Responses to Referee 1:

We are greatly appreciative of your comments and constructive suggestions that are pretty important to further improve our paper from quality to scientific dissemination. In the following text, we will answer all the questions or comments (in italic with black color) one by one in blue color.

This is the second submission of the paper “Assimilation of sea surface salinities from SMOS in an Arctic coupled ocean and sea ice reanalysis”. In brief, the paper looks at the effect of assimilating the latest version (V3.1) of SMOS surface salinity data into the Arctic region. It does this by comparing the results to model runs which either did not assimilate SMOS data or used an earlier version (V2.0) of the data. Validation was done against a variety of in-situ sources. The broad conclusion is that the V3.1 data does bring some benefits.

As before, my comments on the manuscript can be found in the accompanying PDF. The paper is much improved from its first version, and I thank the authors for their efforts. The major points from my first review have all been addressed. That being said, the paper still needs some work; in particular I found the wording in the results section to be confused at times. Thus, I am now recommending the paper for minor revisions.

-A: Thanks for this comment. We further improve the paper through the following aspects:

I. Updating Fig. 4 using the black contour line to better visualize the SSS distributions.

II. Improve the illustration about Eqs. 1 and 2 to reduce misunderstandings.

III. Carefully corrected some errors and then the text consistency has been checked.

Line 2: "Sea Surface Salinity", with capital "S"; this is because you are defining the acronym.
-A: Yes, it is corrected.

Line 12: replace “areas” with “area”.
-A: Yes, it is corrected.

Line 14: " SSS not ‘it’. As it is currently worded the ‘it’ actually refers to FWC.
-A: Sorry for leading to this misunderstanding. Actually, it meanings to SSS products.

Line 20: Wording implies that these are the only two studies, which I don't think is true.

No need to add additional references, but please indicate that these are just some of the relevant work.
-A: Thanks for pointing out this. It should be replaced by “(e.g., Johannessen et al., 1999; Stroeve and Notz, 2018)"
Line 21: "Other sources" is vague. I assume you mean rivers and rain; please just say that.
-A: Yes, it is corrected with "precipitation and river flux".

Line 27: Wording again implies that these are the only two relevant systems. Something like "(such as Dombrowsky 2009; Fujii et al 2019)"
Or "(for example Dombrowsky 2009; Fujii et al 2019)" would be clearer.
-A: Thank you for this comment.
In fact, both are community papers involving many different forecast systems. But to avoid possible misleading, it is corrected as suggested "(such as Dombrowsky 2009; Fujii et al 2019)"

Line 30: Again, you imply these are the only two studies.
-A: Yes, it is corrected with "(e.g., Storto et al., 2019; Uotila et al., 2019)".

Line 41: Captial 'S's as you are defining the acronym.
-A: Yes, it is corrected.

Line 42: delete "comparatively new but"
-A: Thanks for this comment. We prefer to keep it.

Line 45: Here and elsewhere. I think you are miss using the term 'brightness temperature'. Brightness temperature depends on frequency, and SMOS measures brightness temperatures in multiple different frequency bins. It's not just one value as you imply here.
-A: Thank you for this comment. To avoid possible misunderstandings for the readers, it is corrected by "brightness temperatures (T_B) of the sea surface at different frequencies."

Line 61: delete "the salinity retrieval"
-A: replaced by “the SSS retrieval"

Line 62: delete "the lower sensitivity of T_B in cold waters leading to larger SSS error".
-A: Yes, it is replaced by “the lower sensitivity of T_B to salinity in cold waters”.

Line 69: delete “the”.
-A: It should be kept due to the reference SSS from Argo data.

Line 69: You need to define what Argo is - give a reference.
-A: Yes, one reference is added.

Line 71: delete “the”.
-A: Yes, it is corrected.

Line 76: delete “observations”.
-A: Yes, it is corrected by “These two SSS observation sets”.

Line 77: delete “its upgradated product is available to see Greiner et al., 2021”.
-A: It is replaced by “Greiner et al. (2021) describe an updated version of this product”.

Line 79-80: delete two “ref.”
-A: Yes, it is corrected.

Line 87: replace “resolutions” with “resolution”.
-A: Yes, it is corrected.

Line 87: delete “were”.
-A: Replaced by “was”.

Line 88: delete “shows”.
-A: Replaced by “has”.

Line 88: delete “the”.
-A: Yes, it is deleted.

Line 100: delete “identical to”.
-A: Thanks. We prefer to keep it.

Line 120: delete “with”.
-A: Replaced it with “at”.

Line 121: Reference needed for “ERA-Interim”.
-A: Yes, one reference is added “(Dee et al., 2011)”.

