Revisions Based on Comments of Reviwer 1.

I have carefully addressed all comments and suggestions made by you and revised my paper thoroughly. I hope you find the updated version satisfactory and suitable for publication. Please also note that for more clarification, all the changes applied to the paper are highlighted with yellow colour.

Page numbers are based on the p aper **before** revision.

Major Revisions:

1- "…However, There are some major issues with the paper in my opinion. The paper needs to become more distinctive from the Mathematical Geosciences paper from the same team with same lead author. This will be a challenge because most of what this current submission to GMD offers has already been presented in Math Geosci, but does have the addition of Python demonstration codes to show the utility of the methods."

Thanks for this comment. You are right. In the introduction of the updated version of this paper, we mentioned the major difference between this paper and our previous paper. The main difference is that in this paper, we implement the automatic fitting for reconstruction using the augmented Lagrangian method. However, in the previous paper, we used manual fitting. Also, in the new paper, we explained more about the code behind the software.

2- "…state more clearly and concisely why we need these tools in the community for geological applications b) while making the code more accessible, and with a bit of effort on c) better user instructions."

Thanks for this comment. You are right. In the updated version of the paper. We mention the importance of this method (introduction). Besides, we added more information about the code and how to use it.

3- "The paper tends to get into a lot of details without saying why Sub-division surfaces are so important and why our current state of the art (Implicit?) tools is not properly handling these situations."

Thanks for this comment. Jacquemyn et al. (2019) comprehensively explain the advantages of surface-based modelling compared to grid-based representation in geological modelling.

Jacquemyn, C., Jackson, M. D., & Hampson, G. J. (2019). Surface-based geological reservoir modelling using grid-free NURBS curves and surfaces. *Mathematical Geosciences*, *51*, 1-28.

We also addressed their work by mentioning that "Several studies considered a surface-based approach in geological modelling since the outstanding features of the structure (e.g. heterogeneity) are explicitly demonstrated by the surfaces of the boundary (Jacquemyn et al. 2019).".

Besides, we already explained the importance of subdivision surfaces in our previous paper and addressed that paper in this paper as a reference. Since the current paper is about PySubdiv software, we tried to focus mainly on explaining the software while we addressed other references for more details.

4- Why do we need to preserve sharp edges in some cases and smooth edges in others?

Thanks for this comment. We wanted to introduce the tools (subdivision surface method) to the user. Using or not using sharp or smooth edges mostly depends on the engineering judgment of the user. If the user thinks he/she wants to have more sharp edges (e.g., modelling sharp fault bend) or smooth edges, he has the tools. Also, sharp edges are convenient for representing piecewise smooth surfaces.

I agree that real-world surfaces are never infinitely sharp (DeRose et al. 1998, Lavoué et al. 2007). So we may need semi-sharp creases for modeling. In general, it depends on the user and what he wants to mode (sharp, semi-sharp or smooth).

5- Also importantly, when the whole geological modelling community is moving to implicit surfaces (level set derived) are we going to go back to explicit approaches, even if it means we get some accuracy benefits?

Thanks for this comment. We are aware of the importance of implicit modelling. Both Implicit and explicit modelling have their advantages and disadvantages. However, we are not quite sure if the "*whole geological modelling community is moving to implicit surfaces*". Maybe one of the most important papers that motivated us to focus on this topic was the paper of Jacquemyn et al. (2019), which is the winner of "AMG Mathematical Geosciences 2019 Best Paper Award"

Jacquemyn, C., Jackson, M. D., & Hampson, G. J. (2019). Surface-based geological reservoir modelling using grid-free NURBS curves and surfaces. *Mathematical Geosciences*, *51*, 1-28.

This paper is completely on surface-based modelling with NURBS, which is not a new method. There is no doubt that implicit modelling is a very good method. I saw your paper with Mike and Florian Wellmann (my supervisor and colleague in

PySubdiv project) on implicit modelling is “the winner of the Mathematical Geosciences 2021 Best Paper Award" **Congratulation Erik**.

Some words from Jacquemyn et al. (2019) about the advantages of explicit modelling.

