Methane (CH$_4$) is the second major greenhouse gas after carbon dioxide (CO$_2$) which has substantially increased during last decades in the atmosphere, raising serious sustainability and climate change issues. Here, we develop a data assimilation system for in situ and column averaged concentrations using Local ensemble transform Kalman filter (LETKF) to estimate surface emissions of CH$_4$. The data assimilation performance is tested and optimized based on idealized settings using Observation System Simulation Experiments (OSSEs) where a known surface emission distribution (the truth) is retrieved from synthetic observations. We tested three covariance inflation methods to avoid covariance underestimation in the emission estimates, namely; fixed multiplicative (FM), relaxation to prior spread (RTPS) and adaptive multiplicative. First, we assimilate the synthetic observations at every grid point at the surface level. In such a case of dense observational data, the normalized Root Mean Square Error (RMSE) in the analyses over global land regions are smaller by 10-15% in case of RTPS covariance inflation method compared to FM. We have shown that integrated estimated flux seasonal cycles over 15 regions using RTPS inflation are in reasonable agreement between true and estimated flux with 0.04 global normalized annual mean bias. We have then assimilated the column averaged CH$_4$ concentration by sampling the model simulations at GOSAT observation locations and time for another OSSE experiment. Similar to the case of dense observational data, RTPS covariance inflation method performs better than FM for GOSAT synthetic observation in terms of normalized RMSE (2-3%) and integrated flux estimation comparison with the true flux. The annual mean averaged normalized RMSE (normalized mean bias) in LETKF CH$_4$ flux estimation in case of RTPS and FM covariance inflation is found to be 0.59 (0.18) and 0.61 (0.23) respectively. The chi-square test performed for GOSAT synthetic observations assimilation suggests high underestimation of background error covariance in both RTPS and FM covariance inflation.
methods, however, the underestimation is much higher (>100% always) for FM compared to RTPS covariance inflation method.

1. Introduction

Methane (CH₄) is the second major greenhouse gas, after carbon dioxide (CO₂), that has anthropogenic sources. According to the contemporary record of the global CH₄ budget, the total of all CH₄ sources ranges 538–593 Tg yr⁻¹ during 2008–2017 (Saunois et al., 2020). The primary natural sources are from wetlands (~40%). The main anthropogenic CH₄ emissions are from microbial emissions associated with ruminant (livestock and waste), rice cultivation, fugitive emissions (oil and gas production and use), and incomplete combustion of bio and fossil fuels. The major fraction of atmospheric CH₄ sinks (range: 474 - 532 Tg yr⁻¹) occurs in the troposphere by oxidation via reaction with hydroxyl (OH) radicals (Patra et al., 2011; Saunois et al., 2020); other loss processes include oxidation by soil, and reactions with O¹D and Cl. The lifetime of CH₄ in the atmosphere is estimated to be 9.1 ± 0.9 years (Szopa et al. 2021).

Regional CH₄ emissions can be estimated from CH₄ concentration fields and chemistry transport models using Bayesian synthesis approaches based on inverse modeling techniques (e.g., Enting, 2002). In such approach, emissions are optimized on a coarse resolution (e.g., for a limited number of pre-defined regions) mostly using surface-based observations. CH₄ concentrations are provided by the NOAA cooperative air sampling network sites (Dlugokencky et al., 2020) and other networks by the World Data Centre for Greenhouse Gases (WDCGG) website, hosted by the Japan Meteorological Agency. In the recent years, satellite measurements are made from the Greenhouse Gases Observing Satellite (GOSAT) or the TROPOspheric Monitoring Instrument (TROPOMI) (Lorente et al., 2021), covering the globe with fine spatio-temporal scales. GOSAT provide an extensive global observations of column CH₄ concentrations since 2009 (Yoshida et al., 2013). Some of the inverse modeling studies utilize the satellite observations for CH₄ flux estimation (Zhang et al., 2021; Maasakkers et al., 2016), but, it requires enormous computational resources while dealing with more flux regions and more observations.

Grid-based CH₄ flux optimization is also performed using adjoint technique (4-D Var data assimilation) and Ensemble Kalman Filter (EnKF), but was limited to small sets of observations (Houweling et al., 1999; Meirink et al., 2008; Bruhwiler et al., 2014). Bruhwiler et al. (2014) followed the EnKF method of Peters et al. (2005) to estimate the CH₄ surface fluxes that utilizes an off-line ACTM framework. Techniques such as 4-D Var and EnKF are important to estimate CH₄ fluxes since they can assimilate a large number of observations, manage high-resolution fluxes. In the EnKF system, a flow-dependent forecast error covariance structure is provided by ensemble model forecasts, while it does not need an adjoint model that makes it simple but powerful tool for flux estimation.
One of the limitations in EnKF method is the dependence of the resolution of state vector on ensemble size, which can give spurious results if the number of ensemble members is much smaller than the rank of the error covariance matrix (Houtekamer and Zhang, 2016).

LETKF is a type of square-root EnKF that performs analysis locally in space without perturbing the observations (Ott et al., 2002, 2004; Hunt et al., 2007). LETKF is computationally efficient since the observations are assimilated simultaneously, not serially, it is simple to account for observation error correlation. Miyazaki et al. (2011) and Kang et al. (2012) demonstrated the implementation of LETKF data assimilation system by coupling an ACTM for carbon-cycle research using atmospheric CO$_2$ observations. It is also extensively applied for the emission estimation of short-lived species using satellite data (Skachko et al., 2016; Miyazaki et al., 2019; Sekiya et al., 2021). In this work, we will estimate the CH$_4$ fluxes using a LETKF data assimilation system. Assimilation windows ranging from 6 hours (Kang et al., 2012) to several months (Bruhwiler et al., 2014) have been used, depending on the desired time resolution of the estimated emissions, which is often limited by the observational data density. The time frame over which the system behaves linearly, and in what time frame the observations respond to the control variables such as, atmospheric transport, as well as observation abundance, must also be taken into consideration. Within an assimilation window, where and when the fluxes would be constrained by specific observations is to be ascertained by the correlation between ensemble prior fluxes and the ensemble CH$_4$ concentrations simulation from a forward model (Liu et al., 2016).

