion

### 278 4.1 Experiment with dense OSSE

279 The time series of normalized RMSE ( $\sqrt{\sum_{i=1}^n (x_i^a - x_i^t)^2 / n / \bar{x}^t}$ ;  $x_i^a$  and  $x_i^t$  is the analysis and true  
280 state at  $i$ th model grid point,  $n$  is the total number of grid points, and  $\bar{x}^t$  represents the mean of true  
281 flux) in the analyses over global landmass region is shown in Figure 2. The normalized global RMSE  
282 is calculated using FM and RTPS inflation methods (Fig. 2) after assimilating synthetic observation at  
283 every grid (Section 3.4). Noteworthy is that the experiment with FM inflation method shows 10-15%  
284 larger error in estimating the atmospheric surface CH<sub>4</sub> flux compared to RTPS inflation method. One  
285 of the reasons of better RMSE using RTPS inflation method is due to the more degrees of freedom  
286 provided by relaxation ( $\alpha_{\text{RTPS}}$ ) in ensemble spread (Eq. 8) that could nudge the ensemble of CH<sub>4</sub>  
287 concentrations towards observations. The initial flux analysis spread using RTPS and FM is shown in  
288 supporting information (Fig. S1) which shows larger initial analysis flux spread over Brazil, tropical  
289 America, and Asia in RTPS inflation compared to FM inflation method. We performed numerous  
290 sensitivity test with RTPS inflation method and found that uniform relaxation is not substantial, for  
291 some of the regions. Figure 2 shows the RMSE for FM, fixed RTPS ( $\alpha_{\text{RTPS}} = 0.4$ , applied globally,  
292 the optimized value is obtained by manual fine tuning) and conditional RTPS ( $\alpha_{\text{RTPS}} = 0.3-0.7$   
293 applied different  $\alpha_{\text{RTPS}}$  regionally by manual fine tuning). In case of conditional RTPS, the optimal  
294 values of  $\alpha_{\text{RTPS}}$ , i.e., 0.6, 0.3, and 0.7 for the regions south of 20°S, 20°S-20°N, and north of 20°N,  
295 respectively, were obtained from data assimilation sensitivity calculations with varying  $\alpha_{\text{RTPS}}$  for the  
296 three regions separately to best match the true states. We find that the conditional RTPS method

297 improves the accuracy by ~5% compared to fixed RTPS and 10-15% compared to FM. In the  
298 following, we discuss the results obtained using the conditional RTPS and FM inflation methods.

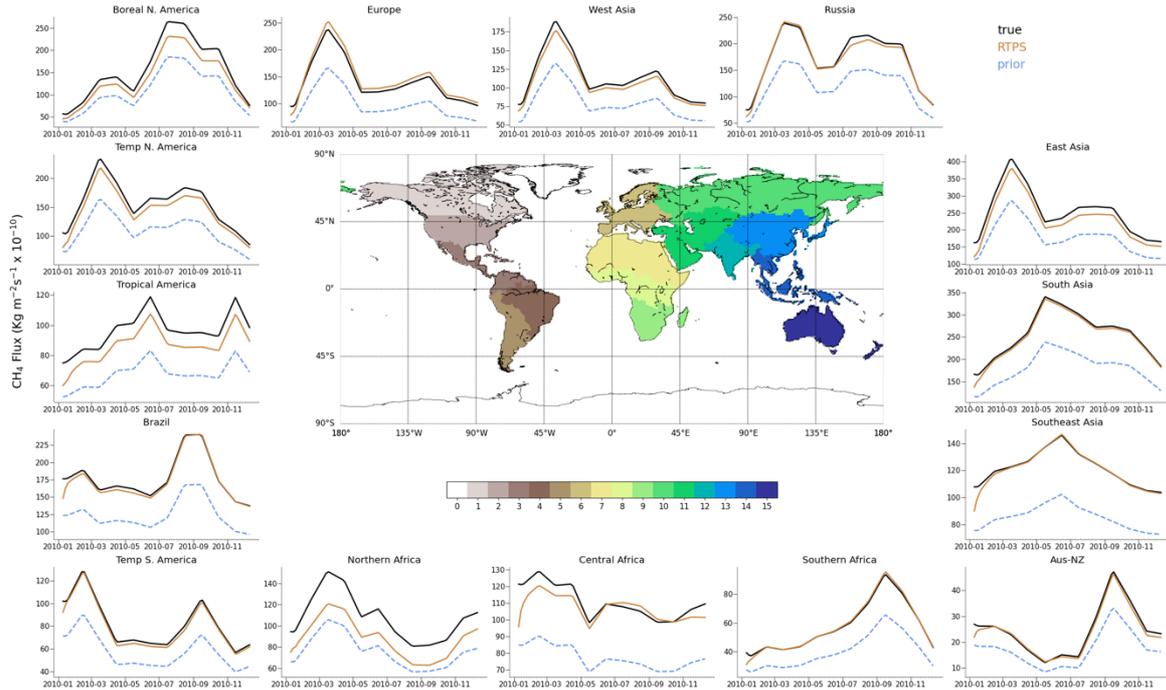
299 We have also shown RMSE (not normalized) of surface flux in supplementary information (Fig. S2).  
300 Flux RMSE has been estimated globally for both the inflation methods, and also for south of 20°N (by  
301 considering only those land grids which fall into south of 20°N; Fig. S2) for comparative purposes. It  
302 could be noticed that (supporting information Fig. S2), above north of 20°N, the flux estimation error  
303 is higher, specifically during spring-summer when CH<sub>4</sub> emissions peak over most of the northern  
304 hemispheric regions (Fig. 3). The high uncertainty during spring-summer (Fig. S2) in the flux  
305 estimation over these regions could appear due to the attenuation of surface observations as a result of  
306 active vertical mixing. The RMSE during autumn (Fig. S2) is comparable in case of global and south  
307 of 20°N, which indicates RMSE arising from southern hemispheric regions, likely over Brazil as it  
308 peaks during autumn (Fig. 3).