Line 125: delete “in”.
-A: Replaced it with “from”.

Line 127: Given how important the domain of the model is to this work, I really think you need to show it. A reader shouldn't need to dig out another paper for this information. Is it not the domain shown in your already existing figures? If it is you could just point to them.
-A: Thank you for this comment. To help the reader quickly perceive the model domain, we have modified it as “The model covers the Arctic Ocean and the whole north Atlantic Ocean (shown in Fig. 1 in Xie et al., 2017)”

Line 130: delete “WAO2013”.
-A: Corrected by “WOA2013”.

Line 133: Please elaborate on why this might be a problem for “stable surface freshwater layers”.
-A: Due to approaching the climate state of salinity, it happens some excessive desalination of the surface seawater locally in the Arctic.

Line 137: Equation 1 cannot be correct. Firstly if H is a linear operation then y-HX would be vector minus a matrix, which you cannot do. Secondly, If H(X) is somehow a vector then dy-H(X) would be a column vector and, so Kd would also be a column vector which you are trying to add to a matrix.

The correct formulation for the DEnKF would be x' a = x' f + K(y-Hx' f) Where I have used prime to indicate mean, or x ai = x fi + K(y i-Hx fi)

For the EnKF, where i indicates the ensemble member. Also, X, y, and K should be bold and not italic. As should H if linear. If H is not linear then it should be written as a function of x not as a matrix operator.

Lastly, you should give an equation showing how the ensemble is involved. I.e K is a function of (X-X')(X-X')T.

-A: Thanks for this correct comment. As the previous explanation, Eqs. 1-2 are only used to quickly draw a picture for the reader about the relationship between the DA step and the model integration. They are not the specific equations used in DEnKF. In addition, all the runs took the same method, so the detailed illustration about DEnKF could be out of the main topic. So I would not repeat many equations presented by other references. To avoid some misunderstanding, this paragraph has been separated into two paragraphs.

More explanation is added, “For the ensemble data assimilation method, the matrix X includes the dynamical members of the model states as different columns and further evolves according to Eqs. 1 and 2 as described in the DEnKF.”

And the rephased sentence, “Like other square root versions of the Ensemble Kalman Filter, the DEnKF splits Eq. 1 into two steps: the K calculation is applied to the ensemble mean, and the anomalies are updated to match a target analysis covariance (more details in Sakov et al., 2012).”

Line 139: You also need to mention that the matrix columns are the ensemble members for “all 3-D and 2-D variables needed by the model forward integration”.

-A: As the above-mentioned, the X term is only one model state. Afterward, when the ensemble DA method is introduced, the X term in DEnKF contains all model members at the same time.

Line 143: delete “matching”.

-A: Corrected by “of”.

Line 145: delete “on”.

-A: Corrected by “in”.
Line 147: delete “On a weekly basis”.

-A: Corrected by “Using a 7-day assimilation cycle”.

Line 160: I still think Exp0 should be called CNTL.

-A: Thanks for this comment. We prefer to use the current names for all experiments.

Line 162: delete “the”.

-A: It should be better to keep “the” because it indicates the specific SSS products.

Line 165: delete “The”.

-A: Yes, it is deleted.

Line 165: delete “Too”.

-A: Corrected by “too”.

Line 166: delete “The”.

-A: Keep it due to it being related to the satellite-based SSS.

Line 167 and 172: delete “Passive Microwaves”. This is the second time you have written “Passive Microwaves”. It should be “passive microwave instruments”, “passive microwave sensors” or something similar. “Passive Microwaves”, sounds like you are talking about the EM wavers themselves, when you are actually talking about measurements. Also, only upper case for the first letter in a sentence, proper nouns, or when defining an acronym. “Passive Microwaves” is none of these things.

-A: Thank you for this suggestion. The relevant “Passive Microwaves” are corrected by “passive microwave instruments”.

Line 182-184: You contradict yourselves here. Table 1 indicates that the obs errors come completely from equation 3; but here you state otherwise. In which case table 1 is incorrect and needs to be changed. Also this is a key part of your experimental setup so I think you should be more explicit on what you have done rather than relying on the Sakov reference.

-A: Thank you for this comment. It is a misunderstanding for Table 1. The Eq. 3 only defines the SSS observation error and is cited in Table 1. However, the observation errors for other assimilated observation types are different from Eq. 3. Furthermore, the concerned sentence can be rewritten as “The settings for all other observation types are identical to those applied in Xie et al. (2017).”

Line 192-193: “location check to ensure observation in the water grid same as the model used;” Badly written, rewrite. I think you are describing a land check, but I could very easily be wrong.