“…it is challenging to represent many common geological features using pillar grids, including faulted domains and non-monotonic surfaces (e.g. recumbent folds, diapir flanks and the margins of intruded or injected bodies). Rock types with diverse petrophysical characteristics are “averaged” within grid cells of arbitrary size and shape, and the continuity and connectivity of low-permeability baffles and barriers or high-permeability zones is often lost. Stair-stepping effects are introduced by cornerpoint grids around any feature that is not aligned to the grid orientation. Such effects are common around faults, but also impact, for example, the continuity and connectivity of sinuous channels…”

6- So just a better context would be good. This is mostly in the Math Geosc paper but could be restated in this submission in a clear and unique manner that motivates the reader to continue.

Thanks for this comment. **You are right**. We implemented major changes to this paper and tried to make it more clear and more understandable for the reader.

7- I have tried to test all the python codes with limited success. The most crucial point was to do interactive editing and updating of the edge values (CSV) for crease sharpness (sharp -1 to smooth-0). I tested with Windows 10, using Pycharm, Anaconda, (Python v3.9 in virtual environment). It is important to provide working code to the perceived user community, which in my opinion would be dominantly windows users. It would also be helpful if a more detailed tutorial could be provided.

Thanks for this comment. **You are right. Please accept my apology**. PyVista uses PyQt for GUI, and unfortunately, due to the limitation of PyQt, the software cannot work well in Windows, Mac and Conda enviroment. Currently, the software can work well in Linux without the Anaconda. We really did our best to remove these limitations. The institute plans to improve the software in the near future by absorbing more sources of funding and hiring more people to help.

8- If the authors choose to update the useability of their codes, it will widen the user community for these tools and provide better discussions for those wishing to implement their proposed solutions. All the SubDivision surface codes performed well but only generated smooth refined meshes or coarse grip frames. Not composite straight and smooth non-manifold object types. This is a big stumbling block at this stage since the users have no way to fully evaluate the utility or advantages of the code without a working application.

Thanks for this comment. Yes. **You are right. We are sorry for it**. In the updated version of the software, we explain more about the details of geological modelling inside the software. Also, We generated a tutorial video about the code, and we hope this will be helpful. Also, we cleaned the code and made it better for the users.

Other comments:

Line 23

Thanks for the comment. The geological model can be constructed by various methods, e.g. marching cube from the implicit model. In this paper, the goal of surface reconstruction is to make the geological model manageable with a few numbers of control points. Of course, it may associate with re-meshing. However, re-meshing is not the goal of reconstruction in this paper.

Line 25

Thanks for the comment. In this paper, the problems known from spline surfaces mainly refer to (1) the limitation of the spline surfaces in supporting arbitrary topology (Cashman 2010) and (2) the requirement of stitching multiple patches (e.g., NURBS patches) for generating the complex structures (Botsch *et al.*, 2010). We already mentioned the problems with related references in lines 38,39, and 40.

Line 27

Thanks for the comment. The goal is to generate the "control mesh" for each geological model. The control mesh contains some control points that can help to investigate the uncertainties of different geological scenarios and, consequently, solve inverse problems. For example, if we have uncertainties in one location of the model, we can assign one control point to that location and generate the control mesh. Then, by changing the control point's location, we can generate different models and evaluate the uncertainty. However, to make the sentence more clear, i deleted the word.

Line 30

Thanks for the comment. We added the references which you mentioned to the updated version of this paper.

Line 31

Thanks for your comment. You are right. However, to reduce the number of pages of the paper, we decided to focus mainly on subdivision surfaces.

Line 34

Thanks for the comment. We implemented your comment. It is worth mentioning that we already mentioned the differences between the NURBS and subdivision surfaces in the following lines. However, to make it more clear, I explain it here.

Line 40

Thanks for the comment. We mentioned the definition of "topology" just to show that when we say "subdivision surfaces support arbitrary topology", what we are speaking about. Speaking about "heat transport" or "fluid flow" is beyond the scope of this paper. We just bring these examples to stress the importance of supporting the arbitrary topology. We deleted this part to avoid misunderstanding the topic.

