Revisions Based on Comments of Erick.

I have carefully addressed all comments and suggestions made by you and revised my paper thoroughly. I hope you find the updated version satisfactory and suitable for publication. Please also note that for more clarification, all the changes applied to the paper are highlighted with yellow colour.

Page numbers are based on the p aper **before** revision.

Major Revisions:

1- "…However, There are some major issues with the paper in my opinion. The paper needs to become more distinctive from the Mathematical Geosciences paper from the same team with same lead author. This will be a challenge because most of what this current submission to GMD offers has already been presented in Math Geosci, but does have the addition of Python demonstration codes to show the utility of the methods."

Thanks for this comment. You are right. In the introduction of the updated version of this paper, we mentioned the major difference between this paper and our previous paper. The main difference is that in this paper, we implement the automatic fitting for reconstruction using the augmented Lagrangian method. However, in the previous paper, we used manual fitting. Also, in the new paper, we explained more about the code behind the software.

2- "…state more clearly and concisely why we need these tools in the community for geological applications b) while making the code more accessible, and with a bit of effort on c) better user instructions."

Thanks for this comment. You are right. In the updated version of the paper. We mention the importance of this method (introduction). Besides, we added more information about the code and how to use it.

3- "The paper tends to get into a lot of details without saying why Sub-division surfaces are so important and why our current state of the art (Implicit?) tools is not properly handling these situations."

Thanks for this comment. Jacquemyn et al. (2019) comprehensively explain the advantages of surface-based modelling compared to grid-based representation in geological modelling.

Jacquemyn, C., Jackson, M. D., & Hampson, G. J. (2019). Surface-based geological reservoir modelling using grid-free NURBS curves and surfaces. *Mathematical Geosciences*, *51*, 1-28.

We also addressed their work by mentioning that "Several studies considered a surface-based approach in geological modelling since the outstanding features of the structure (e.g. heterogeneity) are explicitly demonstrated by the surfaces of the boundary (Jacquemyn et al. 2019).".

Besides, we already explained the importance of subdivision surfaces in our previous paper and addressed that paper in this paper as a reference. Since the current paper is about PySubdiv software, we tried to focus mainly on explaining the software while we addressed other references for more details.

4- Why do we need to preserve sharp edges in some cases and smooth edges in others?

Thanks for this comment. We wanted to introduce the tools (subdivision surface method) to the user. Using or not using sharp or smooth edges mostly depends on the engineering judgment of the user. If the user thinks he/she wants to have more sharp edges (e.g., modelling sharp fault bend) or smooth edges, he has the tools. Also, sharp edges are convenient for representing piecewise smooth surfaces.

I agree that real-world surfaces are never infinitely sharp (DeRose et al. 1998, Lavoué et al. 2007). So we may need semi-sharp creases for modeling. In general, it depends on the user and what he wants to mode (sharp, semi-sharp or smooth).

5- Also importantly, when the whole geological modelling community is moving to implicit surfaces (level set derived) are we going to go back to explicit approaches, even if it means we get some accuracy benefits?

Thanks for this comment. We are aware of the importance of implicit modelling. Both Implicit and explicit modelling have their advantages and disadvantages. However, we are not quite sure if the "*whole geological modelling community is moving to implicit surfaces*". Maybe one of the most important papers that motivated us to focus on this topic was the paper of Jacquemyn et al. (2019), which is the winner of "AMG Mathematical Geosciences 2019 Best Paper Award"

Jacquemyn, C., Jackson, M. D., & Hampson, G. J. (2019). Surface-based geological reservoir modelling using grid-free NURBS curves and surfaces. *Mathematical Geosciences*, *51*, 1-28.

This paper is completely on surface-based modelling with NURBS, which is not a new method. There is no doubt that implicit modelling is a very good method. I saw your paper with Mike and Florian Wellmann (my supervisor and colleague in

PySubdiv project) on implicit modelling is "the winner of the Mathematical Geosciences 2021 Best Paper Award" **Congratulation Erik**.

Some words from Jacquemyn et al. (2019) about the advantages of explicit modelling.