309

310 **Figure 2.** Time series of normalized RMSE of surface CH<sub>4</sub> flux analysis, for 1 year of data  
311 assimilation using FM, fixed RTPS, and conditional RTPS inflation methods over global landmass  
312 region.

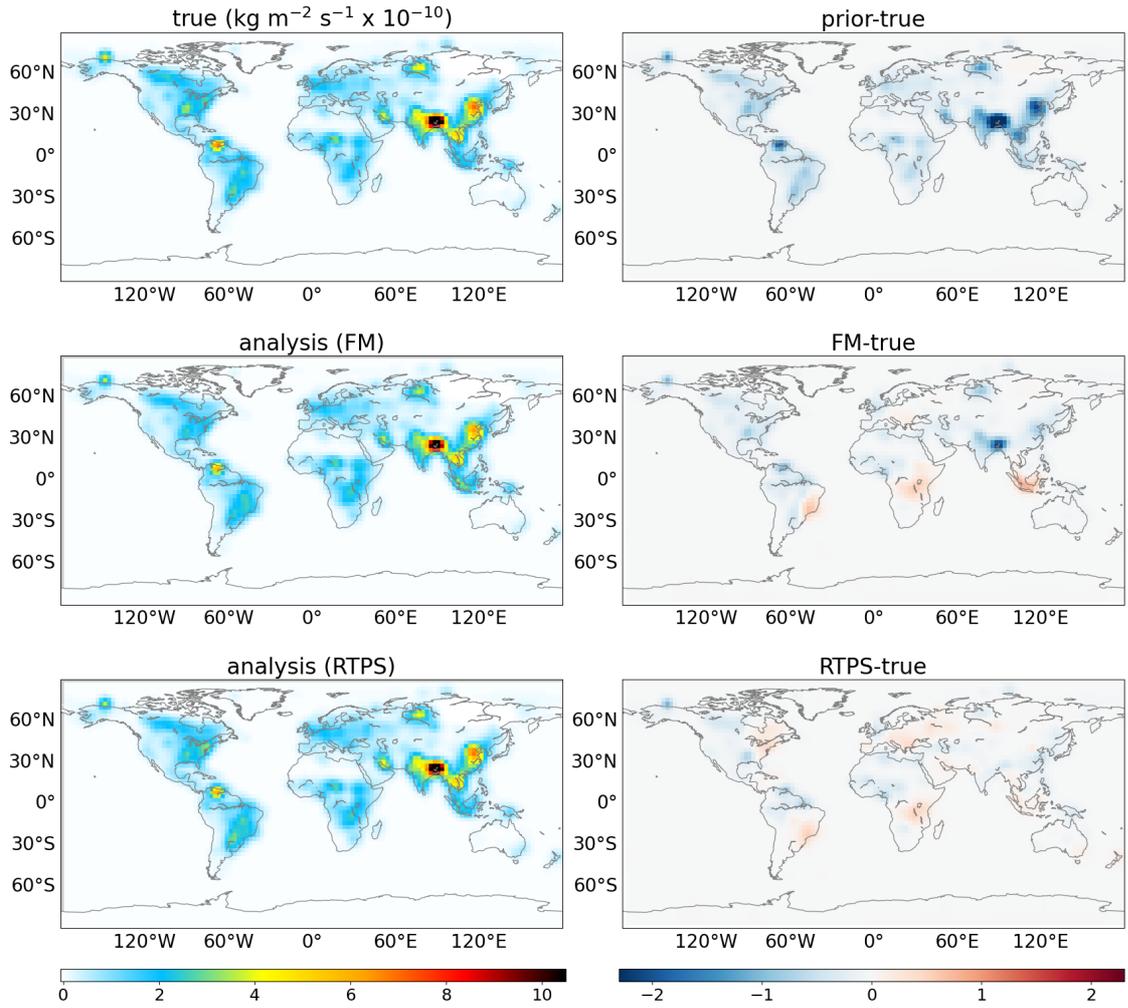
313 Figure 3 shows regional total flux seasonal cycles comparison of the estimated fluxes for 15 terrestrial  
314 regions with those of the prior and true fluxes. The estimated flux retrieved using RTPS inflation  
315 method over different regions agrees well with that of true flux. We intend to show the capability of  
316 LETKF estimated fluxes over these regions using surface observations to mimic the true fluxes in our  
317 understanding of terrestrial biosphere CH<sub>4</sub> cycle. These results are consistent with Figure 2 with  
318 annual global normalized mean bias ( $\sum_{i=1}^n(x_i^a - x_i^t)/\sum_{i=1}^n(x_i^t)$ ) of -0.04. It could also be noticed  
319 from Figure 3 that estimated fluxes converge to true fluxes over most of the regions after about 2-3  
320 months.