Main objective of this work is to develop an advanced 4-D data assimilation system based on LETKF that simultaneously estimates atmospheric distributions and surface fluxes of CH$_4$. OSSEs are conducted to assess the performance of LETKF since it is important to test the system against the known emissions or the truth. The OSSE LETKF set-up of top-down CH$_4$ flux estimation using online ACTM is an essential step before implementing on real in situ and satellite observation.

2. Formulation of LETKF System

We briefly describe the LETKF in the application of CH$_4$ flux estimation, while detailed derivation of equations and code implementation are given elsewhere (Hunt et al., 2007; Miyazaki et al., 2011; Miyoshi et al., 2010). The notation used here for LETKF formulation is adopted from Kotsuki et al. (2017). In the LETKF, the background ensemble (columns of matrix $x^b$) in a local region evolved from a set of perturbed initial conditions. The background ensemble mean, $\bar{x}^b$, and its perturbation, $X^b$, are estimated from the ensemble forecast such as:

$$\bar{x}^b = \frac{1}{m} \sum_{i=1}^{m} x^b_i; \quad X^b_i = x^b_i - \bar{x}^b$$ (1)
Where ‘m’ indicates the ensemble size. The background error covariance matrix $P^b$ in the m-dimensional ensemble is defined as:

$$P^b = \frac{1}{m-1} X^b [X^b]^T$$  \hspace{1cm} (2)

The analysis ensemble mean $\bar{x}^a$ is derived using background ensemble mean $\bar{x}^b$ and ensemble perturbations $X^b$ such as:

$$\bar{x}^a = \bar{x}^b + X^b \bar{P}^a (y^o)^T R^{-1} (y^o - H \bar{x}^b) = \bar{x}^b + X^b w^a$$  \hspace{1cm} (3)

where $H$, $Y$, $R$, and $\bar{P}^a$ denote the linear observation operator, ensemble perturbation matrix in the observation space ($Y = Hx$), observation error covariance matrix, and analysis error covariance matrix in the ensemble space, respectively. The superscripts ‘o’, ‘b’ and ‘a’ denote the observations, background (prior), and analysis (posterior), respectively. $w^a$ defines the analysis increment (or analysis weight) in observation space and is derived using the information about observational increment $y^o - H \bar{x}^b$. The analysis error covariance matrix ($\bar{P}^a$) in the m-dimensional ensemble space is spanned by ensemble perturbation (Hunt et al., 2007) and defined as:

$$\bar{P}^a = [(m-1)1 + (H X^b)^T R^{-1} H X^b]^{-1}$$  \hspace{1cm} (4)

Finally, the analysis ensemble perturbations $X^a$ at the central grid point are derived such as:

$$X^a = X^b \{(m - 1) \bar{P}^a\}^{1/2}$$  \hspace{1cm} (5)

Where, $\{(m - 1) \bar{P}^a\}^{1/2}$ is a multiple of the symmetric square root of the local analysis error covariance matrix in ensemble space and could be computed by singular vector decomposition method. The LETKF solves the analysis update equations 3 and 5 at every model grid point independently by assimilating local observations within the localization cut-off radius.

We have applied a gross error check as a quality control to exclude observations that are far from the first guess, the appropriate degrees of the gross error check are also examined. Figure 1 shows the schematic diagram of our LETKF set-up with two ensemble members for 3 consecutive assimilation cycles with 8 days assimilation window. The analysis is obtained at mid-point time of the assimilation window (Figure 1). The analyzed (updated) surface flux is used for next data assimilation cycle starting from the mid-point time of the previous data assimilation window. The state vector augmentation approach is used to estimate the atmospheric CH$_4$ surface flux (Kang et al., 2012; Miyazaki et al., 2011).
Assimilation window size and ensemble members are chosen based on computational efficiency and estimation accuracy. A larger assimilation window means fluxes are constrained by more observations, however, it requires handling of large matrix optimization which is difficult in cases of dense observation and introduces sampling errors related to transport errors. In this study, few sensitivity experiments performed to demonstrate the choice of assimilation window length and ensemble size when GOSAT synthetic observation are assimilated in Section 4.2.

Figure 1: Schematic represents the temporal evolution of LETKF cycle. In the first assimilation window (Cycle1), the dotted lines show the ensemble forecast of CH$_4$ concentrations (with 2 ensemble members), the solid line shows the linear combination of the forecasts, the filled circles show the observations of CH$_4$ concentration. The data assimilation finds the linear combination of the ensemble forecast by estimating the weight ($w^a$) that best fits the observations throughout the assimilation window. The analysis weight is applied to obtain optimal surface fluxes (F) and the concentration of CH$_4$ at the intermediate time of the data assimilation window. The updated analyzed concentration ensembles are used as initial condition after relaxation ($X^{a,RLX}$) (Eq. 8) for the next ensemble forecast. The spread of the ensemble members represents the forecast error. The schematic is adapted from Kalnay and Yang (2010) and Miyazaki et al. (2011).