"…it is challenging to represent many common geological features using pillar grids, including faulted domains and non-monotonic surfaces (e.g. recumbent folds, diapir flanks and the margins of intruded or injected bodies). Rock types with diverse petrophysical characteristics are "averaged" within grid cells of arbitrary size and shape, and the continuity and connectivity of low-permeability baffles and barriers or high-permeability zones is often lost. Stair-stepping effects are introduced by cornerpoint grids around any feature that is not aligned to the grid orientation. Such effects are common around faults, but also impact, for example, the continuity and connectivity of sinuous channels…"

6- So just a better context would be good. This is mostly in the Math Geosc paper but could be restated in this submission in a clear and unique manner that motivates the reader to continue.

Thanks for this comment. **You are right**. We implemented major changes to this paper and tried to make it more clear and more understandable for the reader.

7- I have tried to test all the python codes with limited success. The most crucial point was to do interactive editing and updating of the edge values (CSV) for crease sharpness (sharp -1 to smooth-0). I tested with Windows 10, using Pycharm, Anaconda, (Python v3.9 in virtual environment). It is important to provide working code to the perceived user community, which in my opinion would be dominantly windows users. It would also be helpful if a more detailed tutorial could be provided.

Thanks for this comment. **You are right. Please accept my apology**. PyVista uses PyQt for GUI, and unfortunately, due to the limitation of PyQt, the software cannot work well in Windows, Mac and Conda enviroment. Currently, the software can work well in Linux without the Anaconda. We really did our best to remove these limitations. The institute plans to improve the software in the near future by absorbing more sources of funding and hiring more people to help.

8- If the authors choose to update the useability of their codes, it will widen the user community for these tools and provide better discussions for those wishing to implement their proposed solutions. All the SubDivision surface codes performed well but only generated smooth refined meshes or coarse grip frames. Not composite straight and smooth non-manifold object types. This is a big stumbling block at this stage since the users have no way to fully evaluate the utility or advantages of the code without a working application.

Thanks for this comment. Yes. **You are right. We are sorry for it**. In the updated version of the software, we explain more about the details of geological modelling inside the software. Also, We generated a tutorial video about the code, and we hope this will be helpful. Also, we cleaned the code and made it better for the users.

Other comments:

Line 23

Thanks for the comment. The geological model can be constructed by various methods, e.g. marching cube from the implicit model. In this paper, the goal of surface reconstruction is to make the geological model manageable with a few

numbers of control points. Of course, it may associate with re-meshing. However, re-meshing is not the goal of reconstruction in this paper.

Line 25

Thanks for the comment. In this paper, the problems known from spline surfaces mainly refer to (1) the limitation of the spline surfaces in supporting arbitrary topology (Cashman 2010) and (2) the requirement of stitching multiple patches (e.g., NURBS patches) for generating the complex structures (Botsch *et al.*, 2010). We already mentioned the problems with related references in lines 38,39, and 40.

Line 27

Thanks for the comment. The goal is to generate the "control mesh" for each geological model. The control mesh contains some control points that can help to investigate the uncertainties of different geological scenarios and, consequently, solve inverse problems. For example, if we have uncertainties in one location of the model, we can assign one control point to that location and generate the control mesh. Then, by changing the control point's location, we can generate different models and evaluate the uncertainty. However, to make the sentence more clear, i deleted the word.

Line 30

Thanks for the comment. We added the references which you mentioned to the updated version of this paper.

Line 31

Thanks for your comment. You are right. However, to reduce the number of pages of the paper, we decided to focus mainly on subdivision surfaces.

Line 34

Thanks for the comment. We implemented your comment. It is worth mentioning that we already mentioned the differences between the NURBS and subdivision surfaces in the following lines. However, to make it more clear, I explain it here.

Line 40

Thanks for the comment. We mentioned the definition of "topology" just to show that when we say "subdivision surfaces support arbitrary topology", what we are speaking about. Speaking about "heat transport" or "fluid flow" is beyond the scope of this paper. We just bring these examples to stress the importance of supporting the arbitrary topology. We deleted this part to avoid misunderstanding the topic.

Line 43

Thanks for the comment. You are right. However, due to several references (e.g., Botsch et al., 2010), subdivision surface method is one of the best methods to tacked the limitation of NURBS, which has received limited attention in geological modelling. Also, the subdivision surface method has more features (e.g., supporting non-manifold topology), making it attractive for geological modelling.