321

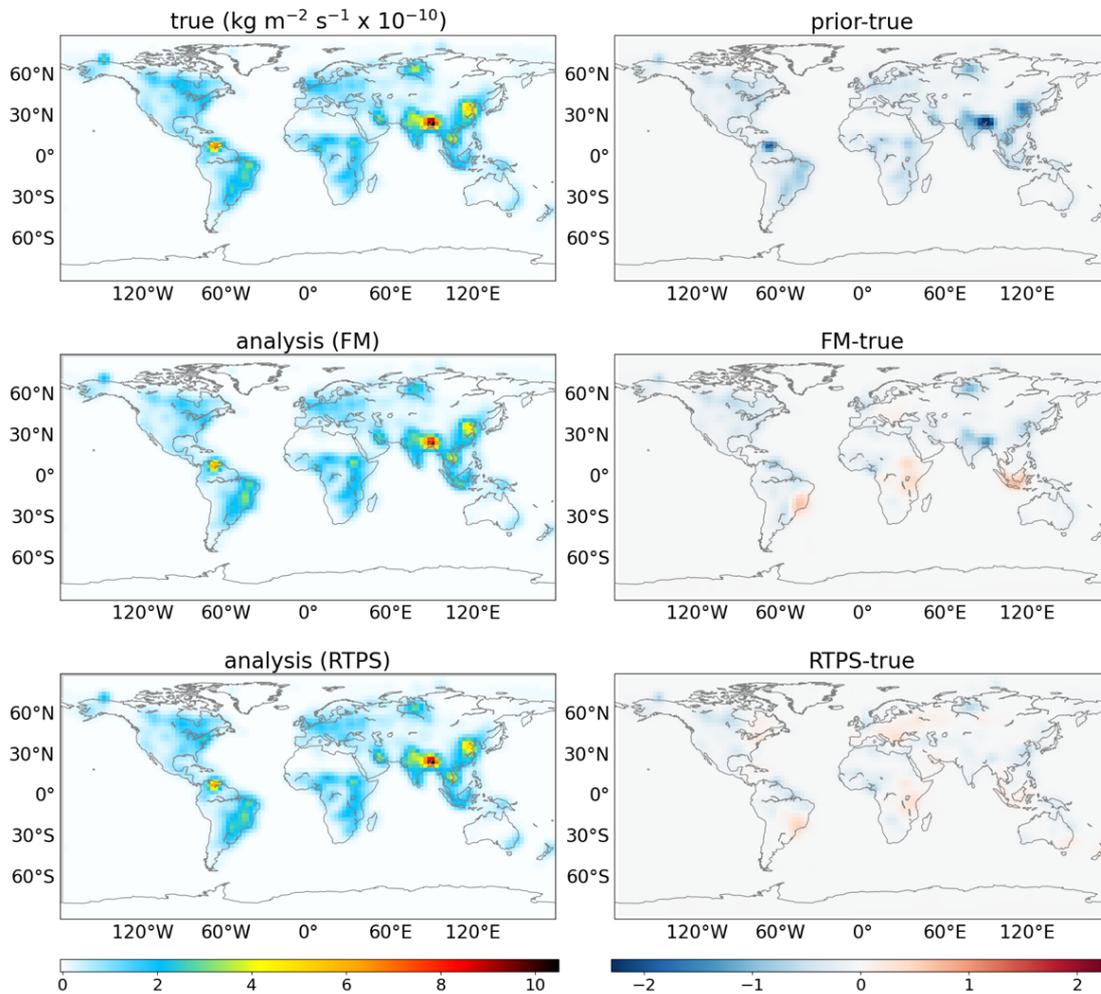
322 **Figure 3.** The 1-year CH<sub>4</sub> total flux seasonal cycles of true (black), prior (blue), and estimated from  
 323 the LETKF (orange) conditional RTPS inflation method in 15 regions after assimilating dense  
 324 synthetic surface CH<sub>4</sub> observations.

325 To see the degree of similarity in the flux distribution between the estimated and true fluxes, we show  
 326 monthly mean spatial flux distribution for June, and November in Figure 4 and 5, respectively, along  
 327 with the bias in prior and estimated flux. As shown in Figures 4 and 5, the general spatial patterns of  
 328 the true flux are estimated well. These results suggest that, our LETKF system is capable of  
 329 reproducing continental spatial flux patterns by using such an idealized dense surface observational  
 330 data. However, some clear differences in flux estimation could be noticed from FM and RTPS  
 331 inflation method (Figs. 4 and 5), for e.g., over Eurasian and American continent, analysis with RTPS  
 332 shows clear improvement compared to FM covariance inflation method. We calculated the global  
 333 mean normalized bias with RTPS and FM covariance inflation method which is found to be -0.04 and  
 334 -0.11, respectively over land regions that shows RTPS significantly improved the flux estimation  
 335 compared to FM covariance inflation method.



336

337 **Figure 4.** Spatial distribution of surface CH<sub>4</sub> fluxes (true; top left panel, FM analysis; middle left  
 338 panel, RTPS analysis; bottom left panel) and the associated bias in prior (prior-true; top right panel)  
 339 and estimated (FM-true; middle right panel, RTPS-true; bottom right panel) fluxes during June, 2010.



340

341 **Figure 5.** Same as Figure 4 but for November, 2010.

## 342 4.2 Experiment by mimicking the real satellite observational data set

343 In this section we discuss the LETKF flux estimation by assimilation of GOSAT synthetic CH<sub>4</sub>  
 344 concentration observations. Figure 6 shows the model simulated mean XCH<sub>4</sub> concentration sampled  
 345 spatiotemporally with GOSAT observations during January and July for the year 2010 (sampling  
 346 method discussed in Section 3.4). In this case we have shown different LETKF sensitivity  
 347 experiments such as; LETKF sensitivity to (1) FM, RTPS, adaptive multiplicative inflation (2)  
 348 assimilation window (3) ensemble size, (4) chi-square test, (5) prior ensemble spread. In the LETKF  
 349 sensitivity experiments from 1-4, the initial ensemble spread provided similar way as Experiment 1  
 350 and conditional RTPS inflation method is used. Conditional RTPS method is also used in Section  
 351 4.2.6 for CH<sub>4</sub> flux estimation.

### 352 4.2.1 LETKF sensitivity to FM, RTPS, and adaptive multiplicative inflation

353 This study mainly emphasizes on FM and RTPS inflation methods used in CH<sub>4</sub> LETKF data  
354 assimilation. The annual average normalized RMSE (absolute bias) with RTPS and FM covariance  
355 inflation is found to be 0.59 (0.18) and 0.64 (0.22), respectively. The RTPS inflation method performs  
356 better than the FM inflation method overall. In addition to RTPS inflation, sensitivity test is also  
357 performed using adaptive multiplicative inflation methods.

