Thank you for taking the time to review our manuscript. We have responded to your comments below.

A more detailed description of the usability of this method is needed. This could well be a useful method for the community to explore sea ice dynamics, but certain features of the method and results are still unclear to me.

Thank you for your suggestion. We have thoroughly expanded the methods section and added an additional appendix section to help the community understand these techniques.

First is the dimensionality of the results. The figures give results in units of d-1, so it seems that TSE metrics are equivalent to standard strain measurements. How do the units given in the results scale in comparison to Itkin (2017) and other measurements? Are the TSE scales and strain measurements of comparable magnitude? Is TSE most comparable to divergence, how does it respond to shearing across the trajectory? A few toy examples of this method, such as included in the appendix will aid this paper. For example, it is currently unclear to me what TSE we expect for a large coherent ice cover, under no deformation, but under acceleration. What TSE do we expect for a divergent flow? How does it respond under rotation or with a flow field experiencing shear or curl? These examples will aid the interpretation of particularly figure 2, where the TSE and triangle methods show different behaviour. What is happening to the ice during these periods?

Thank you for bringing these points to our attention. We have now included an additional section in the appendix where we clearly outline the mathematical connections of TSEs and divergence and shear by way of the rate-of-strain tensor.

The second is that the method relies on the magnitude of the tangential vector to the buoy trajectory. Does this mean that directional information is included within the TSE results, or is it purely a scalar?

TSE is purely a scalar as it is a measure of the rate stretching of trajectory-tangent vectors, the change in magnitudes.

Does this link into the analysis between the TSE and polygon methods on L 199?

I do not understand exactly what link you are referring to, but the differences in Green's theorem-based methods and TSE are thought to be the root of all the differences between our results and previous polygon-based findings. The effects on L199 may be from the choice of triads used, or Green's theorem approximation errors.

The context of figure 1 is difficult to understand. This may be due to the difficulty in interpreting the method and theory presented, but consider adding to this figure the deformation results of Itkin (2017), if comparable.

Thank you for your mention. We have expanded on the explanation and caption of Figure 1 to improve understanding. We find that adding the deformation results of Itkin would make the figure too busy, without adding much value. The connection we are drawing is with the *occurrence of storms that are important for sea ice dynamics, as the storms provide the external validation of TSEs. We do a more thorough comparison with array-based metrics in the MOSAiC example.*

The figure captions all need expanding upon.

Thank you. The figure captions have all been expanded.

Thirdly a greater description of \bar{TSE} is required. This is given as the "hyperbolicity strength" but this definition does not explain to me why this value is positive definite.

Thank you for bringing this to our attention The value is positive because it is a sum of positive values. The term hyperbolicity strength derives from the original manuscript. A greater description has been included in the new methods section.

The equational form and results suggest that this metric gives a longer time scale measurement of stretching, but only for divergence with no expression of compression. In figure 2 it is compared to the strain rate magnitudes, so is it a measure of the total magnitude of change in stretching?

TSE is positive for stretching, and negative for compression along a trajectory. Thus, if the material stretches and then compresses to its original state, TSE = 0. \bar{TSE} does not allow for this cancelation, instead adding up all "hyperbolic"(stretching and compression) action. The term hyperbolic comes from the dynamical systems definition of hyperbolic manifolds that act as attracting and repelling structures, where nearby vectors undergo exceptional stretching or shrinking.

Fourth, the buoy deformation results in figure 2 are unexplained and uncited. Are these the first publication of the triangle based deformation measurements from MOSAIC? If so they need much more documentation than currently included here and possibly a figure or two to allow these results to be adequately interpreted. If not then a further description of the previous publication is required. How reliable are the results? What are the successes of these results?