Thanks for the comment. We implemented the definition of subdivision surfaces in the updated version of this paper. It is worth mentioning that we already mentioned the differences between the NURBS and subdivision surfaces in the previous lines. It is just two lines since we investigated the advantages and disadvantages of both NURB and subdivision surfaces in our previous paper (Moulaeifard et al. 2023).

Thanks for the comment. We comprehensively explained the (1) subdivision surfaces and NURBS methods and (2) the main differences in geological modelling in our previous paper (Moulaeifard et al. 2023). Therefore, in this paper, we just shorty explain them and mostly focus on automatic reconstruction, which is the nobility of this paper.

Also, regarding the DSI, we are pretty aware of the importance of this method which is already implemented in GoCad. In the updated version of this paper, we already spoke about the DSI in the introduction. The goal of reconstruction in this paper is to find the control mesh with **few** control points, which helps us to control the reconstructed mesh. Lévy and Mallet (1999) in their paper (section 3.3), explain how we can use DSI in each iteration of subdivision surfaces while all vertices, except (control nodes), are free to move in each iteration. Also, Lévy and Mallet (1999)mention that "the convergency of the obtained meshes has always been observed in practical application, **but the formal proof of this convergency is still is am an open problem**". There is uncertainty about the convergency when we want to use DSI during subdivision surfaces which is one of the reasons that we did not used DSI in our project. We selected the loop algorithm since the convergence of the series of refined meshes is formally proved (Loop 1987).

Line 57

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 64

Thanks for the comment. I modified the sentences in the updated version of the paper.

Line 95

Thanks for the comment. I modified the sentences in the updated version of the paper.

Line 112

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 113

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 118

Thanks for the comment. I added all of the references to the updated version of the paper.

Line 141

Thanks for the comment. You are right. I modified the sentence and added Table 1 and Table 2 to avoid misunderstanding. Apparently, zero is full smoothing, and one is sharp. However, it is worth mentioning that each vertex is connected to multiple edges and concequently should follow the Table 2. I used the word "resistance" to make it more understandable for the reader.

Line 155

Thanks for the comment. We explained the subdivision surfaces comprehensively in our previous paper (Moulaeifard et al. 2023), which is addressed in this paper. The current paper focused on the reconstruction procedure, which is the main difference between the two papers. We prepared an instruction for modelling inside the software. Our software is very young; unfortunately, it works only in Linux systems.

Line 156 and 161

Thanks for the comment. You are right. We implemented more figures and explanations for the "non-manifold topology" section in the paper's updated version.

Line 165

Thanks for the comment. You are right. We implemented more figures and explanations for the "definition of the reconstruction and algorithm" in the paper's updated version.

Line 167

Thanks for the comment. You are right. I corrected the word in the paper's updated version.

Line 173

Thanks for the comment. You are right. I modified this part in the paper's updated version.

Line 174

Thanks for the comment. I understand your concern regarding the word "control points". However, since the subdivision surfaces method originated from computer graphics, we wanted to preserve the name (due to several GC references) to have a common language between computer graphics researchers and us. Also, paying attention to the other words that came with "control point" can help us to understand the meaning. For example, when the "control points" come with a "control mesh".

Line 191

Thanks for the comment. b is the coefficient for optimization. The user defines it.

Line 192

Thanks for the comment. Since we got the equations of this section from (Wu et al. 2017), we wanted to preserve the notation and definition of the symbols.

Line 192

Thanks for the comment. The penalty term adds a cost for violating each constraint (violation of both crease sharpness value and position of the control points).

Line 198

Thanks for the comment. I modified it in the updated version of the paper.

Line 205

Thanks for the comment. I modified it in the updated version of the paper.

Line 208

Thanks for the comment. I modified it in the updated version of the paper.

Line 212

Thanks for the comment. I modified it in the updated version of the paper.

Line 215

Thanks for the comment. We use Blender to render the final mesh (better quality in rendering) or sometimes to generate triangular input mesh for our software. Blender is not used inside in PySubdiv.

Line 218

Thanks for the comment. I added a better explanation in the updated version of the paper.