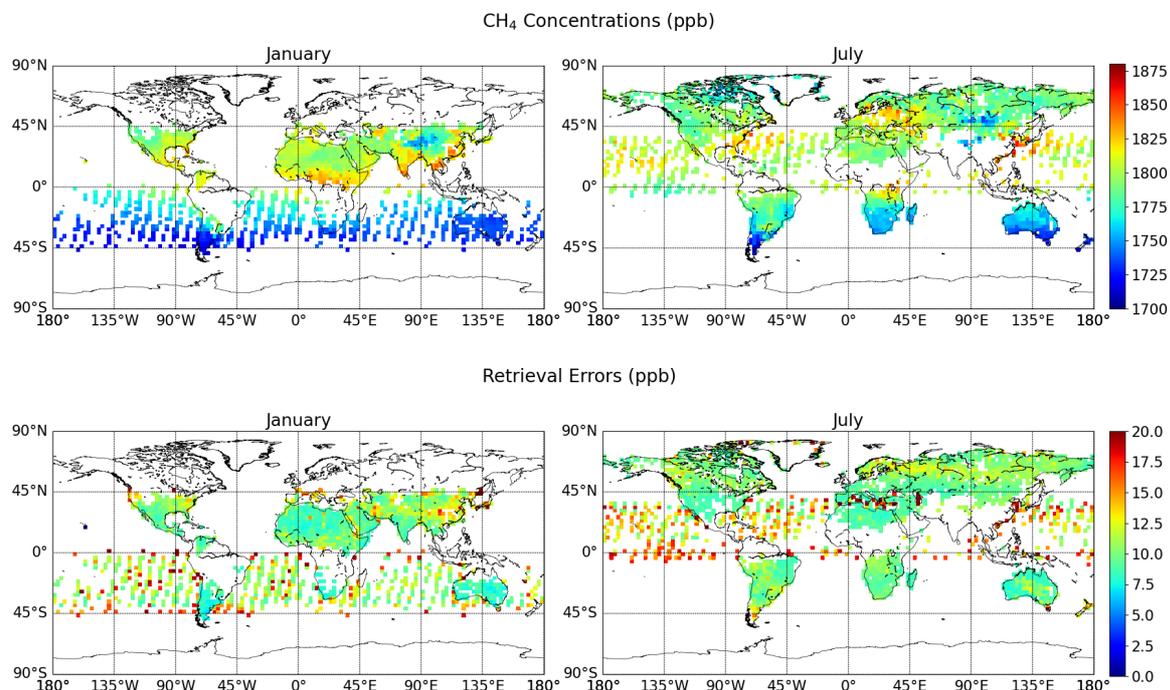
358 In the adaptive inflation, we need to provide an initial multiplicative inflation factor at the beginning  
359 of data assimilation cycle (Cycle 1 in Fig. 1). Following the method of Miyoshi (2011), the  
360 multiplication inflation factor information calculated in previous cycle (i.e. Cycle1 in Fig. 1) is used  
361 for next data assimilation cycle at every grid point (Cycle 2 in Fig. 1). We perform two sensitivity  
362 experiments. In the first (second) case we provided 50% (40%) initial inflation in the beginning of  
363 Cycle 1 (Fig. 1). The normalized RMSE in the both the adaptive inflation sensitivity experiments are  
364 comparable (0.65, Supporting information Fig. S3a) till July, but from the beginning of August,  
365 RMSE increases exponentially in the first experiment. However, in terms of chi-square distribution  
366 CH<sub>4</sub> flux estimation with first sensitivity adaptive multiplicative inflation experiment (50% initial  
367 inflation case) is better than second sensitivity experiment (Supporting information Fig. S3b; chi-  
368 square test described in Section 4.2.4). To identify the regions of high estimated CH<sub>4</sub> flux error, we  
369 have shown the background error spread in CH<sub>4</sub> flux estimation over 15 regions (Supporting  
370 information Fig. S3c) and found that spread over west and south east Asia rises exponentially post  
371 July that indicates the rise of estimated CH<sub>4</sub> flux error over these regions in the first sensitivity  
372 adaptive multiplicative inflation experiment. Our analysis suggests that CH<sub>4</sub> flux estimation is  
373 depending on the initial inflation factor provided in the beginning of data assimilation cycle (Cycle 1,  
374 Fig. 1) in adaptive multiplication method. Also, we need to be very careful to monitor the background  
375 error spread evolution with time to estimate the CH<sub>4</sub> flux with adaptive inflation, chi-square  
376 distribution analysis is not sufficient.

377 In case of RTPP inflation, we found the parameter  $\alpha_{RTPP}$  is very difficult to fine-tuned due to its very  
378 high sensitivity to estimate the CH<sub>4</sub> flux. We fail to obtain an optimized  $\alpha_{RTPP}$  value to estimate the  
379 CH<sub>4</sub> flux. Whitaker and Hamill (2012), also demonstrated the better accuracy in LETKF  
380 meteorological data assimilation with RTPS compared to RTPP covariance inflation method. They  
381 found RTPP method produces very large errors if the inflation parameter exceeds the optimal value.

#### 382 **4.2.2 Assimilation window**

383 The LETKF data assimilation window length determines the time span of the observations assimilated  
384 in each assimilation cycle. We have shown the sensitivity of two assimilation window size  
385 configurations; 3 days and 8 days in supporting information Figure S4. Our sensitivity experiments  
386 with window size configurations show that 8 days long assimilation window estimates the CH<sub>4</sub> flux

387 with better accuracy (~10%) compared to 3 days assimilation window, because more observational  
388 information is incorporated into the system with 8 days long assimilation window. This study uses 8  
389 days assimilation window for CH<sub>4</sub> LETKF data assimilation.