-A: It is rewritten as “location check to remove SSS observations in the model land mask”.

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Line 242: “After skipping the diurnal signals in observed surface salinity”. Please define more clearly what you actually do about diurnal cycles. I don’t know what you mean by “skipping”.
-A: It is rewritten as “After filtering the diurnal signals by daily averages of observed surface salinity”.

Line 244-247: “All the assimilation runs use a weekly assimilation cycle: The model runs forward 7 days after each assimilation step and provides daily averages for each day from the ensemble mean, which we refer to as “forecast” even when using delayed-mode observations and atmospheric forcings.” You’ve already covered this in the DA section, so you don’t need to say it again here.
-A: Thank you for this comment. Deleted this sentence.

Line 250: Vectors and matrixies should be bold and not italic. 
H should also depend on i.
-A: Thanks for this comment. The format of vectors and matrix is corrected as suggested. The operator H can work with different locations indicated by observations y, which means the fixed function for H. So at different times, H can keep the same format here.

Line 255: delete “model daily average at the observation time as the ensemble means of 100 model members.”
-A: It is rewritten by “the model daily average of the ensemble mean at the observation time”.

Line 276: “the number of assimilated observations in the two runs remains identical”. They are not identical. The gray line exceeds 1.3x10^4 in ExpV3, but does not in ExpV2
-A: Thank you for this point. The relevant illustration is corrected by “the number of assimilated observations in the two runs reached a maximum in the middle of September (gray lines in Fig. 3). But the maximal number of SSS in ExpV3 shows is higher than in ExpV2.”

Line 279: delete “the”.
-A: Replaced by “a”.

Line 287: delete “become”.
-A: Replaced by “becomes”.

Line 290: delete “present”.
-A: Replaced by “are shown for the control run (Exp0) in August and September 2016, respectively”.

Line 303: delete “by”.
-A: Replaced by “in”.

Line 306: delete “(Fig. 1)”.
-A: Replaced by “(shown in Fig. 1)”.

Line 309: delete “Then”.
-A: Yes, it is corrected.

Line 410: delete “changes of”.
-A: Yes, delete it.

Line 410: delete “from the three runs”.
A: Yes, it is deleted. The sentence is corrected by “the increment for other surface variables will also be interpreted.”

Line 418-419: “should generally decrease because of the presence of a new SSS term in the denominator of the Kalman Gain”. In general this statement is incorrect. If there are no correlations between the SSS and the other variables, then K will be block diagonal and the salinity will not affect the other variables at all.
(My statement is true mathematically; in reality the conditioning would be changed, so numerically you might get a different answer.)

A: Thanks for this comment. In practice, the Arctic SSS is always correlated with other variables and ensemble covariances always return off-diagonal blocks. This sentence is separated into two sentences for easy understanding.

In addition, we modified the statement into “… (the ensemble covariance matrices contain off-diagonal blocks of correlation between SSS and dynamically related variables and so the assimilated observations usually compete with each other). However, the SSS biases originating either from the model or observations also affect the other model variables and increase the innovations on the following assimilation step and thus the consecutive increments”.

Line 420: delete “spill over”. I don’t follow your argument here. It might just be wording. Reword and add further detail about your proposed mechanism.

A: Thank you for this comment. Replaced with “affect”.

Line 422-423: “Hypothetically, if the SSS were the only source of errors in TOPAZ, the increments of other variables should vanish over time”. True, but you are not doing ‘perfect’ experiments here, and there are lots of other error sources. This just seems irreverent.

A: Thank you for this comment. We think we have made our point clearer now: “wherever the forecast errors are caused by SSS errors, the increments of other variables should diminish”.

Line 423: delete “of”.

A: Yes.

Line 443: delete “of”.

A: Replaced by “in”.

Line 449: delete “the”.

A: Yes, it is deleted.

Line 453, 456, 458, and 470: delete “BS”. I recommend writing ‘Beaufort Sea’ here. I know you have defined BS elsewhere, but it is a non-standard acronym and writing it in full makes the readers job easier.

A: Yes, all of “BS” are replaced by “Beaufort Sea”.

Line 480: delete “products will”.

A: Yes, it is deleted. We modify it into “both versions of SMOS satellite-based acts compensates”.

Line 483: “qualitative”, I’m not sure ‘qualitative’ is the correct word here. You have data and you have shown it. You could say that more data is needed to confirm your hypothesis.

A: Thank you for this comment. We replaced “qualitative” by “preliminary”.
Line 486: delete “weekly”.
A: Yes, it is deleted and modified as “the week-long data assimilation”.