2.1 Covariance inflation

The LETKF data assimilation needs variance inflation to mitigate the under dispersive ensemble. We tested three methods; fixed multiplicative (FM), relaxation-to-prior spread (RTPS), and adaptive multiplicative covariance inflation.
The fixed multiplicative (FM) inflation method (Anderson and Anderson, 1999) inflates the prior ensemble by inflating the background error covariance matrix $P^b$ defined in equation (Eq. 2) such as:

$$p_{\text{inf}}^b = \gamma P_{\text{tmp}}^b$$  \hspace{1cm} (6)

where $P_{\text{tmp}}^b$ represents the temporary background error covariance matrix which is inflated by a factor $\gamma$.

The other inflation methods used to prevent the reduction of ensemble spread are relaxation-to-prior perturbation (RTPP) (Zhang et al., 2004) and relaxation-to-prior spread (RTPS) (Whitaker and Hamill, 2012). The RTPP methods relax the reduction of the ensemble spread after updating the ensemble perturbations which blends the background and analysis ensemble perturbations as:

$$X_{\text{inf}}^a = \alpha_{\text{RTPP}} X^b + (1 - \alpha_{\text{RTPP}})X_{\text{tmp}}^a$$  \hspace{1cm} (7)

where $\alpha_{\text{RTPP}}$ denotes the relaxation parameter of the RTPP.

The RTPS inflation method relaxes the reduction of ensemble spread by relaxing the analysis spread to prior spread such as:

$$X_{\text{RLX}}^a = \left(\frac{\alpha_{\text{RTPS}} \sigma^b + (1 - \alpha_{\text{RTPS}}) \sigma^a}{\sigma^a}\right) X_{\text{tmp}}^a$$  \hspace{1cm} (8)

where $\sigma$ and $\alpha_{\text{RTPS}}$ denote the ensemble spread, and relaxation parameter of the RTPS, respectively. The range of parameter $\alpha_{\text{RTPS}}$ is bounded by $[0, 1]$. This study focuses mainly on the FM and RTPS covariance inflation methods.

In addition, Miyoshi (2011) applied adaptive inflation by determining the multiplicative inflation factors at every grid point at every analysis step using the observation-space statistics derived by Daley (1992) and Desroziers et al. (2005).

$$<dd^T> = H p_{\text{in}}^b H^T + R$$  \hspace{1cm} (9)

Where the operator ‘$<>$’ denotes the statistical expectation and $d = y^o - H \bar{x}^b$ (observation-minus-first-guess), and R is the error observation covariance matrix.

The impact of using the adaptive multiplication inflation method is discussed in the GOSAT synthetic observation assimilation experiments in Section 4.2.

2.2 MIROC4-ACTM

Model for Interdisciplinary Research on Climate, version 4.0 (MIROC4) based ACTM (hereafter referred to as MIROC4-ACTM) (Patra et al., 2018; Bisht et al., 2021) is used here for CH$_4$
concentration simulations. The model simulations have been performed at horizontal grid resolution of approximately $2.8 \times 2.8^\circ$ latitude-longitude grid (T42 spectral truncations) and hybrid vertical coordinate of 67 levels (Earth’s surface to 0.0128 hPa, Watanabe et al., 2008). Bisht et al., 2021 performed the multi-tracer analysis and demonstrated the importance of very well-resolved stratosphere in the MIROC4-ACTM that illustrates better extratropical stratospheric variabilities, and simulated tropospheric dynamical fields. The meteorological fields in MIROC4-ACTM are nudged to the JMA Re-analysis (JRA-55) data (Kobayashi et al., 2015).

3. Experimental set-up

3.1 Construction of known surface emissions (truth)

Present OSSEs intend to develop basic tuning strategies before the actual data to be assimilated which is useful to accelerate the operational use of real observations. The OSSE has been discussed here by exploiting the known “truth”. The synthetic observations to be assimilated in the OSSE are generated from nature runs which uses bottom-up surface emission (true) data to simulate global 3-D CH$_4$ concentrations. The true surface CH$_4$ emissions are prepared on the monthly scale using anthropogenic and natural sectors, minus the surface sinks due to bacterial consumption in the soil (Chandra et al., 2021). The anthropogenic emissions were obtained from the Emission Database for Global Atmospheric Research, version 4.3.2 inventory (EDGARv4.3.2) (Maenhout et al., 2019) that includes the emissions from the major sectors such as; fugitive, enteric fermentation and manure management, solid waste and wastewater handling. The biomass burning emissions are taken from the Global Fire Database (GFEDv4s) (van der Werf et al., 2017) and Goddard Institute for Space Studies emissions (Fung et al., 1991). The wetland and rice emissions are taken from the process-based model of the terrestrial biogeochemical cycle, Vegetation Integrated Simulator of Trace gases (VISIT) (Ito, 2019) that is based on Cao et al. (1996). The other natural emission such as, ocean, termites, mud volcano are taken from TransCom-CH$_4$ inter-comparison experiment (Patra et al., 2011). The total emissions are taken as the truth for the OSSEs and the concentration simulated by MIROC4-ACTM will be referred to as synthetic observations.