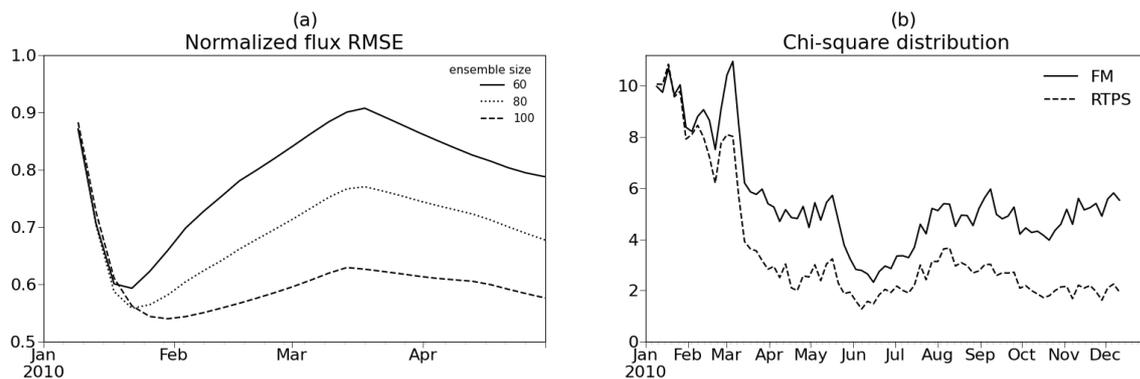


390

391 **Figure 6.** Monthly mean ACTM simulated XCH<sub>4</sub> (ppb) sampled with GOSAT observations to be  
392 assimilated (valid during the year 2010). The actual retrieval errors are added in the synthetic GOSAT  
393 observations. Data are shown for two representative months, depicting the southern and northern  
394 hemisphere differences in data coverage.

#### 395 4.2.3 Ensemble size

396 Figure 7a shows the RMSE using different ensemble members. The RMSE stabilizes gradually as the  
397 ensemble size increases from 60 to 80 to 100 ensemble members. The ensemble size dependency of  
398 flux estimation suggests the further scope of the improvement in flux estimation by increasing the  
399 ensemble members. In this study we stick to 100 ensemble members due to high computational cost  
400 while solving large covariance matrices. The larger error in flux estimation in case of column  
401 averaged synthetic GOSAT CH<sub>4</sub> observations assimilation compared to dense observations (Fig. 2) is  
402 likely due to the weaker constraint on surface fluxes provided by satellite observations and sparse  
403 observations.



404

405 **Figure 7:** (a) Flux estimation RMSE using different ensemble size with RTPS covariance inflation.

406 (b) Chi-square distribution using FM and RTPS covariance inflation methods with the ensemble size

407 of 100.

#### 408 4.2.4 Chi-square test

409 We have carried out chi-square test for the evaluation of background error covariance matrix

410 (Miyazaki et al., 2012). For the  $\chi^2$  test, the innovation statistics are diagnosed from the observation

411 minus forecast ( $y^o - Hx^b$ ), the estimated error covariance in the observation space ( $HP^bH^T + R$ ),

412 and the number of observations  $k$ , such as:

$$Y = \frac{1}{\sqrt{k}} (HP^bH^T + R)^{-1/2} (y^o - Hx^b) \quad (11)$$

413 Using this statistic, the  $\chi^2$  is defined as follow:

$$\chi^2 = \text{trace}YY^T \quad (12)$$

414 The performance of background error covariance matrix determined based on the high and lower

415 value of chi-square. Chi-square value should converge to 1, a value higher (lower) than 1 indicates

416 underestimation (overestimation) of the background error covariance matrices. Our results suggest

417 that, background error covariance matrix is highly underestimated in both RTPS and FM covariance

418 inflation methods (Fig. 7b). However, the chi-square values convergence towards 1 is better in the

419 case of RTPS compared to FM covariance inflation method which indicates the improved

420 representation of background errors and then more appropriate data assimilation corrections in the

421 case of the RTPS inflation method. The chi-square distribution starts saturating after the month of

422 March. Post March analysis shows the background error covariance matrix underestimation is much

423 higher (>100%) in case of FM compared to RTPS covariance inflation method.

#### 424 4.2.5 CH<sub>4</sub> LETKF sensitivity to initial ensemble spread

425 A test case for CH<sub>4</sub> LETKF data assimilation has been performed where the initial spread is provided  
426 by considering the initial perturbation on each model grid with spatial error correlation between grid  
427 points among ensemble members, with global mean correlation of 20%. In this case, we found that  
428 the analysis fluxes are extremely sensitive to the initial ensemble spread if prior fluxes perturbed with  
429 more than 5% prior uncertainty. Therefore, we used initial ensemble perturbation with only 2% prior  
430 uncertainty. Reducing the initial ensemble spread reduces the CH<sub>4</sub> flux estimation sensitivity (>60%).  
431 However, it also poses a challenge to mitigate the under-dispersive background error covariance  
432 matrix. We performed LETKF data assimilations in this case with RTPS covariance inflation method  
433 ( $\alpha_{\text{RTPS}} = 0.9$  optimized value is used here uniformly) with 8-days long assimilation window and 100  
434 ensemble members and calculated the normalized RMSE between analysis and true fluxes  
435 (Supporting information Fig. S5). Noteworthy that, the estimated error between analysis and true  
436 fluxes (Fig. S5) with this setting (grid-wise initial ensemble spread) is still larger (25%) than the case  
437 when region-wise initial ensemble spread provided (Fig. 7a; 100 ensemble size). It suggests that,  
438 initial ensemble spreads among ensemble members needs to be carefully provided that best represents  
439 CH<sub>4</sub> variability among ensembles to estimate the CH<sub>4</sub> flux.

