Point to point response to reviewer RC1

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We thank the reviewer for helpful and constructive comments. Below you will find detailed responses to all the reviewer's comments. The reviewer's comments are highlighted in red, our comments are in black.

The manuscript describes a modeling approach of the flow in unsaturated sand in order to explain so-called Bauters' paradox. A semi-continuum model approach is presented that can imitate the finger flow phenomena with the oversaturation in the tip and the whole transition towards a diffusion-type infiltration plume as function of the initial soil moisture condition. The model was successfully tested by description of the Bauters' experiment.

General Comments: The manuscript tackles an important problem that is in the focus of the journal. Methods and results are novel and can much contribute to progress in describing unsaturated flow phenomena.

Major issue 1: My main critical comment is that for in-depth understanding the physical basis of the modeling approach and finally also of the results, a better explanation of the semi-continuum's model concept would help.

A detailed explanation of the concept of the semi-continuum model is published in [1]. However, we agree with the reviewer that a better explanation of the semi-continuum model will improve the manuscript. Since the reviewer's comment (**major issue 3**) addresses the same issue, we refer to the explanation in the response to **major issue 3**.

Major issue 2: In addition, I found the introduction is much too far reaching, of course, everything is somehow connected. The review on global water cycle, the flow phenomena, sample volume dependency, REV concepts and Richards' equations, and other aspects, all does not help much to better understand the problem, and are not discussed later any more.

We will update the introduction. First, the review of global water cycle is not relevant for the main point of the manuscript, and therefore the first paragraph in the Introduction will be replaced by the following text: "Infiltration of rainwater into soil forms an essential part of the hydrological cycle. Therefore, research on the movement of water in soil has long been a focus of attention. The origins of infiltration research were substantially influenced by the idea to describe the movement of water in soil by diffusion-like models [2]. Later, it was discovered that – even in homogeneous porous materials – flow may become spatially very inhomogeneous. Most of the infiltrating water flows through preferential pathways (fingers) leaving islands of dry material behind. In the fingers, the so-called saturation overshoot often occurs – the finger tip becomes much more saturated than the finger tail. This type of flow cannot be described by the diffusion-like models [3]. However, it is well described by a semi-continuum model introduced in [1]. In this paper, we demonstrate that this semi-continuum model captures infiltration into an unsaturated homogeneous porous medium comprehensively, in the sense that it correctly describes the experimentally observed complicated transition between the finger-like and diffusion-like flow regimes."

Second, sample volume dependency, the concept of REV, Richards' equation, etc. are closely related to the semi-continuum model, specifically the scaling of the retention curve. This will be better explained in the manuscript and the introduction will be improved accordingly. Please, see the response to **major issue 3**, where this is explained in detail.

Major issue 3: What I did not understand was the semi-continuum model concept, especially what is different from a numerical discretization of a continuum model? Perhaps you did it already in other papers. It seems relatively simple and more empirical because of the block size selection and the scaling relations. Maybe it helps to include an illustration? The idea of scaling the retention function with block size is also unclear to me. I did not read the cited references, but the present paper seems to apply the previously developed model concept to describe the specific experiment, yes?

Let us first answer the last question: *I did not read the cited references, but the present paper seems to apply the previously developed model concept to describe the specific experiment, yes?* Indeed, this paper uses a previously developed semi-continuum model to describe the specific experiments of Bauters et al. [4]. For clarity, this statement will be included in the manuscript. We chose the experimental results of Bauters et al. because no other model has been able to reproduce them so far.

Another comment by the reviewer relates to understanding the semi-continuum model concept that is closely related with the scaling of the retention curve (see Figure 1 in the manuscript). Understanding this concept may not be clear, so we provide a brief discussion below that we plan to include in the manuscript. However, the detailed mathematical and physical justification is already published in [1], hence for a deeper understanding we refer to this paper.

