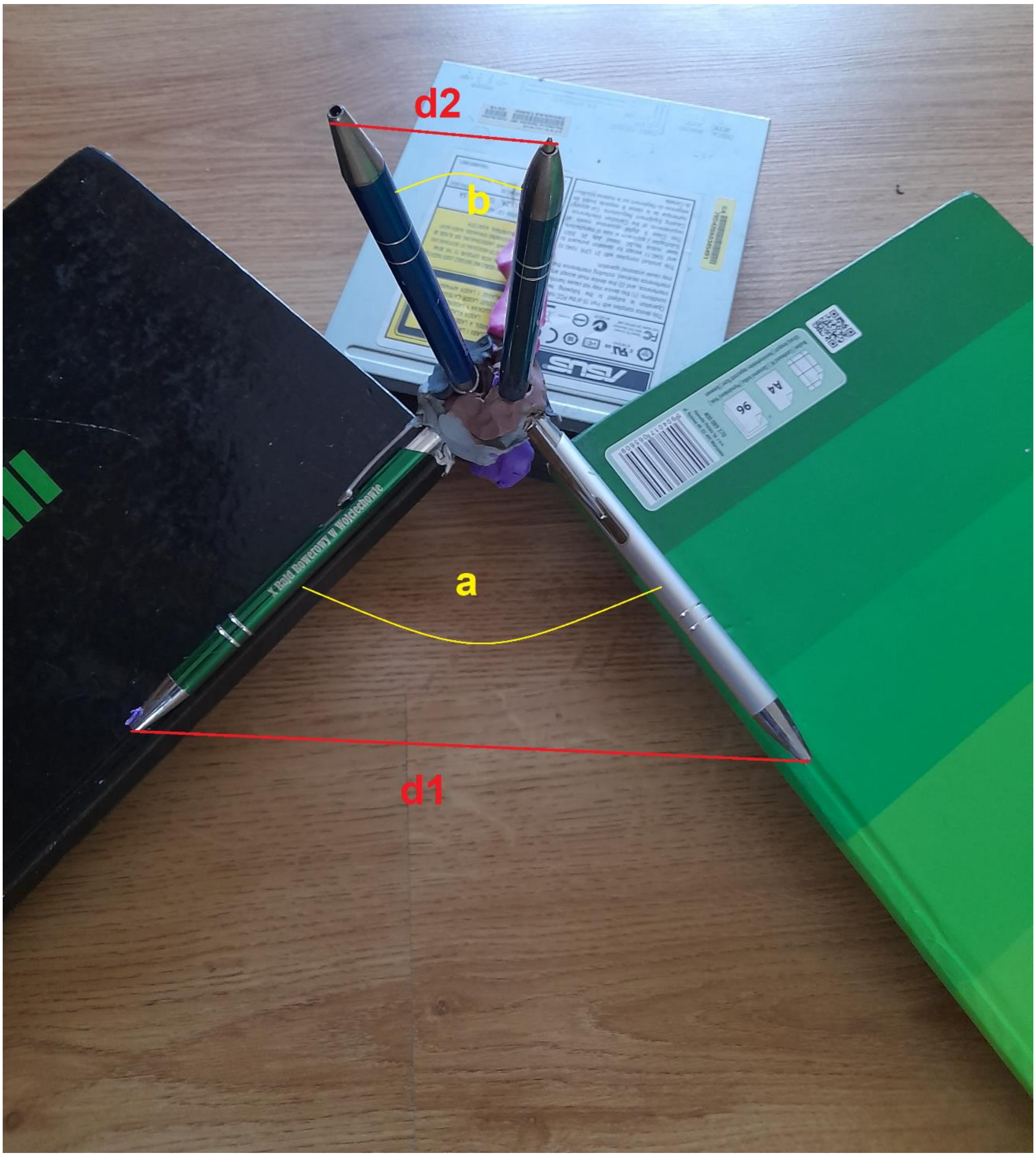
We thank reviewers Dr Guillaume Duclaux and Professor Thomas Blenkinsop for their constructive criticism. Both reviews have a similar question: can the results (e.g. elbow method metrics, clustering on stereonets, structural interpretations etc.) be different for normal and dip vectors if these two representations convey the same geometric information? Our answer is: yes, and it can be demonstrated with Photograph 1 which shows differences in Euclidean distances and angles between dip and normal vectors of two subhorizontal observations (a black and a green notebook) that differ in their dip direction.