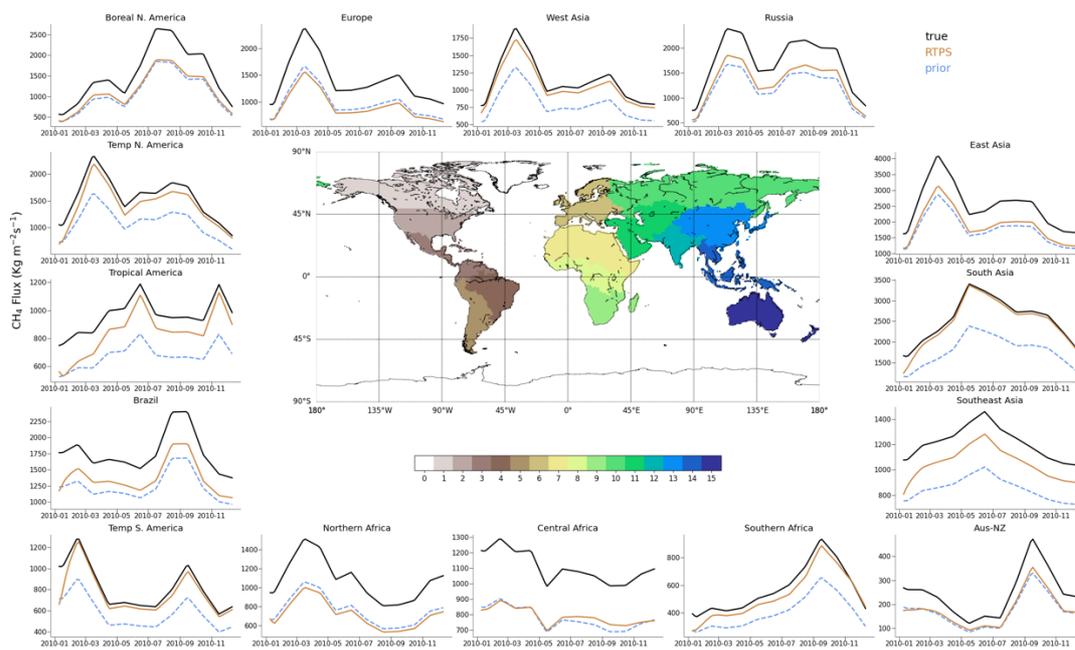
440 Note that, the OSSEs used in this study did not consider the effects of model errors other than CH<sub>4</sub>  
441 fluxes, such as model transport errors. In real situations, model errors can have a substantial impact on  
442 flux estimates (Locatelli et al., 2013), which needs to be taken into account in background  
443 covariances. Therefore, the optimal data assimilation setting can differ between the OSSEs presented  
444 in this study and real observation cases. Further efforts, e.g., by conducting a more comprehensive  
445 OSSE that accounts for various model errors and by performing various sensitivity calculations in real  
446 cases, would provide an improved understanding of the optimal inflation settings to improve CH<sub>4</sub> flux  
447 estimates in following study.

#### 448 **4.2.6 Estimated CH<sub>4</sub> flux analysis**

449 Figure 8 shows the regional fluxes seasonal cycle comparison for the estimated fluxes over 15  
450 terrestrial regions with those of the prior and true fluxes. We have also shown assimilation results in  
451 case of FM inflation method in supporting information (Fig. S6), which shows the flux estimation  
452 disagreement over more regions compared to RTPS inflation method; e.g., for Tropical and North  
453 America, whole African continent, Australia-New Zealand.

454 We have shown the GOSAT observations in Figure 6 and supporting information Figure S7. We  
455 found very marginal flux estimation improvement over Central Africa after May (Fig. 8), that could  
456 be associated with the less GOSAT coverage over this region (Fig. 6). On the other hand, over  
457 Northern Africa, no improvement in flux estimation is found. In case of dense OSSE too (Fig. 3), we  
458 didn't find satisfactory flux estimation over Northern Africa which is most probably related to the

459 insufficient initial spread among ensemble members over this region (we have used same initial  
 460 ensemble spread in both OSSE cases). Over Europe, GOSAT observations are remarkably less,  
 461 specifically for first few months (January-April; supporting information Fig. S7). Therefore, the flux  
 462 update over Europe would be influenced by the observations from neighboring regions falling under  
 463 the chosen cutoff radius that are mainly in Northern Africa where the flux estimation itself not  
 464 satisfactory. It could also be noticed that the retrieval error added in this OSSE case are high over  
 465 Europe (September-October; supporting information Fig. S7,) and its adjacent Sea (Mediterranean  
 466 Sea; June-August) which could also affect the surface CH<sub>4</sub> flux estimation.