Line 491: “remains qualitative”. "not fully verified"
A: Replaced by “not fully verified.”

Line 491: “in better agreement”. You don’t state what you are comparing against. Please state the product that is a worse match to the ITP data than your own.
A: The FWC reaches the maxima in October which is consistent with the seasonal Freshwater storage presented by the ITP data in Fig. 4a of Rosenblum et al. (2021).

So the revised sentence is shown as “the timing of the peak is in better agreement with the seasonal freshwater storage presented by the Ice-Tethered Profiler (ITP) data in Fig. 4a of Rosenblum et al. (2021)”.

Line 491: “ITP”. Undefined acronym.
A: It is corrected by “Ice-Tethered Profiler (ITP)”.

Line 490-494: I found the entire section discussing the ITP data confused. It needs rewording. The sentence is far too long; consider splitting it.
A: Thank you for this comment. The last part of this sentence is replaced by another sentence: “In addition, we also notice that the amplitude of the seasonal FWC seems too small in all experiments in Fig. 11, which can be related to insufficient thick ice in TOPAZ (Uotila et al., 2019).”.

Line 502: delete “the”.
A: Replaced by “a”.

Line 503-504: delete “and without the assimilation of two successive SMOS SSS products from BEC”.
A: Replaced by “with two new reanalysis runs which additionally assimilated two versions of the SMOS SSS products from BEC”.

Line 506: delete “the cruises”.
A: Replaced by “cruise tracks”.

Line 506: delete “the”.
A: Deleted.

Line 520: replace “coasts” with “coast”.
A: Replaced as the suggestion.

Line 534: “more than 9%”. You need to state why you are using 9% as your cutoff. It would seem more sensible to use statistical significance.
A: Thanks for this comment. The additional state is added as “The threshold value of 9% refers to the reduction of SSS RMSD in GS, where the bias reduction is significant (through the t-test) but relatively minimal of the RMSD reductions in ExpV3 in the Arctic marginal seas.”

Line 534: delete “the”.
A: Deleted.

Line 625: The indentation is wrong.
A: Yes, it is corrected.
Line 641: Fix the formatting
-A: Thank you for this comment. The reference format is automatically varied so we would keep its form to ensure these texts could be correctly resolved by different editing platforms.

Line 670: Formatting.
-A: Same as the above.

Line 720: Formatting.
-A: Same as the above.

P25, Table 2: “are reduced by at least 9%”. 9% seems arbitrary. You need to explain why it was chosen. To me it would seem more sensible just stick to the T test results.
-A: Thank you for this comment, it is not fully arbitrary. The threshold value of 9% refers to the reduction of SSS RMSD in GS, where the bias reduction is significant (through the t-test).

P25, Table 2: delete “subscript”.
-A: Replaced by “superscript”.

P26, Fig. 1: This looks like the 'jet' colour map. It has well documented problems (see https://agilescientific.com/blog/2017/12/14/no-more-rainbows). I suggest using something different.
-A: Thank you for this comment. For the concerned illustration of Fig. 1 the purpose is to discuss the extremely low and high salinities, so we prefer to keep it. One of the co-authors is a typical colour-blind and has ensured that the main point of the figure comes well across.

P28, Fig.4: The pink contour is difficult to see against the red background. And the caption of “The black isolines” is not consistent with them.
-A: Yes, it is corrected by the new figure in the revision.

P31, Fig. 7: I still think the outliers need to be removed. I appreciate that in your response to my first review you said that you couldn't find a sensible way to do this. However, just by eye I can see that if you remove the two points to the left of the 29PSU line the r values will increase significantly.
-A: Thank you for this comment. I agree that the datasets could be more significant if ticking off the two outliers. However, all valid observations should be objectively presented, especially without enough physical reasons to refuse them.

P33, Fig. 8: I'm not sure what alpha is. Either remove it or explain it. Applies to other plots as well.
-A: It wants to remind a more strictly significant test 99% also accepted. To avoid a possible misunderstanding, we delete it, which is samelly used in other figures.

P34, Fig. 9: As I said in my previous review. To see changes you should really show Difference plots. You have already done this in Figure 3, it should be done here as well.

Specificially I think this figure should be split in two: a SIC figure and an SST figure. Both figures should show the absolute fields and the difference fields.

If it has to be one figure, then show the absolute fields for Exp0 and difference fields for ExpV2 and ExpV3.
-A: Thank you for this comment. We prefer to keep this current figure which much more highlights the increment changes for SIC, SST.
P36, Fig. 11: "averaged". Please state how the average was calculated? At the moment the reader doesn't know if this is the median, mean, or something else.

-A: It is corrected by "Arctic-wide means".