Line 43

Thanks for the comment. You are right. However, due to several references (e.g., Botsch et al., 2010), subdivision surface method is one of the best methods to tacked the limitation of NURBS, which has received limited attention in geological modelling. Also, the subdivision surface method has more features (e.g., supporting non-manifold topology), making it attractive for geological modelling.

Line 44

Thanks for the comment. We implemented the definition of subdivision surfaces in the updated version of this paper. It is worth mentioning that we already mentioned the differences between the NURBS and subdivision surfaces in the previous lines. It is just two lines since we investigated the advantages and disadvantages of both NURB and subdivision surfaces in our previous paper (Moulaeifard et al. 2023).

Line 49

Thanks for the comment. We comprehensively explained the (1) subdivision surfaces and NURBS methods and (2) the main differences in geological modelling in our previous paper (Moulaeifard et al. 2023). Therefore, in this paper, we just shorty explain them and mostly focus on automatic reconstruction, which is the nobility of this paper.

Also, regarding the DSI, we are pretty aware of the importance of this method which is already implemented in GoCad. In the updated version of this paper, we already spoke about the DSI in the introduction. The goal of reconstruction in this paper is to find the control mesh with **few** control points, which helps us to control the reconstructed mesh. Lévy and Mallet (1999) in their paper (section 3.3), explain how we can use DSI in each iteration of subdivision surfaces while all vertices, except (control nodes), are free to move in each iteration. Also, Lévy and Mallet (1999)mention that "the convergency of the obtained meshes has always been observed in practical application, **but the formal proof of this convergency is still is am an open problem**". There is uncertainty about the convergency when we want to use DSI during subdivision surfaces which is one of the reasons that we did not used DSI in our project. We selected the loop algorithm since the convergence of the series of refined meshes is formally proved (Loop 1987).

Line 57

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 64

Thanks for the comment. I modified the sentences in the updated version of the paper.

Line 95

Thanks for the comment. I modified the sentences in the updated version of the paper.

Line 112

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 113

Thanks for the comment. I modified the sentence in the updated version of the paper.

Line 118

Thanks for the comment. I added all of the references to the updated version of the paper.

Line 141

Thanks for the comment. You are right. I modified the sentence and added Table 1 and Table 2 to avoid misunderstanding. Apparently, zero is full smoothing, and one is sharp. However, it is worth mentioning that each vertex is connected to multiple edges and concequently should follow the Table 2. I used the word "resistance" to make it more understandable for the reader.

Line 155

Thanks for the comment. We explained the subdivision surfaces comprehensively in our previous paper (Moulaeifard et al. 2023), which is addressed in this paper. The current paper focused on the reconstruction procedure, which is the main difference between the two papers. We prepared an instruction for modelling inside the software. Our software is very young; unfortunately, it works only in Linux systems.

Line 156 and 161

Thanks for the comment. You are right. We implemented more figures and explanations for the "non-manifold topology" section in the paper's updated version.

Line 165

Thanks for the comment. You are right. We implemented more figures and explanations for the "definition of the reconstruction and algorithm" in the paper's updated version.

Line 167

Thanks for the comment. You are right. I corrected the word in the paper's updated version.

Line 173

Thanks for the comment. You are right. I modified this part in the paper's updated version.

Line 174

Thanks for the comment. I understand your concern regarding the word "control points". However, since the subdivision surfaces method originated from computer graphics, we wanted to preserve the name (due to several GC references) to have a common language between computer graphics researchers and us. Also, paying attention to the other words that came with "control point" can help us to understand the meaning. For example, when the "control points" come with a "control mesh".

Line 191

Thanks for the comment. b is the coefficient for optimization. The user defines it.

Line 192

Thanks for the comment. Since we got the equations of this section from (Wu et al. 2017), we wanted to preserve the notation and definition of the symbols.

Line 192

Thanks for the comment. The penalty term adds a cost for violating each constraint (violation of both crease sharpness value and position of the control points).

Line 198

Thanks for the comment. I modified it in the updated version of the paper.