Photograph 1: Two subhorizontal dip vectors (green and grey pens) with slightly different dip directions. Euclidean distance between these vectors is d1. Two subvertical normal vectors (both blue pens) of the same observations with the Euclidean distance between them d2. We can see that d1 is greater than d2 and a similar effect is applicable for angles a and b (the angle between dip vectors is greater than the angle between normal vectors: a>b).

Reviewer 2

Comment #1

This is a difficult paper to read, because it contains a lot of jargon about geometry, and because of vague general statements, some of which are unnecessary (e.g. “dip angle is not capable of showing the dip direction of faults and vice-versa” and “Geology is considered to be a subjective science (Curtis, 2012)”).

Agree. We wanted to emphasize that we do a three-dimensional analysis of outliers.

Perhaps it is better to give only examples of subjectivity without writing these general statements.

Change in the manuscript: we deleted the sentences:

- „Geology is considered to be a subjective science (Curtis, 2012)”, and
- „dip angle is not capable of showing the dip direction of faults and vice-versa”

Comment #2

A further problem for understanding the paper is that some of the methods section is couched in the technical language of the CGAL library. This is unhelpful to the general reader, and needs to be explained in simple terms.

Clarify. This remark is about our chapter „3.4 Irregular and regular trend maps”. We believe that not everyone needs these details, but we wanted to include as many details as possible to allow reproducibility of the regular version. Please note that the general message of this chapter is also presented in Fig. 2. We can summarise the method as follows:

„A summary of the regularization method: information about clustering labels of triangles must be attached to points from the regular grid. This transfer of information is possible via CGAL query functions which allow to identify triangles that have points in their interiors (the points are arguments of the query functions). Please note that executing the query functions and clustering are done in separate environments, therefore two datasets (Table X and Y) need to be merged using unique elements (ids of the vertices of triangles).”

Change in the manuscript: we improved Fig. 2 and caption to it

Comment #3

One of the main conclusions, that applying clustering methods to normal vectors and dip direction vectors from the same data set results in different interpretations of the structure (Fig. 15), seems unlikely to be correct. There is no material difference between the geometrical significance and information contained in a normal vector compared to a dip direction vector. If there is a difference in the outcome of the clustering methods, that must be an artefact of the way the methods have been applied to each data set.

Disagree/Clarify.

Disagree. We disagree that clustering results must be the same for the dip and normal vectors (see Photograph 1 or the explanation below). Please be informed that data sets with required data (coordinates of normal and dip vectors) are available to reviewers so that they can independently verify the results.

Clarify. Because of the differences in values of coordinates, we cannot assume that the squared Euclidean distance (which is the squared distance between tips of the vectors and which determines clustering results) calculated for two different representations of two observations will be equal.

Clarify. Consider Photograph 1 and the following examples:

I. Intuitevely: If you have a subhorizontal surface, then the dip direction vectors will be subhorizontal as well, while the normal vectors will be subvertical (Fig. 1 in this file). And if subhorizontal dip direction vectors dip in opposite directions, then the distance between the tips of such vectors (d2) will be high compared to the distance between tips of subvertical normal vectors (d1) (see Fig. 2).

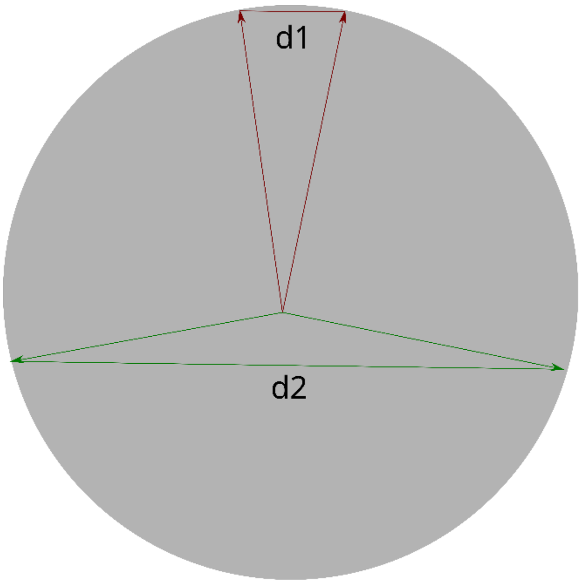


Fig 1. An example showing that Euclidean distances (and then squared Euclidean distances) d1 and d2 for subvertical (brown normal vectors) and subhorizontal (green dip vectors) representations of two observations can be different.