3.2 Prior flux preparation and LETKF setting

Based on our understanding of CH$_4$ inverse modelling, the uncertainty in regional flux estimation is found to be 30% or lower (Chandra et al., 2021). Therefore, we attempted to reproduce the true flux by starting with a prior flux that is lower by 30% of the true flux (prior flux has same seasonal cycles as true flux). The MIROC4-ACTM is initialized with the spin-up of 3 years (2007 – 2009) with prior flux distribution. The initial CH$_4$ distribution on 01 January 2007 was taken from an earlier simulation of 27 years. An initial perturbation with standard deviation of approximately 6-8% spread is applied.
to the a priori flux as the initial ensemble spread, whereas no ensemble perturbation was applied to the initial CH$_4$ concentration. The sensitivity of the initial ensemble spread to CH$_4$ flux estimation is discussed in Section 4.2. The uncertainty to perturb prior fluxes is generated based on random positive values with normal distribution. The monthly scale prior emission is linearly interpolated at 6 hourly intervals to be used in the MIROC4-ACTM simulation for data assimilation. This study performs two LETKF data assimilation experiments. In these experiments, we provided initial perturbation on regional basis over land (53 different land regions; Chandra et al., 2021) and at every grid over ocean, no spatial error correlation between grid points is considered among ensemble members. However, in Section 4.2.5, we also discussed the sensitivity of CH$_4$ data assimilation by providing initial ensemble spread at every grid by considering horizontal spatial error correlation between grid points among ensemble members, with a global mean correlation of 20%.

### 3.3 Experiment 1: Synthetic dense observation formulation

The OSSE setting with very accurate and dense observation surface data is an attempt to demonstrate that data assimilation system works reasonably in the estimation of the true surface flux. Errors in the estimated flux could arise due to the insufficient ensemble size and also the implemented inflation methods to overcome the under-sampling, along with simplified forecast process of emissions. In real data assimilation, there are additional sources of potential errors, such as, atmospheric transports, and inappropriate prior or observation uncertainties. In our OSSEs, CH$_4$ fluxes as mentioned in Section 3.2 are used as “true” fluxes in generating synthetic observations (CH$_4$ concentrations). In the experiment 1, the simulated surface layer CH$_4$ concentrations at each grid for the entire globe were used as synthetic assimilated observations. We added a constant measurement uncertainty of 5 ppb, which is typically achieved by the present-day measurement systems (e.g., Dlugokencky et al., 2020).

In this study, the CH$_4$ observations are assimilated by applying the observation error covariance localization (Kotsuki et al., 2020) to reduce the spurious spatial correlation due to smaller ensemble size than the degrees of freedom of the system ($R \leftarrow R \times \exp\left(-\frac{1}{2} \left((d_h/\sigma_h)^2 + (d_v/\sigma_v)^2\right)\right)$). Where $d_h$ and $d_v$ denote the horizontal distance (km) and vertical difference ($\log[Pa]$) between the analysis model grid point and observation location. The tunable parameters $\sigma_h$ and $\sigma_v$ are the horizontal localization scale (km) and vertical localization scale ($\log[Pa]$), respectively. Using the spatial localization technique, we have estimated the CH$_4$ flux for each grid by choosing the CH$_4$ observations that influence the grid point using optimal cutoff radius ($\approx 3.65\sigma_{h,v}$; Miyoshi et al., 2007) with horizontal covariance localization ($\sigma_h$) of 2200 km and vertical covariance localization ($\sigma_v$) of 0.3 in the natural logarithmic pressure ($\log[Pa]$) coordinate. The localization is performed to improve the signal to noise ratio of ensemble-based covariance. Numerous sensitivity experiments have been performed by varying the horizontal and vertical localization length in order to obtain the optimized CH$_4$ flux that best compare with the truth. The LETKF assimilates the observations within
the specified radius to solve the analysis state at each grid point independently (Liu et al., 2016; Kotsuki et al., 2020). State vector of the analysis includes the atmospheric CH$_4$ concentration, which is the prognostic variable of forecast model and the state vector is further augmented by surface CH$_4$ flux, which is not a model prognostic variable. This augmentation enables the LETKF to directly estimate the parameter through the background error covariance with observed variables (Baek et al., 2006). The state vector augmentation is implemented similar to that used by Miyazaki et al. (2011). This approach analyses CH$_4$ flux during the analysis step. The purpose of the simultaneous CH$_4$ emission and concentration optimization is to reduce the uncertainty of the initial CH$_4$ concentrations on the CH$_4$ evolution during the assimilation window and to maximize the observations potential (Tian et al., 2014).

The atmospheric CH$_4$ concentration is changed during both the analysis and forecast steps. A challenge of this scheme is that, the analysis increment is added to the model state at each analysis step, without considering the global total CH$_4$ mass conservation in the model, but consistent with the observed local CH$_4$ abundance.

In this case, surface flux at every model grid point is analyzed with 8-days assimilation window during the year 2010 with the 100 ensemble members. The ensemble size and assimilation window are chosen based on the CH$_4$ flux estimation accuracy calculated by performing sensitivity experiment for ensemble size (60, 80, and 100) and assimilation window (3-days and 8-days), respectively (not shown).

3.4 Experiment2: synthetic satellite observation formulation

One way to address the real-world CH$_4$ flux estimation problem is to first make the OSSE dataset like real observations. In this OSSE experiment, we have assimilated synthetic column average CH$_4$ concentrations with a coverage mimicking GOSAT satellite observations. We prepared a model simulated column averaged CH$_4$ concentrations (XCH$_4$) dataset that is spatiotemporally sampled with GOSAT-observations as follows:

$$XCH_4 = XCH_4^{(a\ priori)} + \sum_j h_j a_j (CH_4^{(ACTM)} - CH_4^{(a\ priori)})_j$$  \hspace{1cm} (10)

Where, $XCH_4$ is the column-averaged model simulated CH$_4$ concentration. $XCH_4^{(a\ priori)}$ is a priori column-averaged concentration. $CH_4^{(ACTM)}$ and $CH_4^{(a\ priori)}$ are the CH$_4$ profile from ACTM and a priori, respectively. $h_j$ is the pressure weighting function ($j$ is the vertical layer index), and $a_j$ represents averaging kernel matrix for the column retrieval which is the sensitivity of the retrieved total column at the various (‘$j$’) atmospheric levels. In the next step, we added the same retrieval
(XCH₄) error as GOSAT to the XCH₄ (ACTM simulated) to make the OSSE more realistic and then attempt to estimate the true fluxes.