Line 205

Thanks for the comment. I modified it in the updated version of the paper.

Line 208

Thanks for the comment. I modified it in the updated version of the paper.

Line 212

Thanks for the comment. I modified it in the updated version of the paper.

Line 215

Thanks for the comment. We use Blender to render the final mesh (better quality in rendering) or sometimes to generate triangular input mesh for our software. Blender is not used inside in PySubdiv.

Line 218

Thanks for the comment. I added a better explanation in the updated version of the paper.

Line 260

Thanks for the comment. You are right. However, since the subdivision surface method originated from computer graphics, we wanted to preserve the name (due to several GC references) to have a common language between computer graphics researchers and us.

Line 272

Thanks for the comment. I modified it in the updated version of the paper.

Line 278

Thanks for the comment. We added more explanation ins section 3.1.2 of the updated paper. Hoppe et al. (1994), Wu et al. (2017) offer to consider a threshold angle (θ_0) as a criterion for tagging the edges of the initial control mesh. For each edge, the edge is tagged as sharp (crease sharpness value equal to one) if the angle between the normal of two adjacent faces (θ_e) is more than the threshold angle ($\theta_e > \theta_0$). It is worth reminding that this value is just the initial value and may be changed during the optimization.

Page 14 (top comment)

Thanks for the comment. We added a new section (section 3.1) in the updated version of the paper to show how the software works, such as generating the control mesh (3.1.1), setting the crease sharpness values (3.1.2) and reconstruction (3.1.3).

Regarding the numbers, the input data of URG model is in a grid-point format URG model, which contains 616464 individual nodes. One has to connect the points to get the mesh of the input data.

Control mesh and control points (vertices of the control mesh) are generated inside the PySubdiv software, and the goal is to manage the mesh with a few control points. There is a big difference between control points and grid points since changing the location of the control points makes smooth changes on the surfaces. However, the Grid-points are just the input data.

Line 295

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 303

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 309

Thanks for the comment. I understand your concern regarding the word "control points". In order to avoid misunderstanding, I modified the sentence by adding "control points of the control mesh". I think w cannot really change the word "control points" since the subdivision surface method originated from computer graphics (due to several GC references) and concequently, for having a common language between computer graphics researchers and us.

Line 342

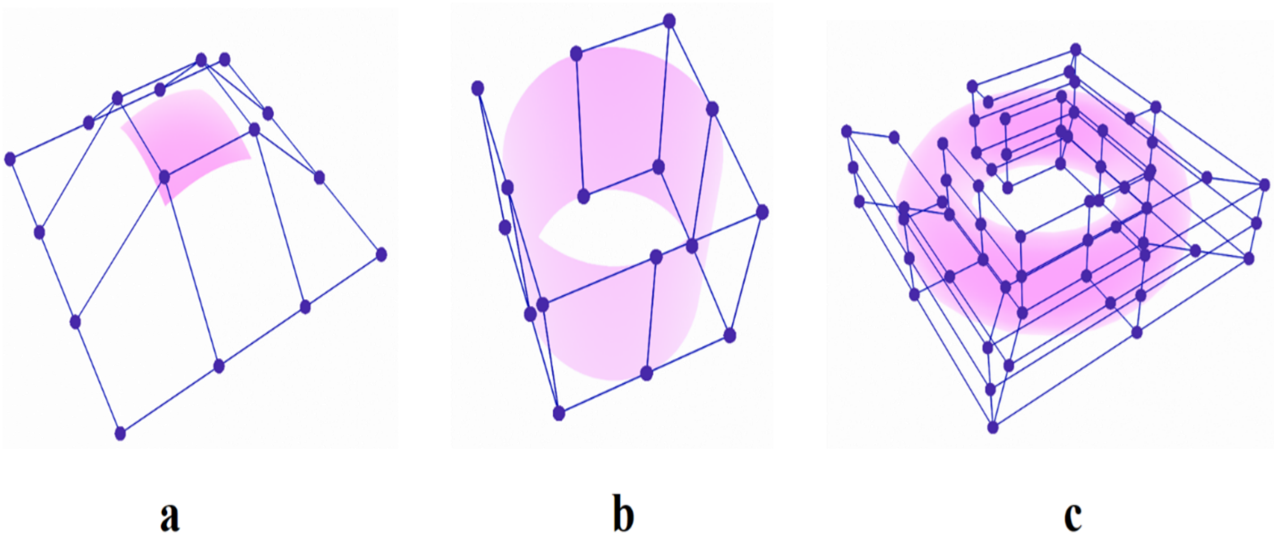
Thanks for the comment. As mentioned in line 340, the "iteration" is related to the subdivision algorithm (not the iteration inside the reconstruction algorithm). First, we have one control mesh, and then we apply the subdivision surfaces (Loop scheme), which results in the generation of smooth mesh. Then, we will use this smooth subdivided mesh inside the algorithm for reconstruction. The algorithm itself contains several iterations in which the user does not specify the number (we have one while loop to reach the best fit). By increasing the number of iterations of the subdivision methods (not the iteration inside the algorithm), the smooth mesh will have more vertices. Therefore by importing the smooth mesh inside the reconstruction algorithm, we have to deal with more vertices and, consequently, more iteration inside the reconstruction process.