II. A spreadsheet with a numerical example similar to this in Fig. 1 (of this response file)

	A	B	C	D	E	F	G	H	I
1		X_N	Y_N	Z_N	X_D	Y_D	Z_D	Dip_ang	Dip_dir
2	triangle v	0,0140094	0,0145931	0,999795	0,692391	0,721239	-0,02023	1,15913	46,16906
3	triangle u	-0,0217465	-0,025059	0,999449	-0,65506	-0,754845	-0,03318	1,901383	229,0482
4									
5									
6		between v_n and u_n	between v_d and u_d						
7	squared Euclidean distance	0,002850893	3,99462396						
8	dot product = cosine	0,998573769	-0,997312185						
9	1 - dot product = 1- cosine)	0,001426231	1,997312185						
10	2*(1-cosine)	0,002852462	3,994624369						
11	angle	3,060442257	175,7982068						

Fig 2 An example showing differences in squared Euclidean distance calculated for unit normal and dip vectors of two observations v and u. It can be seen that the squared Euclidean distance between normal vectors of v and u is ***0.002850893****, while the same distance between dip vectors of v and u is* ***3.99462396****.*

Change in the manuscript:

Comment #4

Another main conclusion is that optimisation methods must be applied to investigate clustering. This is relatively trivial: any clustering algorithm requires a similarity index, and the one used here (cosine distance) is a standard metric for assessing orientation differences.

Clarify. We agree that clustering algorithms use similarity functions, but we didn't argue in the Conclusion that „optimisation must be applied to investigate clustering”. Clustering in this case is optimisation, so the latter doesn't have to be „applied” to the former. Again, we didn't have such a sentence in the manuscript. We argued, however, that:

1) in the first bullet point of the Conclusion: that if you use a color pallette for dip angle or dip diretion available in the GIS software, then the boundaries between colors may be subjective and without optimization significance, so it may be better to use clustering (thus optimisation)
2) We argued that theorems about Voronoi diagrams are useful to explain meaning of the clustering results.

Change in the manuscript: none

Comment #5

Further to the previous point, this metric should not result in significant differences between normal and dip direction vectors, because the cosine distance between two normal vectors must be the same as the cosine difference between the two dip vectors of the same surface.

Disagree. We disagree that squared Euclidean distances, angles and cosine distances between two normal vectors (v_n, u_n) and two dip vectors (v_d, u_d) of two observations u and v must be the same – see Photograph 1. We are also a bit confused with the term „of the same surface” because if there are two normal and dip vectors, then in both cases they represent two distinct observations, so we would argue that they don't represent the same entity.

Clarify. In Fig. 3 (in this response file) you can see a couple of pairs of vectors with a constant directional separation (to honour that dip and normal vectors point to the same direction). The distances between tips of vectors that intersect the hemisphere are greatest (d1) at the bottom and lowest (d5) at the top of the hemisphere (the same applies to angles). This illustrates that if you rotate (or lift) a subhorizontal dip vector to get a subvertical normal vector, then you should expect changes in squared Euclidean distances, angles and cosine distances as well (see also Photograph 1).