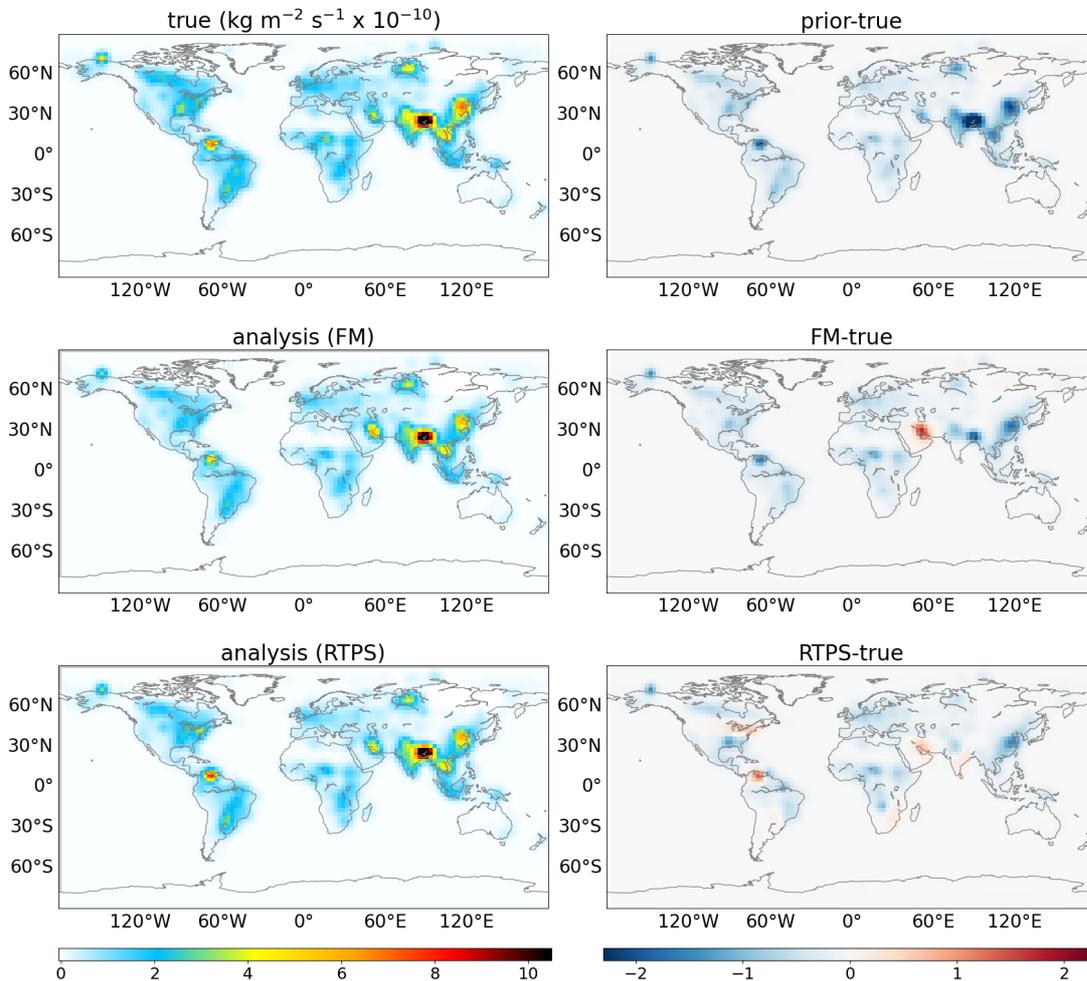
467

468 **Figure 8.** Same as Figure3 but after assimilating synthetic GOSAT observations.

469 Figure 9 and 10 show spatial patterns of the true and estimated fluxes by assimilating the column  
 470 averaged CH<sub>4</sub> concentrations during June and November (Fig. 6). It may be noticed that RTPS  
 471 covariance inflation method better able to estimate the true flux pattern compared to FM covariance  
 472 inflation method. The spatial pattern shown using RTPS inflation method emphasizes the positive and  
 473 negative bias in the estimated flux (Figs. 9 and 10), but generally agrees with the flux seasonal cycle  
 474 plots shown in Figure 8.

475 Our LETKF CH<sub>4</sub> data assimilation experiment by assimilating GOSAT synthetic observation with the  
 476 implementation of the advanced RTPS covariance inflation method better estimate the time-evolving  
 477 surface CH<sub>4</sub> fluxes compared to FM covariance inflation method. The difficulty to estimate the  
 478 surface CH<sub>4</sub> flux over a few regions may be overcome by applying additional methodologies, such as  
 479 the assimilation of surface observations simultaneously, and the use of information about the CH<sub>4</sub>  
 480 fluxes climatology. A correction factor derived based on empirical formulation that could use CH<sub>4</sub>

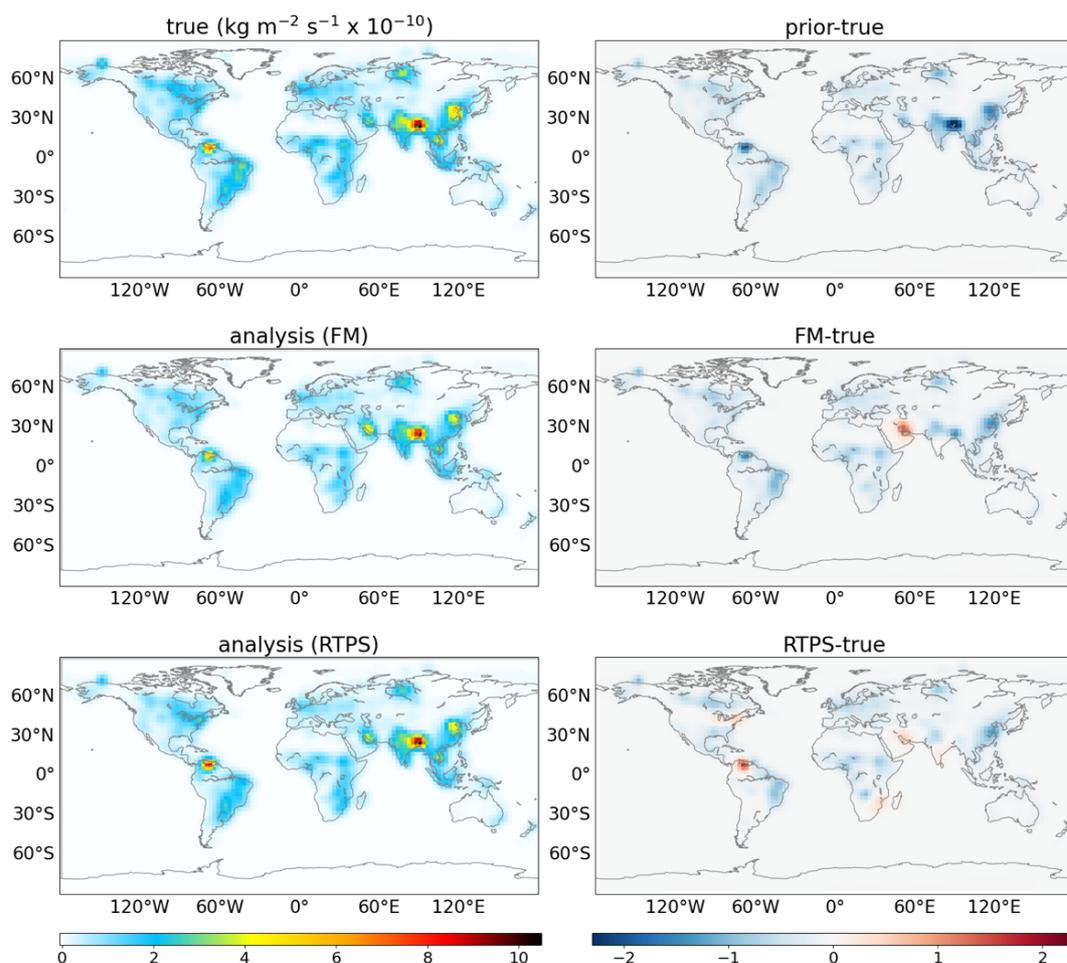
481 flux climatology information is needed to apply to maintain the CH<sub>4</sub> mass conservation. This could be  
482 implemented by the checking the simulated CH<sub>4</sub> burden gain between years in comparison with the  
483 observed CH<sub>4</sub> growth rates.