In this case, the CH₄ flux has been estimated for each grid by choosing the CH₄ observation with cutoff radius ($\approx 3.65\sigma_{h,v}$) with horizontal covariance localization ($\sigma_h$) of 5000 km and vertical covariance localization ($\sigma_v$) of 0.35 in the natural logarithmic pressure (log[Pa]) coordinate. The optimal horizontal and vertical covariance localization values are chosen based on trial and error method (those best fits to estimate CH₄ flux when compared with truth). A long cutoff radius has been chosen due to sparse observational coverage of GOSAT. Covariance localization is necessary to remove long-range erroneous correlations and for mitigating sampling errors in the ensemble-based error covariance with a limited ensemble size (Miyoshi et al., 2007; Greybush et al., 2011; Kotsuki et al., 2020). The surface flux is analyzed at every model grid point with 8-days assimilation window and 100 ensemble members those are chosen based on sensitivity experiments discussed in Section 4.2.

4. Results and Discussion

4.1 Experiment with dense OSSE

The time series of normalized RMSE ($\sum_{i=1}^{n}(x_i^a - x_i^f)^2/n/\bar{x}^f$; $x_i^a$ and $x_i^f$ is the analysis and true state at $i$th model grid point, $n$ is the total number of grid points, and $\bar{x}^f$ represents the mean of true flux) in the analyses over global landmass region is shown in Figure 2. The normalized global RMSE is calculated using FM and RTPS inflation methods (Fig. 2) after assimilating synthetic observation at every grid (Section 3.4). Noteworthy is that the experiment with FM inflation method shows 10-15% larger error in estimating the atmospheric surface CH₄ flux compared to RTPS inflation method. One of the reasons of better RMSE using RTPS inflation method is due to the more degrees of freedom provided by relaxation ($\alpha_{\text{RTPS}}$) in ensemble spread (Eq. 8) that could nudge the ensemble of CH₄ concentrations towards observations. The initial flux analysis spread using RTPS and FM is shown in supporting information (Fig. S1) which shows larger initial analysis flux spread over Brazil, tropical America, and Asia in RTPS inflation compared to FM inflation method. We performed numerous sensitivity test with RTPS inflation method and found that uniform relaxation is not substantial, for some of the regions. Figure 2 shows the RMSE for FM, fixed RTPS ($\alpha_{\text{RTPS}} = 0.4$, applied globally, the optimized value is obtained by manual fine tuning) and conditional RTPS ($\alpha_{\text{RTPS}} = 0.3$-0.7 applied different $\alpha_{\text{RTPS}}$ regionally by manual fine tuning). In case of conditional RTPS, the optimal values of $\alpha_{\text{RTPS}}$, i.e., 0.6, 0.3, and 0.7 for the regions south of 20°S, 20°S-20°N, and north of 20°N, respectively, were obtained from data assimilation sensitivity calculations with varying $\alpha_{\text{RTPS}}$ for the three regions separately to best match the true states. We find that the conditional RTPS method
improves the accuracy by ~5% compared to fixed RTPS and 10-15% compared to FM. In the following, we discuss the results obtained using the conditional RTPS and FM inflation methods.

We have also shown RMSE (not normalized) of surface flux in supplementary information (Fig. S2). Flux RMSE has been estimated globally for both the inflation methods, and also for south of 20°N (by considering only those land grids which fall into south of 20°N; Fig. S2) for comparative purposes. It could be noticed that (supporting information Fig. S2), above north of 20°N, the flux estimation error is higher, specifically during spring-summer when CH₄ emissions peak over most of the northern hemispheric regions (Fig. 3). The high uncertainty during spring-summer (Fig. S2) in the flux estimation over these regions could appear due to the attenuation of surface observations as a result of active vertical mixing. The RMSE during autumn (Fig. S2) is comparable in case of global and south of 20°N, which indicates RMSE arising from southern hemispheric regions, likely over Brazil as it peaks during autumn (Fig. 3).

Figure 2. Time series of normalized RMSE of surface CH₄ flux analysis, for 1 year of data assimilation using FM, fixed RTPS, and conditional RTPS inflation methods over global landmass region.

Figure 3 shows regional total flux seasonal cycles comparison of the estimated fluxes for 15 terrestrial regions with those of the prior and true fluxes. The estimated flux retrieved using RTPS inflation method over different regions agrees well with that of true flux. We intend to show the capability of LETKF estimated fluxes over these regions using surface observations to mimic the true fluxes in our understanding of terrestrial biosphere CH₄ cycle. These results are consistent with Figure 2 with annual global normalized mean bias \( \frac{\sum_{i=1}^{n}(x_i^a - x_i^f)}{\sum_{i=1}^{n}(x_i^f)} \) of -0.04. It could also be noticed from Figure 3 that estimated fluxes converge to true fluxes over most of the regions after about 2-3 months.
Figure 3. The 1-year CH$_4$ total flux seasonal cycles of true (black), prior (blue), and estimated from the LETKF (orange) conditional RTPS inflation method in 15 regions after assimilating dense synthetic surface CH$_4$ observations.