Line 342

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Line 345

Thanks for the comment. Here, the word "grid" indicates the grid structure of the spline surfaces (Figure below).



To create a model with a complex topology, many NURBS patches have to be smoothly connected (by stitching NURBS patches together). Multiple connections between surface patches, in addition to topological or geometrical constraints, make the modelling procedure difficult (Botsch et al. 2010, Cashman 2010). This grid I think this grid is different from the grid which we use for flow simulation or FEM. With both NURBS and subdivision surfaces, we generate our mesh (boundary of the object), and then we can import it into any grid-based system, e.g. FEM, for other processing computations.

Line 364

Thanks for the comment. You are right. I deleted this sentence since it sounded vague, and maybe it is not always correct.

Line 378

Thanks for the comment. You are right. I modified it in the updated version of the paper.

Lines 402 and 402

Thanks for the comment. I modified the references in the updated version of the paper.

Botsch, M., L. Kobbelt, M. Pauly, P. Alliez, and B. Lévy. 2010. Polygon mesh processing. CRC press.

Cashman, T. J. 2010. NURBS-compatible subdivision surfaces. Cashman, Thomas J.

DeRose, T., M. Kass, and T. Truong. 1998. Subdivision surfaces in character animation. Pages 85-94 *in* Proceedings of the 25th annual conference on Computer graphics and interactive techniques.

Hoppe, H., T. DeRose, T. Duchamp, M. Halstead, H. Jin, J. McDonald, J. Schweitzer, and W. Stuetzle. 1994. Piecewise smooth surface reconstruction. Pages 295-302 *in* Proceedings of the 21st annual conference on Computer graphics and interactive techniques.

Jacquemyn, C., M. D. Jackson, and G. J. Hampson. 2019. Surface-based geological reservoir modelling using grid-free NURBS curves and surfaces. Mathematical Geosciences **51**:1-28.

Lavoué, G., F. Dupont, and A. Baskurt. 2007. A framework for quad/triangle subdivision surface fitting: Application to mechanical objects. Pages 1-14 *in* Computer Graphics Forum. Wiley Online Library.

Lévy, B., and J.-L. Mallet. 1999. Discrete smooth interpolation: Constrained discrete fairing for arbitrary meshes. Available on www at http://www. loria. fr/levy/Papers/1999/s99dsi. pdf.

Loop, C. 1987. Smooth subdivision surfaces based on triangles.

Moulaeifard, M., F. Wellmann, S. Bernard, M. de la Varga, and D. Bommes. 2023. Subdivide and Conquer: Adapting Non-Manifold Subdivision Surfaces to Surface-Based Representation and Reconstruction of Complex Geological Structures. Mathematical Geosciences **55**:81-111.

Wu, X., J. Zheng, Y. Cai, and H. Li. 2017. Variational reconstruction using subdivision surfaces with continuous sharpness control. Computational Visual Media **3**:217-228.