Line 260

Thanks for the comment. You are right. However, since the subdivision surface method originated from computer graphics, we wanted to preserve the name (due to several GC references) to have a common language between computer graphics researchers and us.

Line 272

Thanks for the comment. I modified it in the updated version of the paper.

Line 278

Thanks for the comment. We added more explanation ins section 3.1.2 of the updated paper. Hoppe et al. (1994), Wu et al. (2017) offer to consider a threshold angle (θ_0) as a criterion for tagging the edges of the initial control mesh. For each edge, the edge is tagged as sharp (crease sharpness value equal to one) if the angle between the normal of two adjacent faces (θ_e) is more than the threshold angle ($\theta_e > \theta_0$). It is worth reminding that this value is just the initial value and may be changed during the optimization.

Thanks for the comment. We added a new section (section 3.1) in the updated version of the paper to show how the software works, such as generating the control mesh (3.1.1), setting the crease sharpness values (3.1.2) and reconstruction (3.1.3).

Regarding the numbers, the input data of URG model is in a grid-point format URG model, which contains 616464 individual nodes. One has to connect the points to get the mesh of the input data.

Control mesh and control points (vertices of the control mesh) are generated inside the PySubdiv software, and the goal is to manage the mesh with a few control points. There is a big difference between control points and grid points since changing the location of the control points makes smooth changes on the surfaces. However, the Grid-points are just the input data.

Line 295

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 303

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 309

Thanks for the comment. I understand your concern regarding the word "control points". In order to avoid misunderstanding, I modified the sentence by adding

"control points of the control mesh". I think w cannot really change the word "control points" since the subdivision surface method originated from computer graphics (due to several GC references) and concequently, for having a common language between computer graphics researchers and us.

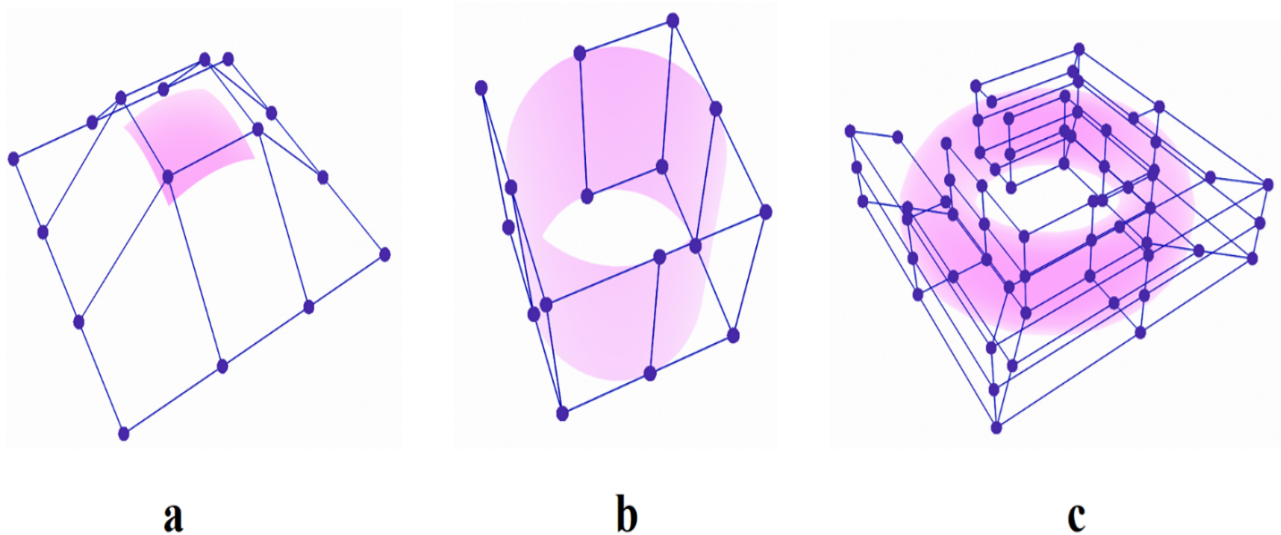
Line 342

Thanks for the comment. As mentioned in line 340, the "iteration" is related to the subdivision algorithm (not the iteration inside the reconstruction algorithm). First, we have one control mesh, and then we apply the subdivision surfaces (Loop scheme), which results in the generation of smooth mesh. Then, we will use this smooth subdivided mesh inside the algorithm for reconstruction. The algorithm itself contains several iterations in which the user does not specify the number (we have one while loop to reach the best fit). By increasing the number of iterations of the subdivision methods (not the iteration inside the algorithm), the smooth mesh will have more vertices. Therefore by importing the smooth mesh inside the reconstruction algorithm, we have to deal with more vertices and, consequently, more iteration inside the reconstruction process.