To see the degree of similarity in the flux distribution between the estimated and true fluxes, we show monthly mean spatial flux distribution for June, and November in Figure 4 and 5, respectively, along with the bias in prior and estimated flux. As shown in Figures 4 and 5, the general spatial patterns of the true flux are estimated well. These results suggest that, our LETKF system is capable of reproducing continental spatial flux patterns by using such an idealized dense surface observational data. However, some clear differences in flux estimation could be noticed from FM and RTPS inflation method (Figs. 4 and 5), for e.g., over Eurasian and American continent, analysis with RTPS shows clear improvement compared to FM covariance inflation method. We calculated the global mean normalized bias with RTPS and FM covariance inflation method which is found to be -0.04 and -0.11, respectively over land regions that shows RTPS significantly improved the flux estimation compared to FM covariance inflation method.
**Figure 4.** Spatial distribution of surface CH$_4$ fluxes (true; top left panel, FM analysis; middle left panel, RTPS analysis; bottom left panel) and the associated bias in prior (prior-true; top right panel) and estimated (FM-true; middle right panel, RTPS-true; bottom right panel) fluxes during June, 2010.
Figure 5. Same as Figure 4 but for November, 2010.

4.2 Experiment by mimicking the real satellite observational data set

In this section we discuss the LETKF flux estimation by assimilation of GOSAT synthetic CH$_4$ concentration observations. Figure 6 shows the model simulated mean XCH$_4$ concentration sampled spatiotemporally with GOSAT observations during January and July for the year 2010 (sampling method discussed in Section 3.4). In this case we have shown different LETKF sensitivity experiments such as; LETKF sensitivity to (1) FM, RTPS, adaptive multiplicative inflation (2) assimilation window (3) ensemble size, (4) chi-square test, (5) prior ensemble spread. In the LETKF sensitivity experiments from 1-4, the initial ensemble spread provided similar way as Experiment 1 and conditional RTPS inflation method is used. Conditional RTPS method is also used in Section 4.2.6 for CH$_4$ flux estimation.

4.2.1 LETKF sensitivity to FM, RTPS, and adaptive multiplicative inflation
This study mainly emphasizes on FM and RTPS inflation methods used in CH₄ LETKF data assimilation. The annual average normalized RMSE (absolute bias) with RTPS and FM covariance inflation is found to be 0.59 (0.18) and 0.64 (0.22), respectively. The RTPS inflation method performs better than the FM inflation method overall. In addition to RTPS inflation, sensitivity test is also performed using adaptive multiplicative inflation methods.

In the adaptive inflation, we need to provide an initial multiplicative inflation factor at the beginning of data assimilation cycle (Cycle 1 in Fig. 1). Following the method of Miyoshi (2011), the multiplication inflation factor information calculated in previous cycle (i.e. Cycle 1 in Fig. 1) is used for next data assimilation cycle at every grid point (Cycle 2 in Fig. 1). We perform two sensitivity experiments. In the first (second) case we provided 50% (40%) initial inflation in the beginning of Cycle 1 (Fig. 1). The normalized RMSE in both the adaptive inflation sensitivity experiments are comparable (0.65, Supporting information Fig. S3a) till July, but from the beginning of August, RMSE increases exponentially in the first experiment. However, in terms of chi-square distribution CH₄ flux estimation with first sensitivity adaptive multiplicative inflation experiment (50% initial inflation case) is better than second sensitivity experiment (Supporting information Fig. S3b; chi-square test described in Section 4.2.4). To identify the regions of high estimated CH₄ flux error, we have shown the background error spread in CH₄ flux estimation over 15 regions (Supporting information Fig. S3c) and found that spread over west and south east Asia rises exponentially post July that indicates the rise of estimated CH₄ flux error over these regions in the first sensitivity adaptive multiplicative inflation experiment. Our analysis suggests that CH₄ flux estimation is depending on the initial inflation factor provided in the beginning of data assimilation cycle (Cycle 1, Fig. 1) in adaptive multiplication method. Also, we need to be very careful to monitor the background error spread evolution with time to estimate the CH₄ flux with adaptive inflation, chi-square distribution analysis is not sufficient.

In case of RTPP inflation, we found the parameter 𝛼_{RTPP} is very difficult to fine-tuned due to its very high sensitivity to estimate the CH₄ flux. We fail to obtain an optimized 𝛼_{RTPP} value to estimate the CH₄ flux. Whitaker and Hamill (2012), also demonstrated the better accuracy in LETKF meteorological data assimilation with RTPS compared to RTPP covariance inflation method. They found RTPP method produces very large errors if the inflation parameter exceeds the optimal value.

### 4.2.2 Assimilation window

The LETKF data assimilation window length determines the time span of the observations assimilated in each assimilation cycle. We have shown the sensitivity of two assimilation window size configurations; 3 days and 8 days in supporting information Figure S4. Our sensitivity experiments with window size configurations show that 8 days long assimilation window estimates the CH₄ flux...
with better accuracy (~10%) compared to 3 days assimilation window, because more observational information is incorporated into the system with 8 days long assimilation window. This study uses 8 days assimilation window for CH$_4$ LETKF data assimilation.

**Figure 6.** Monthly mean ACTM simulated XCH$_4$ (ppb) sampled with GOSAT observations to be assimilated (valid during the year 2010). The actual retrieval errors are added in the synthetic GOSAT observations. Data are shown for two representative months, depicting the southern and northern hemisphere differences in data coverage.