The triad analysis we use in this paper for the MOSAiC data has not been previously published. A data paper for the buoys is currently under review (Bliss et al., 2022), and has now been referenced in the revised manuscript. The triad analysis was used in AGU 2020 and EGU 2021 presentations by Jenny Hutchings, and is chosen from buoy tracks that lasted for the full period from November 2019 deployment until June 2020. This is so we can create a time series for the full time without needing to account for changes in the array. We do have another paper we are working on that will improve upon this methodology by triangulating the array over shorter time periods, to provide a more detailed, potentially higher accuracy, and complete timeseries of total deformation and spatial variability in this deformation within the MOSAiC distributed network. However this paper is not ready for submission in the near future. The data we use here was chosen as it was created with a similar method to the past campaigns and that used for the IABP analysis. We do note that the MOSAiC buoy array is not well suited to automated triangulation methods. Delaunay triangulation creates skinny arrays that are less accurate for calculating deformation with. Hence similar to previous campaigns (SEDNA and ISPOL for *example) we hand-picked triangles within the array that ensured each triangle maintained as best a non-skewed shape as possible over the time period. This was achieved by checking triangle shapes by eye in November, March and June. We agree that it would be best to document this fully here, and have included figures that show the triads and their evolution during the time series. Please do note, this is not the definitive MOSAiC data set for sea ice deformation, it was simply chosen as a best representation of deformation to compare against the new method presented in this paper. We do not wish this paper to provide the definitive MOSAiC sea ice deformation time series for the triad method, but do believe that using a time series that was created with the method documented in Hutchings et al. (2012) is reasonable to show the utility of the new TSE method.*

Related to this issue: a separate data section is required. This needs to include all descriptions of the data used and the previous results repeated in this study. Currently this information is within the introduction, method and results and is difficult to follow.

A new data section has been included.

L 14 a more up to date reference for this is desirable.

We have now included a more recent reference.

L 15 -16. Does this feedback come directly from Serreze and Francis? A little more expansion on how they discovered and documented is needed. The current description is too brief to show the importance of ice dynamics.

Thank you. The original reference was a bit confusing. This sentence has been changed to the follow

"As the ice warms in spring, melt is accelerated around existing fractures due to a reduction in albedo and the presence of more open water. Arctic amplification, the disproportionate warming of the arctic in a changing global climate, has been partially attributed to the enhanced oceanic heating and ice-albedo feedback caused by diminishing sea ice (Screen and Simmonds, 2010; Dai et al., 2019; Thackeray and Hall, 2019; Jenkins and Dai, 2021). "

L 28-29 this sentence doesn't fit the flow of the paragraph. Consider moving it before the description of SAR data.

Thank you. This has been relocated.

L 35 This paragraph will benefit from an expanded definition of a Lagrangian coherent structure, in particular why this perspective results in the difficulties in using 'gridded sea ice displacement fields' mentioned later.

We have expanded the introduction and appendix to better explain Lagrangian coherent structures and related Lagrangian diagnostics to identify them.

L 46 What characteristics of a LCS make it a hyperbolic LCS? And what makes sea ice applicable to a hyperbolic LCS? Additional arguments and descriptions of Haller et al. 2021 could be incorporated.

We have expanded the introduction and appendix to better explain how Lagrangian coherent structures are relevant for the study of sea ice dynamics.

L 50 Does the period have the 'much larger influence' or is it the identification that has it? 'Much larger influence' than what?

Thank you. 'Much larger influence' has been removed.

L 63 'the the' -> 'that the'

Thank you. Corrected.

L 63, 64 I find that this sentence does not give enough background on Haller et al. 2021 to allow for any understanding of the following equations. Please include a sentence each, with terms, on 'material stretching', 'hyperbolicity strength' and 'initial material tangent vector'. At the moment the reader is required to also read a large part of Haller et al. 2021 in order to understand these equations. It is also unclear what is represented in the two equations.

Thank you, we have significantly expanded the introduction and methods section to better explain our approach to quantifying sea ice dynamics.

L 69 My interpretation of the appendix does not show that this is "verified". If I indeed it is only likely that sea ice drift is slowly varying, then this needs to be stated as such.