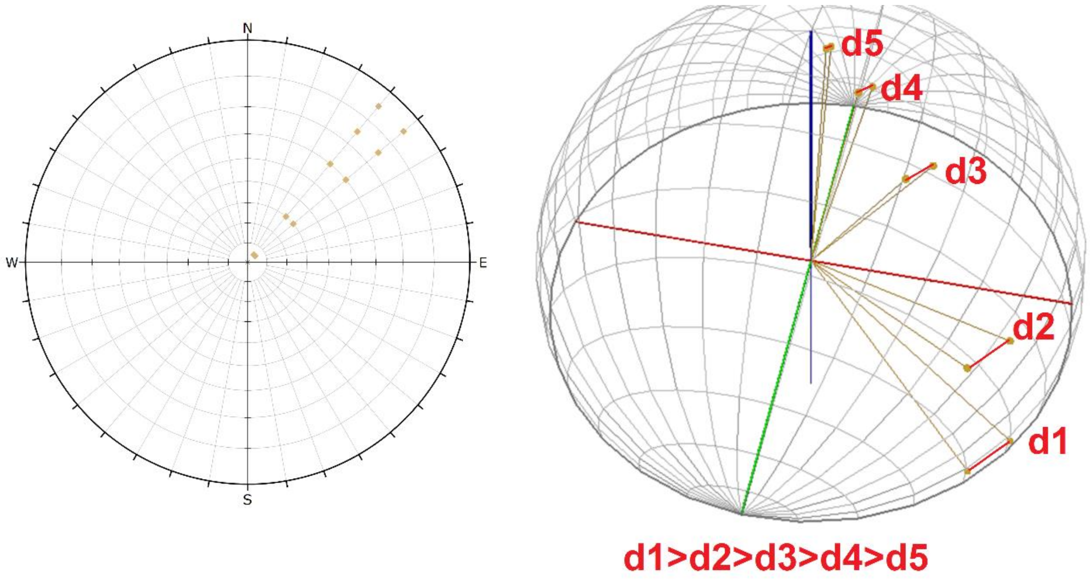


Fig 3 This illustration presents synthetic data: pairs of observations with constant directional separation (10 degrees of separation, see on the left). On the right, a 3D view is presented which shows that the Euclidean distance between tips of the vectors decreases when the pairs are approaching the upper vertex of a hemisphere. This can serve to illustrate that k-means clusters with subhorizontal representations of observations (in our case: dip vectors of a subhorizontal surface) can indicate greater within dissimilarity than subvertical representations of observations (in our case: normal vectors of a subhorizontal surface).

Change in the manuscript: none

Comment #6

There is some discussion about anomalous results:

“The above effect could be explained by several competitive hypotheses. For example, the fault plane could have been drilled, 365 thus broadening the zone of triangles genetically related to the fault (Michalak et al., 2021). Assuming the tectonic origin of the related structures, it can be hypothesized that fault drags on the hanging wall contribute to subsidiary elevation differences that must be consumed by nearby triangles. It could also be argued that an unusual lowering of the contact surface is due to a deformation zone composed of many smaller faults. Another hypothesis could be that the related feature is not a fault but rather a sedimentary slope, which would explain the gradual lowering of the contact surface.”

Such hypotheses are useful, but would be better illustrated with specific examples and some reasoning about which is the preferred hypothesis.

Clarify. We can agree that from the viewpoint of a structural geologist, it is better to select only one hypothesis and provide arguments. But please note that our paper is classified as a „Method article”, so we put „equal probability” to all possible hypotheses. Otherwise, we are afraid that the reviewers or readers could think that we aim to solve a specific geological problem, and it would no longer be a „Method article”.

Change in the manuscript: none

Comment #7

The determination of the optimum number of clusters is explained in Figure 7, but the results sections shows results from 2, 3 and 4 numbers of clusters. This is unnecessary: only the optimum results should be shown.

Agree/Clarify. Figures related to non-optimum results such as 2 clusters for CEBS and 4 clusters for KSH can be indeed removed in both versions. In our opinion, some non-optimum results should remain (3 clusters for CEBS – dip vectors, 4 clusters for CEBS – normal vectors, 2 clusters for KSH – dip vectors, 3 clusters for KSH – normal vectors) because they are still useful for comparing representation results and for proposing models for clustering results.

Change in the manuscript: We decided to remove figures related to non-optimum results such as 2 clusters for CEBS and 4 clusters for KSH. Some non-optimum results remained (3 clusters for CEBS – dip vectors, 4 clusters for CEBS – normal vectors, KHS) because they are still useful for comparing representation results and for proposing models for clustering results.

Comment #8

The figures could be substantially improved. The use of such a dark background does not help (e.g. Fig. 6c). In most cases the grid is the most dominant and least important aspect of the maps, obscuring the detail of the clustering. The stereoplots are not explained in the figure captions.

Agree.