484

485 **Figure 9.** Monthly mean true (true; top left panel) and estimated (FM analysis; middle left panel,  
486 RTPS analysis; bottom left panel) CH<sub>4</sub> flux after assimilating column averaged synthetic CH<sub>4</sub>  
487 concentrations (Fig. 6) during June using FM and RTPS inflation methods. The associated bias with  
488 prior and estimated fluxes is also shown (prior-true; top right panel; FM-true; middle right panel,  
489 RTPS-true; bottom right panel).

490



491

492 **Figure 10.** Same as Figure 9 but for November.

493 **5. Summary**

494 In this study, we have introduced 4D-LETKF data assimilation system that utilizes MIROC4-ACTM  
 495 as a forward model for CH<sub>4</sub> flux estimation. This study has extensively tested both FM and RTPS  
 496 inflation methods for the LETKF CH<sub>4</sub> flux estimation. We have conducted two experiments to  
 497 demonstrate the ability of LETKF system to estimate the CH<sub>4</sub> surface flux globally. In Experiment1,  
 498 we have assimilated the synthetic dense surface CH<sub>4</sub> observations. While in Experiment2, synthetic  
 499 GOSAT CH<sub>4</sub> observations are assimilated. Based on the results of the sensitivity tests using FM and  
 500 RTPS inflation methods in Experiment1, we have found that RTPS inflation produces significantly  
 501 less normalized RMSE (10-15%) compared to FM inflation method. In Experiment2, we discussed,  
 502 LETKF parameters such as, different inflation techniques, ensemble size, assimilation window, initial  
 503 ensemble spread sensitivity, and chi-square test. The ensemble size (this study uses maximum 100  
 504 ensemble members) sensitivity test suggests that more ensemble members could help to accurately  
 505 represent the covariance matrix with more degrees of freedom. The assimilation window sensitivity

506 test exhibits that 8 days assimilation window reduces the normalized flux RMSE by about 10%  
507 compared to 3 days assimilation window in case of GOSAT synthetic observations assimilation.

508 Our approach of assimilation with RTPS inflation could provide more degrees of freedom to fit the  
509 ensemble of CH<sub>4</sub> concentrations to the observed ones, resulting the improved analyzed fluxes. The  
510 RTPS inflation method is capable of obtaining reasonable flux estimates with normalized annual  
511 mean bias of 0.04, and 0.61 in case of dense surface synthetic observations and GOSAT synthetic  
512 observations, respectively. We demonstrated in our sensitivity OSSE experiment with synthetic  
513 GOSAT observations that, over American and African continents and also over Australia - New  
514 Zealand, the LETKF data assimilation with FM inflation method does not show much improvement in  
515 the true flux estimation, but RTPS inflation method reasonably estimate the true flux over most of  
516 these regions. One of the reasons for better flux estimates from RTPS inflation method is the  
517 prevention of analysis spread drastically. In the CH<sub>4</sub> LETKF flux estimation, surface CH<sub>4</sub> flux is not a  
518 prognostic state vector in the ACTM, which results in the decay of spread continuously in analysis  
519 steps. RTPS inflation method could mitigate such under disperse spread problem. This study finds  
520 that spatially homogeneous relaxation is not sufficient. It needs to be fine-tuned and applied  
521 conditionally.

522 The sensitivity of LETKF CH<sub>4</sub> flux estimation to initial ensemble spread needed to be carefully dealt  
523 with when applied to real data assimilation system. A future OSSE with additive covariance inflation  
524 technique could be interesting while applied with RTPS inflation method for CH<sub>4</sub> LETKF data  
525 assimilation since in additive covariance inflation initial estimated flux error cannot propagate. The  
526 state vector augmentation technique used here updates the flux after each data assimilation cycle but it  
527 doesn't conserve the total atmospheric CH<sub>4</sub> amount which is one of the limitations of this work. A  
528 correction factor needs to be implemented to conserve the total atmospheric CH<sub>4</sub> amount after  
529 completion of a few data assimilation cycles. We have not accounted for the transport error due to  
530 meteorological fields in this work (Patra et al., 2011b), in case of real observations data assimilation a  
531 week-long window may introduce transport errors in CH<sub>4</sub> analysis because of nonlinear growth of  
532 ensemble perturbations.

533 *Code and data availability.* The LETKF source codes can be accessed from  
534 <https://doi.org/10.5281/zenodo.7127658>. All the scripts for running the LETKF data assimilation  
535 software, input and output results data files are available at <https://doi.org/10.5281/zenodo.7098323>.  
536 CH<sub>4</sub> ACTM simulation module coupled with MIROC4-AGCM can be accessed from  
537 <https://doi.org/10.5281/zenodo.7118365>. The source code of MIROC4-AGCM is archived at  
538 <https://doi.org/10.5281/zenodo.7274240> with restriction because of the copyright policy of the  
539 MIROC developer community, and no contribution of this work to the MIROC4 source code  
540 development.

541 *Author contributions.* The LETKF data assimilation experiments were designed by JSHB. PKP, MT  
542 and TS help to set the LETKF code on MIROC4-ACTM for CH<sub>4</sub> data assimilation. The manuscript is  
543 prepared by JSHB and analysis interpretation input and feedback are provided by PKP, TS, KM. All  
544 coauthors, KM, TS, PKP, NS, MT and YK contributed to writing and revision of the paper.

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548 LETKF code on MIROC4-ACTM for CH<sub>4</sub>.

549

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