Thank you for bringing this to our attention. The phrasing has been changed from verified to assessed.

L 78 including a definition of a "steady flow" will benefit this section.

Thank you. "(does not change with time)" has been added for clarity.

L 87 Again a definition of hyperbolicity in this context will aid the understanding here.

Thank you. This section has been changed to:

"TSE is positive for stretching, and negative for compression along a trajectory. Thus, if the material surrounding a buoy stretches and then compresses back to its original state, $\mathrm{TSE} = 0$. $\overline{\mathrm{TSE}}$ does not allow for this cancellation as the summand is strictly positive. It instead adds up all hyperbolic (stretching and compression) action. The term hyperbolic here comes from the dynamical systems definition of hyperbolic

manifolds that act as attracting and repelling structures, where nearby vectors undergo exceptional stretching or shrinking."

L 90 Can you add in this paragraph a description on the units of the two equations and TSE? A further description of how this relates to usual deformation units and how to interpret the two values would be of help (you may want to put this elsewhere).

Thank you, a discussion of units and relation to usual deformation metrics has been added to the appendix.

L 91 A citation is needed here.

Thank you. A citation has been included.

L 95 - 97 What is meant by this sentence? Will this technique be used later, or is it a note on the context of TSE methods and the use of existing stress vs train rheology methodologies?

Thank you for bringing this to our attention. This is not a technique, rather a statement about what TSE is measuring. We have clarified this further in Remark 2.

L 112 It is not immediately obvious why equations 6-9 are included as they are not referenced. Consider removing them. Do they apply directly to the example A3? If so put them there too. The following paragraph gives a detail dsicussion on the limitations of polygon based approaches, these extra questions don't bring anything useful here.

Equations 6-9 explicitly detail the calculations used for Green's theorem array-based diagnostics discussed throughout the manuscript. We have made this connection clearer by referencing the equations later. We bring up the polygon-based approaches as both the N-ICE and MOSAiC examples rely on these techniques. The Green's theorem technique is also standard for ice dynamics studies using buoys. They are the best reference to ground our new diagnostics and included as such.

L 123 An example of such a long time series is needed here.

Citations have been added.

L 149 An extra summary sentence here showing plain words rational for this technique would be beneficial.

Thank you. We have significantly changed our introduction and now explain why these datasets were chosen to validate our new approach.

L 169 how is the slowly varying nature of sea ice drift relate to these storm periods? Is it more or less likely that the slowly vary criterion holds?

The slowly varying assessment (see Appendix) is pointwise in time, so it is not related to the timescale or period of a storm.

L 175 I'm not sure why the beginning part of this sentence is needed, as the second part of it does not logically follow. It's fine that the diagnostic has a time window, and this description is a sensible choice.

Thank you. Lagrangian diagnostics are distinct from more common Eulerian diagnostics, and we are highlighting one such practical difference when comparing the findings of each.

L 179 here may be good place to refer to equations 6-9, if they are needed at all.

Thank you. This has been added.

L 186 a quick summary of the Itkin cleaning method would be a beneficial addition here.

We have rephrased this section to the following:

"For our analysis, we focus 24 buoy trajectories in two time windows previously examined by \citet{Itkin2017}. GPS positions were primarily sampled at 1-hour intervals, though some sampled every three hours. \citet{Itkin2017} resampled all trajectories to a 1 hr^{-1} sampling frequency using a linear interpolant, and we follow this convention for our N-ICE2015 analysis. Buoy speeds that exceeded $5 km/day$ were removed and positions were resampled using a linear interpolant."

Figure 1. This caption requires extensive expansion. All lines need to be defined. It is currently impossible to interpret this figure without extensive reading in the text. The figure needs to be interpretable from the caption alone assuming a knowledge on the papers aims and method. Addional lines at d-1 = 0 will allow

Thank you. This caption has been extensively expanded.

L 217 - 221 Has this data been analysed by this method previously? If so citations and a summary of results is required. If not then this paper needs expansion as a presentation of these new results too. A least a discussion of previous use of these results or method is required.