Change in the manuscript: We improved Figs 4, 6 and 8.

Other comments (in the annotated pdf):

We were requested by the Editor to address geological issues. It is possible that we will follow also other pieces of advice after discussing with Editors.

Comment #9

There is no need for these section headings in the Introduction

Clarify: we will discuss the issue with Editors because we believe that it can be helpful

Change in the manuscript: none

Comment #10

This is a trivial point whihc all strcutural geologist will understand, and none wouold make this mistake

Agree. We wanted to emphasise that we do a 3D analysis.

Change in the manuscript: the second part of the sentence was deleted

Comment #11

Figure reference out of order

Agree. But we would like to ask for an exception.

Change in the manuscript: none

Comment #12

Explain the colours and the colour bar. What units are these?

Clarify. Units correspond to elevation.

Change in the manuscript: figure was corrected

Comment #13

What is the CGAL library?

Clarify. Computational Geometry Algorithms Library – it contains many algorithms related to computational geometry.

Change in the manuscript: CGAL was expanded

Comment #14

This is the first time that boreholes have been mentioned. What boreholes are these?

Clarify. Well, the word „boreholes" is unnecessary because we also have geophysical surface data.

Change in the manuscript: „boreholes" deleted

Comment #15

Sentence does not make sense: replace as well as by and?

Agree.

Change in the manuscript: we replaced „as well as" by „and"

Comment #16

underwent

Agree.

Change in the manuscript: we replaced „were undergone" by „underwent"

Comment #17

In b and c the grid is much too prominent and obscures the data

Agree.

Change in the manuscript: We improved the figure.

Comment #18

Ores of what minerals?

Clarify. iron

Change in the manuscript: we added clarification

Comment #19

The clustering map is poor. Only one cluster memebrship (te magneta one) can be clearly seen. The grid obscures much of this diagram

Clarify. We deleted this figure because it doesn't present optimum results – see your comment #7.

Change in the manuscript: we removed the figure

Comment #20

Wxplain the stereoplots. Why are there two versions in each figure? What advantage does the secind one have

Clarify. The stereonet on the left presents the projection of points from the unit lower hemisphere (tips of unit dip vectors) onto the horizontal plane. The stereonet on the right presents the projection of points from the upper hemisphere (tips of unit normal vectors) onto the horizontal plane. One advantage of using two versions is that in case of a subhorizontal surfaces, it is difficult to see boundaries of clusters when the combination of normal vector representation and upper hemipshere is used.

Change in the manuscript: we added explanations about stereonets in the captions

Comment #21

The grid is too prominent

Agree.

Change in the manuscript: we corrected the figure

Comment #22

I have no idea what this sentence means

Clarify. In case of triangles genetically related to discontinuities, dip angles are affected by the density of borehole network

Change in the manuscript: we added a clarification

Comment #23

This paragraph is extremely difficult to follow and would benefit from an additional diagram

Agree/Clarify. We agree that some rearrangements of the text are needed but we didn’t include a new figure

Change in the manuscript: we’ve made some rearrangements of the text

Comment #24

Which method?

Clarify. We meant the general workflow.

Change in the manuscript: we replaced „methods” by „workflow”

Comment #25

This is not really a conclusion but just speculation.

Clarify. Because we don’t speculate which version is „correct”, we would argue that it is a conclusion

Change in the manuscript: none

We’ve introduced also our own changes:

Own change #1 (the end of section 4.1):

We added clarification regarding the details of the CEBS surface: "The investigated Jurassic horizon represents the base of the Jurassic in places where the Jurassic sediments are present (Maystrenko et al., 2013, 2012). Within the rest of the model area, this horizon corresponds to the top of pre-Jurassic sediments or to the top of the crystalline basement."

Own change #2 (throughout the manuscript):

We replaced „megafolds” with „megacylinders” because one can say about conical folds so using „megafolds” for cylinders only may be discriminative for conical folds

Own change #3 (in discussion)

We added a limittion regarding singular input data: „We note that dip vectors are not uniquely defined for horizontal observations (the dip direction cannot be specified), so we recommend removing horizontal observations prior to conducting clustering. A similar problem and proposed solution applies to normal vectors of vertical observations, for which two possible dip directions can be given.”