### 4.2.3 Ensemble size

Figure 7a shows the RMSE using different ensemble members. The optimal $\alpha_{eq}$ (Eq. 8) value ranging from 0 to 1, is applied based on flux estimation accuracy achieved by fine tuning $\alpha_{eq}$ value over different regions. The RMSE stabilizes gradually as the ensemble size increases from 60 to 80 to 100 ensemble members. The ensemble size dependency of flux estimation suggests the further scope of the improvement in flux estimation by increasing the ensemble members. In this study we stick to 100 ensemble members due to high computational cost while solving large covariance matrices. The larger error in flux estimation in case of column averaged synthetic GOSAT CH$_4$ observations assimilation compared to dense observations (Fig. 2) is likely due to the weaker constraint on surface fluxes provided by satellite observations and sparse observations.
Figure 7: (a) Flux estimation RMSE using different ensemble size with RTPS covariance inflation. (b) Chi-square distribution using FM and RTPS covariance inflation methods with the ensemble size of 100.

4.2.4 Chi-square test

We have carried out chi-square test for the evaluation of background error covariance matrix (Miyazaki et al., 2012). For the $\chi^2$ test, the innovation statistics are diagnosed from the observation minus forecast $(y^o - Hx^b)$, the estimated error covariance in the observation space $(H P^b H^T + R)$, and the number of observations $k$, such as:

$$Y = \frac{1}{\sqrt{k}} (H P^b H^T + R)^{-1/2} (y^o - Hx^b)$$  \hspace{1cm} (11)

Using this statistic, the $\chi^2$ is defined as follow:

$$\chi^2 = \text{trace}YY^T$$  \hspace{1cm} (12)

The performance of background error covariance matrix determined based on the high and lower value of chi-square. Chi-square value should converge to 1, a value higher (lower) than 1 indicates underestimation (overestimation) of the background error covariance matrices. Our results suggest that, background error covariance matrix is highly underestimated in both RTPS and FM covariance inflation methods (Fig. 7b). However, the chi-square values convergence towards 1 is better in the case of RTPS compared to FM covariance inflation method which indicates the improved representation of background errors and then more appropriate data assimilation corrections in the case of the RTPS inflation method. The chi-square distribution starts saturating after the month of March. Post March analysis shows the background error covariance matrix underestimation is much higher (>100%) in case of FM compared to RTPS covariance inflation method.

4.2.5 CH$_4$ LETKF sensitivity to initial ensemble spread
A test case for CH$_4$ LETKF data assimilation has been performed where the initial spread is provided by considering the initial perturbation on each model grid with spatial error correlation between grid points among ensemble members, with global mean correlation of 20%. In this case, we found that the analysis fluxes are extremely sensitive to the initial ensemble spread if prior fluxes perturbed with more than 5% prior uncertainty. Therefore, we used initial ensemble perturbation with only 2% prior uncertainty. Reducing the initial ensemble spread reduces the CH$_4$ flux estimation sensitivity (>60%).

However, it also poses a challenge to mitigate the under-dispersive background error covariance matrix. We performed LETKF data assimilations in this case with RTPS covariance inflation method ($\alpha_{RTPS} = 0.9$ optimized value is used here uniformly) with 8-days long assimilation window and 100 ensemble members and calculated the normalized RMSE between analysis and true fluxes (Supporting information Fig. S5). Noteworthy that, the estimated error between analysis and true fluxes (Fig. S5) with this setting (grid-wise initial ensemble spread) is still larger (25%) than the case when region-wise initial ensemble spread provided (Fig. 7a; 100 ensemble size). It suggests that, initial ensemble spreads among ensemble members needs to be carefully provided that best represents CH$_4$ variability among ensembles to estimate the CH$_4$ flux.

Note that, the OSSEs used in this study did not consider the effects of model errors other than CH$_4$ fluxes, such as model transport errors. In real situations, model errors can have a substantial impact on flux estimates (Locatelli et al., 2013), which needs to be taken into account in background covariances. Therefore, the optimal data assimilation setting can differ between the OSSEs presented in this study and real observation cases. Further efforts, e.g., by conducting a more comprehensive OSSE that accounts for various model errors and by performing various sensitivity calculations in real cases, would provide an improved understanding of the optimal inflation settings to improve CH$_4$ flux estimates in following study.

4.2.6 Estimated CH$_4$ flux analysis

Figure 8 shows the regional fluxes seasonal cycle comparison for the estimated fluxes over 15 terrestrial regions with those of the prior and true fluxes. We have also shown assimilation results in case of FM inflation method in supporting information (Fig. S6), which shows the flux estimation disagreement over more regions compared to RTPS inflation method; e.g., for Tropical and North America, whole African continent, Australia-New Zealand.

We have shown the GOSAT observations in Figure 6 and supporting information Figure S7. We found very marginal flux estimation improvement over Central Africa after May (Fig. 8), that could be associated with the less GOSAT coverage over this region (Fig. 6). On the other hand, over Northern Africa, no improvement in flux estimation is found. In case of dense OSSE too (Fig. 3), we didn’t find satisfactory flux estimation over Northern Africa which is most probably related to the
insufficient initial spread among ensemble members over this region (we have used same initial ensemble spread in both OSSE cases). Over Europe, GOSAT observations are remarkably less, specifically for first few months (January-April; supporting information Fig. S7). Therefore, the flux update over Europe would be influenced by the observations from neighboring regions falling under the chosen cutoff radius that are mainly in Northern Africa where the flux estimation itself not satisfactory. It could also be noticed that the retrieval error added in this OSSE case are high over Europe (September-October; supporting information Fig. S7), and its adjacent Sea (Mediterranean Sea; June-August) which could also affect the surface CH$_4$ flux estimation.

Figure 8. Same as Figure3 but after assimilating synthetic GOSAT observations.

Figure 9 and 10 show spatial patterns of the true and estimated fluxes by assimilating the column averaged CH$_4$ concentrations during June and November (Fig. 6). It may be noticed that RTPS covariance inflation method better able to estimate the true flux pattern compared to FM covariance inflation method. The spatial pattern shown using RTPS inflation method emphasizes the positive and negative bias in the estimated flux (Figs. 9 and 10), but generally agrees with the flux seasonal cycle plots shown in Figure 8.