We have changed this section to the following:

"We focus here on the paths of 101 buoys deployed within 40 km of the Polarstern. This public data set documented by \citep{Bliss2022}. The half-hourly buoy track data was cleaned following \citep{Hutchings2012}. Triads were also handpicked from the MOSAiC buoys with data spanning October 2019 to June 2020, and is the focus of a forthcoming publication. The arrays were selected to maintain reasonable shapes (no small angles, area greater than 1km^2) from the beginning to the end of the time series and resampled to uniform 6-hourly intervals. Handpicking triads, however, does require user discretion. Buoy tracks were resampled to match the triad sampling rate. The arrays used are shown in Figure \ref{fig:MOSAiC Array}. A deeper comparison and refinement of geometrically suitable arrays in

the MOSAiC data is a current topic of research. The method we use here is in line with previous work \citep{Hutchings2011, Hutchings2012}."

L225 please refer div, D back to the equations previously and change the labels on the plot to directly match the text. Using 2a, 2b will help too. Please also add from an improved method why TSE is compared to div, and \bar{TSE} to total D.

These equations are now referenced at the beginning of the N-ICE results section. We have thoroughly expanded the methods sections, as well as added an additional section in the appendix detailing the relationship between TSE, div, \bar{TSE}, and total D.

L 240 which source do these numbers come from? The polygon of triangle based methods? What numbers come from the other method? Can the two methods be dimensionally compared in this way? Is there any method that allows for the integral of all deformation and TSE over the period discussed?

These values come from the array-based methods, as explained in the new data section and at the beginning of the MOSAiC results. The values of the diagnostics are not interchangeable, as detailed in the new appendix section comparing TSE and array-based diagnostics, but dimensionally they have the same units. We could integrate the total deformation and calculate \bar{TSE} for the entire period, but that would provide us with only two scalars without a direct comparison, instead of looking for distinct temporal deformation features, as is our goal.

L 241 Is this value significant? Which line plot does it come from? Do all significant deformations have a higher value?

We have added a reference to "subplot b". We have changed the text to the following:

"In the 3-day window following the Apr 17 TSE and TSE peak, the mean buoy divergence oscillated around zero (Figure 6b), with the magnitude staying below 0.1d−1. This is approximately 1% of peak values of mean divergence, suggesting a relatively insignificant period of divergence. This is in contrast to TSE and TSE on April 17 which sits at approximately 50% of their total peak values, suggesting a relatively motion with a larger contribution to ice dynamics at the same time. "

L 242 Is shear plotted anywhere? How do we interpret shear against the TSE metrics?

Shear is not plotted, but can be inferred as it is loosely difference between the magnitude of divergence and total deformation. Its value is not particularly relevant for the present analysis and adding it does not reveal any additional insights, while making the figures busier.

Figure 2. Caption needs expanding. (a-d) are referenced in the text but do not appear. What is the black line in b and d? A scale is required for the colourbar.

Thank you. All captions in the manuscript have been expanded.

L 258 Please comment on how the spacing of the buoys and the time handling of this data (linear sampling) affects the dimensionality of the calculated TSE in comparison to high time resolution data from the other sources.

It is not clear to me what the reviewer is referring to with this comment. The dimension of TSE does not change, it is always a scalar value. We have however included a comment regarding the possible effects of shorter sampling periods. Linear subsampling would have the same effect on TSE as it does on decreasing the spacing the for a Riemann sum that is approximating an integral.

L 259 Shown in black where?

Thank you, we have clarified which plot we were referring to.

L 267 In the line plot I see that at the beginning of the period the TSE is distributed about zero, and then towards these events the spread of values reduces to oscillating peaks. Is this what you mean?

We have rephrased this sentence to

"The first event corresponds to stretching from March 26 to March 29, 2017. Previous mean TSE gradually increased built up until the absolute maximum of mean TSE on March 26."