The scaling of the retention curve, i.e. the dependence of the capillary pressure-saturation relation on the block size, is not a common approach in flow modelling. However, the dependence of the experimentally determined retention curve on the porous medium sample size has been observed for a long time [5, 6, 7, 8, 9, 10]. The concept of REV is essential in this case because if the sample of porous medium is smaller than REV, key physical quantities, such as the retention curve, are strongly dependent on the sample size. The crucial idea of the semi-continuum model is to include this dependency in the model, i.e. to scale the retention curve according to the block size. In the semi-continuum model, a block represents a real sample of the porous material. This makes the semi-continuum model fundamentally different from numerical schemes for solving partial differential equations where the block plays only a discretization (i.e. mathematical) role and regardless of the block size, the retention curve remains the same. In the semi-continuum model, the computational mesh (the blocks) takes into account the dependence of the physical parameters on the size of the blocks. Surprisingly, the idea of taking REV size into account in modelling porous media has been around for a long time. For instance, in [11], the authors estimated the size of the REV and used it as a lower limit for the size of the finite elements. They argue that the use of smaller elements would lead to violation of continuum assumptions and thus the continuum approximation would no longer be appropriate. The same idea is used in the semi-continuum model: For blocks smaller than the REV, scaling of the retention curve must be included because the continuum approximation is no longer adequate. Because we are interested in the description of flow phenomena below the REV scale, we need to include the dependence of the retention curve on the block size. This scaling of the retention curve must meet a physically justified requirement that the nature of the flow is preserved across all levels of block size. This means that the fluxes between neighboring blocks must not change when Δx changes. Given equation (4) in the manuscript, if Δx decreases by half, the fluxes increase by a factor of two if the scaling of the retention curve is not included. Therefore, a linear scaling of the retention curve is introduced in equation (6) in the manuscript, so the fluxes between blocks remain the same as Δx decreases. For more details, see figures Fig. 4–6 in [1] that show the numerical convergence of the semi-continuum model in 1D and 2D.

The natural question is what the limit of the semi-continuum model would be as $\Delta x \to 0$. We tried to answer this question in [1] and derived the limit equation in a single spatial dimension:

$$(K_{PS}\partial_t S - \partial_t P_H)(P_H - v) \geq 0, \qquad \text{for all } v \in [C_2, C_1], \qquad \text{and } P_H \in [C_2, C_1],$$

$$\theta \partial_t S + \partial_x \left(\frac{\kappa}{\mu} \sqrt{k(S^-)} \sqrt{k(S^+)} (\rho g - \partial_x P_H) \right) = 0, \qquad S^{\pm}(x_0, t) = \lim_{x \to x_0^{\pm}} S(x, t).$$

In this equation, κ denotes the intrinsic permeability, ρ the fluid density, g acceleration due to gravity, μ the dynamic viscosity of fluid, and S the saturation. The values C_1 [Pa] and C_2 [Pa] denote the constant limits of the main wetting and draining branches, respectively. The limit is a partial differential equation containing a Prandtl-type hysteresis operator P_H under the space derivative. If we are located on the main wetting or draining branches, the limit equation becomes a hyperbolic differential equation. Between the two main branches (i.e., we are located on the scanning curve), the limit represents a parabolic differential equation. It means the limit switches between parabolic and hyperbolic types of equation. The limit equation is a new type of mathematical model – we are not aware of any research that has investigated equations of this type. Note that the Richards' equation is a parabolic type equation – that is why it is only able to simulate the diffusion-like flow regime [3].

Major issue 4: It is suggested in the manuscript that the initial water saturation is the variable controlling the finger formation. What about the wettability (i.e., surface tension), which is of course connected with water saturation but can change with time?

The experiments by Bauters et al. [4] that we have reproduced in the manuscript were carried out on pure quartz sand. This sand does not change its wettability according to the duration of contact with distilled water.

Therefore, we did not consider the effect of the change in wettability on the flow of water through the sand because this effect is not the cause of the Bauters' paradox.

Overall, this valuable contribution could become even better if more focused and with more specific explanations on the physical basis of the approaches.

Thank you!

References

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