Our LETKF CH$_4$ data assimilation experiment by assimilating GOSAT synthetic observation with the implementation of the advanced RTPS covariance inflation method better estimate the time-evolving surface CH$_4$ fluxes compared to FM covariance inflation method. The difficulty to estimate the surface CH$_4$ flux over a few regions may be overcome by applying additional methodologies, such as the assimilation of surface observations simultaneously, and the use of information about the CH$_4$ fluxes climatology. A correction factor derived based on empirical formulation that could use CH$_4$
flux climatology information is needed to apply to maintain the CH$_4$ mass conservation. This could be implemented by the checking the simulated CH$_4$ burden gain between years in comparison with the observed CH$_4$ growth rates.

**Figure 9.** Monthly mean true (true; top left panel) and estimated (FM analysis; middle left panel, RTPS analysis; bottom left panel) CH$_4$ flux after assimilating column averaged synthetic CH$_4$ concentrations (Fig. 6) during June using FM and RTPS inflation methods. The associated bias with prior and estimated fluxes is also shown (prior-true; top right panel; FM-true; middle right panel, RTPS-true; bottom right panel).
5. Summary

In this study, we have introduced 4D-LETKF data assimilation system that utilizes MIROC4-ACTM as a forward model for CH\textsubscript{4} flux estimation. This study has extensively tested both FM and RTPS inflation methods for the LETKF CH\textsubscript{4} flux estimation. We have conducted two experiments to demonstrate the ability of LETKF system to estimate the CH\textsubscript{4} surface flux globally. In Experiment1, we have assimilated the synthetic dense surface CH\textsubscript{4} observations. While in Experiment2, synthetic GOSAT CH\textsubscript{4} observations are assimilated. Based on the results of the sensitivity tests using FM and RTPS inflation methods in Experiment1, we have found that RTPS inflation produces significantly less normalized RMSE (10-15\%) compared to FM inflation method. In Experiment2, we discussed, LETKF parameters such as, different inflation techniques, ensemble size, assimilation window, initial ensemble spread sensitivity, and chi-square test. The ensemble size (this study uses maximum 100 ensemble members) sensitivity test suggests that more ensemble members could help to accurately represent the covariance matrix with more degrees of freedom. The assimilation window sensitivity
test exhibits that 8 days assimilation window reduces the normalized flux RMSE by about 10% compared to 3 days assimilation window in case of GOSAT synthetic observations assimilation. Our approach of assimilation with RTPS inflation could provide more degrees of freedom to fit the ensemble of CH₄ concentrations to the observed ones, resulting the improved analyzed fluxes. The RTPS inflation method is capable of obtaining reasonable flux estimates with normalized annual mean bias of 0.04, and 0.61 in case of dense surface synthetic observations and GOSAT synthetic observations, respectively. We demonstrated in our sensitivity OSSE experiment with synthetic GOSAT observations that, over American and African continents and also over Australia - New Zealand, the LETKF data assimilation with FM inflation method does not show much improvement in the true flux estimation, but RTPS inflation method reasonably estimate the true flux over most of these regions. One of the reasons for better flux estimates from RTPS inflation method is the prevention of analysis spread drastically. In the CH₄ LETKF flux estimation, surface CH₄ flux is not a prognostic state vector in the ACTM, which results in the decay of spread continuously in analysis steps. RTPS inflation method could mitigate such under disperse spread problem. This study finds that spatially homogeneous relaxation is not sufficient. It needs to be fine-tuned and applied conditionally.

The sensitivity of LETKF CH₄ flux estimation to initial ensemble spread needed to be carefully dealt with when applied to real data assimilation system. A future OSSE with additive covariance inflation technique could be interesting while applied with RTPS inflation method for CH₄ LETKF data assimilation since in additive covariance inflation initial estimated flux error cannot propagate. The state vector augmentation technique used here updates the flux after each data assimilation cycle but it doesn’t conserve the total atmospheric CH₄ amount which is one of the limitations of this work. A correction factor needs to be implemented to conserve the total atmospheric CH₄ amount after completion of a few data assimilation cycles. We have not accounted for the transport error due to meteorological fields in this work (Patra et al., 2011b), in case of real observations data assimilation a week-long window may introduce transport errors in CH₄ analysis because of nonlinear growth of ensemble perturbations.

**Code and data availability.** The LETKF source codes can be accessed from [https://doi.org/10.5281/zenodo.7127658](https://doi.org/10.5281/zenodo.7127658). All the scripts for running the LETKF data assimilation software, input and output results data files are available at [https://doi.org/10.5281/zenodo.7098323](https://doi.org/10.5281/zenodo.7098323). CH₄ ACTM simulation module coupled with MIROC4-AGCM can be accessed from [https://doi.org/10.5281/zenodo.7118365](https://doi.org/10.5281/zenodo.7118365). The source code of MIROC4-AGCM is archived at [https://doi.org/10.5281/zenodo.7274240](https://doi.org/10.5281/zenodo.7274240) with restriction because of the copyright policy of the MIROC developer community, and no contribution of this work to the MIROC4 source code development.
Author contributions. The LETKF data assimilation experiments were designed by JSHB. PKP, MT and TS help to set the LETKF code on MIROC4-ACTM for CH₄ data assimilation. The manuscript is prepared by JSHB and analysis interpretation input and feedback are provided by PKP, TS, KM. All coauthors, KM, TS, PKP, NS, MT and YK contributed to writing and revision of the paper.

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