Line 342

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Thanks for the comment. Here, the word "grid" indicates the grid structure of the spline surfaces (Figure below).

a   **b**   **c**

To create a model with a complex topology, many NURBS patches have to be smoothly connected (by stitching NURBS patches together). Multiple connections between surface patches, in addition to topological or geometrical constraints, make the modelling procedure difficult (Botsch et al. 2010, Cashman 2010). This grid I think this grid is different from the grid which we use for flow simulation or FEM. With both NURBS and subdivision surfaces, we generate our mesh (boundary of the object), and then we can import it into any grid-based system, e.g. FEM, for other processing computations.

Thanks for the comment. You are right. I deleted this sentence since it sounded vague, and maybe it is not always correct.

Line 378

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Lines 402 and 402

Thanks for the comment. I modified the references in the updated version of the paper.

Botsch, M., L. Kobbelt, M. Pauly, P. Alliez, and B. Lévy. 2010. Polygon mesh processing. CRC press.

Cashman, T. J. 2010. NURBS-compatible subdivision surfaces. Cashman, Thomas J.

DeRose, T., M. Kass, and T. Truong. 1998. Subdivision surfaces in character animation. Pages 85-94 *in* Proceedings of the 25th annual conference on Computer graphics and interactive techniques.

Hoppe, H., T. DeRose, T. Duchamp, M. Halstead, H. Jin, J. McDonald, J. Schweitzer, and W. Stuetzle. 1994. Piecewise smooth surface reconstruction. Pages 295-302 *in* Proceedings of the 21st annual conference on Computer graphics and interactive techniques.

Jacquemyn, C., M. D. Jackson, and G. J. Hampson. 2019. Surface-based geological reservoir modelling using grid-free NURBS curves and surfaces. Mathematical Geosciences **51**:1-28.

Lavoué, G., F. Dupont, and A. Baskurt. 2007. A framework for quad/triangle subdivision surface fitting: Application to mechanical objects. Pages 1-14 *in* Computer Graphics Forum. Wiley Online Library.

Lévy, B., and J.-L. Mallet. 1999. Discrete smooth interpolation: Constrained discrete fairing for arbitrary meshes. Available on www at http://www. loria. fr/levy/Papers/1999/s99dsi. pdf.

Loop, C. 1987. Smooth subdivision surfaces based on triangles.

Moulaeifard, M., F. Wellmann, S. Bernard, M. de la Varga, and D. Bommes. 2023. Subdivide and Conquer: Adapting Non-Manifold Subdivision Surfaces to Surface-Based Representation and Reconstruction of Complex Geological Structures. Mathematical Geosciences **55**:81-111.

Wu, X., J. Zheng, Y. Cai, and H. Li. 2017. Variational reconstruction using subdivision surfaces with continuous sharpness control. Computational Visual Media **3**:217-228.

Revisions Based on the Comments of Prof. Caumon

I have carefully addressed all comments and suggestions made by you and revised my paper thoroughly. I hope you find the updated version satisfactory and suitable for publication. Please also note that for more clarification, all the changes applied to the paper are highlighted with yellow colour.

Page numbers are based on the paper **before** revision.

Major Revisions:

1. The contributions of the paper would need to be made clearer, in particular wrt previous work on subdivision surfaces. This includes the recent paper of the authors (Moulaeifard et al, 2023) and also Chapter 7 of Mallet (2002), which briefly mentions variable edge sharpness in subdivision surfaces. As a strong focus is put on semi-sharp creases, some motivating geological examples could be mentioned early on (e.g., to model the hinges of kink folds, see, for instance, Caumon et al 2013).

