Reply to Editor comments

Before this manuscript can be accepted for formal publication, I have a few minor comments and would like the authors to clarify.

Thank you for reviewing the manuscript and for your comments and suggestions. We clarified your comments and revised the manuscript accordingly. Kindly find our reply in black fonts for your comments in red.

1. Please clarify the adaptive inflation method you used in this work. Is it the Gaussian approach to adaptive covariance inflation proposed by Miyoshi (2011)? Or, the authors used the simple adaptive inflation method, which adaptively adjusts the background error covariance term to fulfill the relationship of the total error covariance in observation space. If it's the latter, please use the correct reference.

Reply: The adaptive covariance inflation proposed by Miyoshi (2011) is used here which is the Gaussian approach to adaptive covariance inflation. We cited the Miyoshi (2011) paper in the Section 4.2.1 (Line No. 359) in the revised manuscript.

2. Line 199: It should be clarified whether the model prognostic variables are perturbed in the initial ensemble. If they are not perturbed, please explain.

Reply: Model prognostic variables are not perturbed in the initial perturbation. We only perturbed the CH_4 prior fluxes. The surface CH_4 flux perturbation is then propagated into the prognostic variables (in this chase, atmospheric CH_4 concentrations) during forecast processes. The sentence has been revised as follows (Section 3.2; Line No. 198-200):

“An initial perturbation with standard deviation of approximately 6-8% spread is applied to the a priori flux as the initial ensemble spread, whereas no ensemble perturbation was applied to the initial CH4 concentration.”

3. Manual regional fine turning of RTPS has been mentioned several times (Lines 287, 385). Please clarify how the tuning is conducted to optimize the inflation for different regions.

Reply: The tuning was conducted based on data assimilation sensitivity calculations with varying α_{RTPS} which were performed for the following three regions separately; south of 20°S, 20°S-20°N, and north of 20°N. We added following lines in the revised manuscript.

Section 4.1; Lines 293-296
“In case of conditional RTPS, the optimal values of α_{RTPS}, i.e., 0.6, 0.3, and 0.7 for the regions south of 20°S, 20°S-20°N, and north of 20°N, respectively, were obtained from data assimilation sensitivity calculations with varying α_{RTPS} for the three regions separately to best match the true states.

4. Are the results with RTPS shown with the experiment using the conditional RTPS? (e.g., Fig. 3 and section 4.2.1)

Reply: Yes. To clarify this setting earlier in the manuscript, we mentioned this in revised manuscript in "(Section 4.1; Line no. 297-298)" and "(Section 4.2; Line no. 350-351)".

5. Line 367: Please use the correct format for citation.

Reply: Corrected "Section 4.2.1; Line No. 379" in the revised manuscript

6. In section 4.2.5, the authors should briefly comment on how the amplitude of variation of the ensemble spread may be limited by model error and how this may affect the performance of the CH4 flux estimation.

Reply: We added the following lines at the end of Section 4.2.5:

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Note that, the OSSEs used in this study did not consider the effects of model errors other than CH_4 fluxes, such as model transport errors. In real situations, model errors can have a substantial impact on flux estimates (Locatelli et al., 2013), which needs to be taken into account in background covariances. Therefore, the optimal data assimilation setting can differ between the OSSEs presented in this study and real observation cases. Further efforts, e.g., by conducting a more comprehensive OSSE that accounts for various model errors and by performing various sensitivity calculations in real cases, would provide an improved understanding of the optimal inflation settings to improve CH_4 flux estimates in following study.

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Locatelli, R., Bousquet, P., Chevallier, F., Fortems-Cheney, A., Szopa, S., Saunois, M., Agusti-Panareda, A., Bergmann, D., Bian, H., Cameron-Smith, P., Chipperfield, M. P., Gloor, E., Houweling, S., Kawa, S. R., Krol, M., Patra, P. K., Prinn, R. G., Rigby, M., Saito, R., and Wilson, C.: Impact of transport model errors on the global and regional methane emissions estimated by inverse modelling, Atmos. Chem. Phys., 13, 9917–9937, https://doi.org/10.5194/acp-13-9917-2013, 2013.

Reply to Reviewer comments

The authors have replied to my previous comments and revised the manuscript accordingly. I now have only one minor comment: I think the description of the LETKF method could be a bit clearer still. Especially, I think the authors must mention in section 2 that the LETKF solves the analysis update equations at every model grid point independently (I think this is mentioned later but is a bit hidden). This aspect of the method explains why the localisation (described in sections 3.3 and 3.4 is needed). In section 3.4, the purpose of localisation could also be made clearer for those not already familiar with the LETKF method. Kotsuki et al. (2020) explain this very nicely in their paper (section 2.1) and perhaps the authors can cite some of this description.

Thank you for reviewing the manuscript again and for comments and suggestions. We have revised the manuscript accordingly. Kindly find our reply as follows:

Reply: We added the following lines in Section 2 (Line 111-112):
"The LETKF solves the analysis update equations 3 and 5 at every model grid point independently by assimilating local observations within the localization cut-off radius."

We added following lines in Section 3.4 (Line 271-274):

"Covariance localization is necessary to remove long-range erroneous correlations and for mitigating sampling errors in the ensemble-based error covariance with a limited ensemble size (Miyoshi et al., 2007; Greybush et al., 2011; Kotsuki et al., 2020)"

Kotsuki, S., Pensoneault, A., Okazaki, A., and Miyoshi, T.: Weight structure of the Local Ensemble Transform Kalman Filter: A case with an intermediate atmospheric general circulation model, Q. J. R. Meteorol. Soc., 146, 3399–3415, https://doi.org/10.1002/qj.3852, 2020.

Miyoshi, T., Yamane, S., & Enomoto, T. (2007). Localizing the error covariance by physical distances within a local ensemble transform Kalman filter (LETKF). *Sola*, *3*, 89-92.

Greybush, S. J., Kalnay, E., Miyoshi, T., Ide, K., & Hunt, B. R. (2011). Balance and ensemble Kalman filter localization techniques. *Monthly Weather Review*, *139*(2), 511-522.