Thanks for the comment. You are right. The updated version of this paper is more clear and more understandable. We submitted the initial version of the paper before 22 July 2022. After that time, we gradually realised the problems of this paper. We do agree with you.

Also, in the introduction of the updated version of this paper, we mentioned the major difference between this paper and our previous paper. The main difference is that in this paper, we implement the automatic fitting using the augmented Lagrangian method. However, in the previous paper, we used manual fitting.

2. In the discussion, some references to TSplines would also be fair, as TSplines is another way to address some of the shortcomings of splines in terms of spatial adaptivity.

Thanks for the comment. You are right. However, based on your another suggestions for discussion sections, I completely update the discussion.

3. Some Figures could be improved: edge colors are difficult to see on Fig. 1; match caption and colors in Figs. 1 and 4; it would help to highlight the mesh on Figs 3 and 6. Also, it is strange to see a quad mesh on these two figures, whereas the paper defines only the triangular subdivision scheme.

Thanks for the comment. You are right. I modified the figures.

4. Maybe some of the details on the subdivision could be moved to an Appendix, as it seems redundant with Moulaeifard et al (2023). I have two questions on the subdivision. First, is there an implicit assumption about mesh quality? It seems to me that the edge refinement could lead to overlaps in some pathological mesh configurations (See drawing on page 6 of the annotated manuscript). I also suspect that some specific subdivision scheme is used for border edges, as Eq (3) is not directly applicable in that case.

Thanks for the comment. You are right. However, based on the comments of the another reviewer, I decided to keep the details in this paper.

Loop (1987) already mentioned the mask for subdividing the border edges (crease edge mask). However, in this paper we assume that our control mesh is watertight

in geological modelling. Therefore, we do not have any border edges; consequently, having the (mask) inside this paper is unnecessary.

Regarding the possibility of overlapping during the refinement. Honestly, since I have never faced this problem during my work. I searched to see if any other study has reported this problem or not. I could not find any paper.

However, if you mean, "Maybe some undesired intersections between surfaces can occur during modelling with the subdivision surface method?". Then we have to say most subdivision schemes (all with only positive subdivision weights) have the feature that the final subdivided surface strictly lies within the convex hull of the control mesh. Therefore, one could easily subdivide until the convex hulls are intersection-free to verify that there is no intersection. With a similar procedure, it is possible to detect intersections and resolve them (e.g., progressively increasing precision). Several papers investigated the intersection aspects in subdivision surfaces (DeRose et al. 1998, Grinspun and Schroder 2001, Severn and Samavati 2006)

5. I don't understand Section 2.4, which seems to be the most original aspect of the work. Probably adding (i) and (i-1) superscripts to symbols and merging with the iterative algorithmic description of Section 4 would help making the presentation more clear (in particular the meaning and the motivation for the constraint used for the optimisation, as the constraint of Eq. (5) seems to be always honored from the definition of k.

Thanks for this comment. You are right. I completely rewrote this part and made it more clear in the updated version of this paper. Also, I added Algorithm 1, which contains the superscriptions.

6. In Section 4, it would be nice to clarify what functions / features of PyVista are used. In particular, I don't understand if the code needs an input non-manifold model from the start, or if it is able to compute non-manifold and watertight features from a "soup" of intersecting and non-conformable surfaces. Some more elements would be helpful.

Thanks for this comment. The code can get any type of mesh as an input (non-manifold, manifold, non-watertight, watertight). Then the user should select the control points based on the vertices of the input mesh and generate the control mesh. In the updated version of this paper, I added one detailed example (anticline structure) and explained all details there.

By the way, PyVista exploits the PyQt for GUI, which is one of the sources of the crashes in our software.

7. In the application section, it would help to compute how much the points were moved by the optimisation process, as this is difficult to see from the figure. Plotting the error (before and after the optimisation) on the initial mesh would probably be more meaningful than showing it on the output mesh, which probably has a coarser resolution. As visualisation is challenging, this could maybe be provided as supplementary material in VTK format or similar?

Thanks for your comments. You are right. I tested all solutions for better representation (w.r.t our software limitations). Unfortunately, to make the results better, sometimes we need more vertices or calculations, which is a bit more than the limitations of the software. However, to make the figures better, I updated the results and added exploded view on the distance map of the four extracted sub-horizontal layers. It is planned to make the visualization part of the software better in future.

8. The discussion could possibly be enriched and expanded. For instance, are there convergence metrics in subdivision surfaces? Are there challenges to obtain correct watertight geological surfaces, and to what extent can PySubdiv help with that (or not)? Is there a trade-off to be found between the crease sharpness value and the density of the initial control mesh (i.e., could one compensate for the inappropriate choice of the other)?

Thanks for this comment. You are right. I updated the current part of the discussion with some of your suggested topics. We plan to add your other suggested topics for the discussion in future papers.

9. In the conclusion, I agree that there is a strong potential to the method to serve for geological inverse problems, but this should be phrased as a perspective.

Thanks for this comment. You are right. I updated the conclusion section.

10. There is room for improvement on the form (equation symbols, integration of equations in the text), english writing style, see detailed remarks in the annotated manuscript.

Thanks for this comment. Please accept my apology for it. I did my best to minimise the writing errors in the updated version of this paper. Also, I used premium Grammarly in the updated version.

Comments on PDF:

Lines 10 and 11

Thanks for the comment. I modified the picture and the sentence in the updated version of the paper.

Line 20

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 34

Thanks for the comment. I modified the sentence in the updated version of the paper. Also, I deleted the vague part.

Thanks for the comment. Modifying classical NURBS surfaces, e.g. adding more control points, will influence an entire row or column of control points (Botsch et al. 2010). Indeed, preserving the grid structure of NURBS surfaces during local refinement is challenging (Fig. 1) (DeRose et al. 1998).

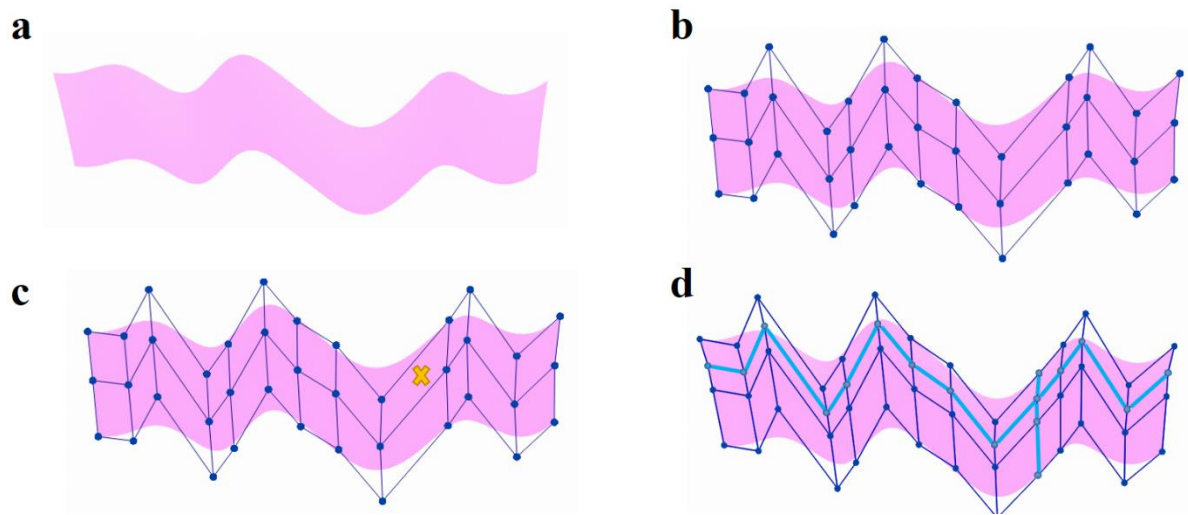



Fig. 1 Representation of the third limitation of NURBS surfaces: adding new control points affects the entire rectangular grid of control points. **a** Smooth NURBS surface with 39 control points. **b** Rectangular grid of control points (3 rows and 13 columns). **c** Considering the position of a new control point (yellow multiple). **d** Adding a new control point at a specific place increases the number of rows and columns to 4 and 14, respectively.

Thanks for the comment. I added the works of Lévy and Mallet (1999) and Mallet (2002) to the updated version of the paper.

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 46

Thanks for the comment. I want to indicate "supporting arbitrary topology".

Line 51

Thanks for the comment. You are right. I modified the sentence in the updated version of the paper.

Line 53

Thanks for the comment. You are right. I modified the sentence in the updated version of the paper.

Line 54

Thanks for the comment. You are right. I modified the sentence in the updated version of the paper.

Line 56

Thanks for the comment. You are right. I modified the sentence in the updated version of the paper.

Line 66

Thanks for the comment. You are right. I modified the sentence in the updated version of the paper.

Line 75

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 78

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 80

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 82

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 84

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 95

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 101

Thanks for the comment. I modified the word in the updated version of the paper.

Line 112

Thanks for the comment. I modified the reference in the updated version of the paper.

Line 117

Thanks for the comment. I modified the word in the updated version of the paper.

Line 118

Thanks for the comment. I added all of the references to the updated version of the paper.

Line 125

Thanks for the comment. I modified the word in the updated version of the paper.

Line 128

Thanks for the comment. I modified the word in the updated version of the paper.

Line 130

Thanks for the comment. I modified the word in the updated version of the paper.

Line 132

Thanks for the comment. The "q" is already showen on the Fig. 4.

Thanks for the comment. Three dots (…) indicate that we can have more triangles in between.

Thanks for the comment. The word "more" is extra. I deleted the word in the updated version of this paper.

Thanks for the comment. You are right. We added more figures in the updated version of this paper which consists of geological examples.

Thanks for the comment. I added "the" in the updated version of the paper.

Thanks for the comment. I modified the word in the updated version of the paper.

Thanks for the comment. I mean, the classic subdivision surfaces cannot support arbitrary topology. I added the "*classic*" in the updated version of the paper.

Line 165 to 205

Thanks for the comment. I already updated this section in the updated version of the paper.

Line 214

Thanks for the comment. I modified it in the updated version of the paper.

Lines 224 and 227

Thanks for the comment. Yes, this includes moving from non-watertight non-manifold mesh to watertight non-manifold mesh.

Line 230

Thanks for the comment. I modified it in the updated version of the paper.

Line 233 to 251

Thanks for the comment. I already updated this section based on your comments in the updated version of the paper.

Line 265

Thanks for the comment. We could just generate each surface separately by using Pyvista, which consequently results in 18 different surfaces and 10 volumes.

Line 273

Thanks for the comment. I deleted the sentence to avoid misunderstanding the reader (it was not necessary).

Line 278

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 289

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 290

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 292

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 295

Thanks for the comment. I modified it in the updated version of the paper.

Line 298

Thanks for the comment. I updated the image in the updated version of the paper.

Line 302

Thanks for the comment. It is compared to the input mesh. I modified it in the updated version of the paper.

Line 313

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 314

Thanks for the comment. However, to avoid misunderstanding, we prefer to keep the sentence.

Line 315 to 322

Thanks for the comment. I modified this part in the updated version of the paper.

Line 326

Thanks for the comment. I deleted the "carefulness" in the updated version of the paper.

Line 329 to 333

Thanks for the comment. I modified this part in the updated version of the paper.

Line 336

Thanks for the comment. I added the "control mesh" in the updated version of the paper.

Line 339

Thanks for the comment. I modified this part in the updated version of the paper.

Line 341

Thanks for the comment. I modified this part in the updated version of the paper.

Line 343

Thanks for the comment. I modified this part in the updated version of the paper.

Line 349

Thanks for the comment. I modified this reference in the updated version of the paper.

Line 353

Thanks for the comment. I modified this sentence in the updated version of the paper.

Line 355

Thanks for the comment. I modified this sentence in the updated version of the paper.

Line 361

Thanks for the comment. The input data can be watertight or non-watertight. In the updated version of the paper, I added one example (anticline example), which is not watertight and showed how we generated watertight control mesh and consequently reconstructed it.

Line 364

Thanks for the comment. You are right. I deleted this sentence since it sounded vague, and maybe it is not always correct.

Line 385

Thanks for the comment. I took all references from google scholar and added them with EndNote software. For this reference, this is the only version.

Lines 408

Thanks for the comment. I modified the reference in the updated version of the paper.

Lines 436

Thanks for the comment. I modified the reference in the updated version of the paper.

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