L 270 Red is positive TSE? So equivalent to net divergence?

No, comparisons of TSE and Eulerian rate-of-strain diagnostics have now been shown in a new appendix section.

L271 high positive or negative values?

We have clarified we mean positive.

L 275 This paragraph will be aided by a previous discussion of what TSE we expect for certain dynamics events. For the accelerating ice described here, what TSE is expected? For constant but rotating flow as described at the end of this paragraph what idealised TSE is expected?

Thank you for bringing this up. We would need to do a climatic analysis of TSE values to identify expected values of TSE. This is beyond the scope of the present analysis and a topic of future research. The value of TSE is that we can locally identify significant events compared with surrounding time periods. We have however included a section in the appendix where we calculate many diagnostics for a simple analytic flow with both high shear and rotational regions.

Figure 3 You have chose not to include \bar{TSE} int his plot. Can you explain why? Please use a divergent colour scale for the divergence, with white at zero and different colours for positive/ negative TSE.

Thank you for bringing this to our attention. The color scale has been changed to a divergence color scale. \bar{TSE} is not included as it does not reveal any additional information.

L 281 This paper has provided not explantation I could interpret so far on how TSE provides insight in to "distant fracture events". Please expand, as this is a useful contribution if true.

Thank you. The following paragraph has now been added to the end of the IABP analysis:

"One particular benefit displayed in this example is the significant spatial extent of the large positive TSE values prior to each fracture in the Beaufort gyre. Not only was the edge of the gyre identified in the gap between positive and negative TSE, but positive TSE was also found thousands of miles from the Prince Patrick and Banks Island fractures. This supports the ability of TSE to identify the Lagrangian coherent structures in the mobile pack ice as whole, not just locally highlight a fracture."

L 285 I have seen the deformation events captured in the new technique, but nothing on prediction. Perhaps the wrong word to use here.

Thank you for bringing this to our attention. We have added the following paragraph to indicate build-up of stress that was measured by TSE prior to the major fracturing in spring 2017:

"The first event corresponds to stretching from March 26 to March 29, 2017. Previous mean TSE oscillations gradually increased to the absolute maximum of mean TSE on March 26. Prior to any evidence of detachment of the mobile pack ice in the Beaufort sea, TSE values were indicating an ongoing increase of stress and strain leading to the major fracturing in March and April, 2017."

L 291 "predicted" again this is the wrong word I think. Captured or similar is more accurate.

This sentence has been rephrased to more accurately reflect our findings:

"Approaching sea ice dynamics through quasi-objective stretching, we were able to capture coherent deformation events in concentrated buoy experiments, and even predict spring breakup in large sparsely-sampled IABP data."

L 296 "Buildup of stress" is this your hypothesised stress state prior to the break up, or does the TSE measure stress? Are there other measurements of stress during this period that can back up this claim?

There are no stress measurements available for this analysis, but we find this is a plausible explanation to the material failure that occurs after significant periods of stretching.

L298 Again is this a hypothesis of what internal stresses are expected within the pack? If these assessments of ice stress are speculation, please be very clear about this, or remove them.

Thank you, this particular speculation on stress has been removed.

L 317 This is the first mention of the technical methodology used in this study. Please include this information earlier in the study also. A data section detailing the exact values taken from the buoy data is needed.

We now have included a statement regarding the simplicity of TSE calculations in the methods section:

"TSE is calculated using only buoy speed and does not require projection to orthogonal velocity components as in Green's theorem approximations from arrays. Speed can be easily calculated using geodesics between GPS locations, which prevents any inconsistencies of results due to map projections. Furthermore, TSE is parameter-free with integration time being the only user-chosen value."

Information about the buoy data is now provided in the new data section.

Appendices

L 362-365 Can you comment on how figure A1 suggests that the slowly varying criterion is met, but not conclusively? I see from figure A1 that $|vt|/|a(t)| < 1$ in most cases (positive tail greater than 10^0), but not strictly or $\ll 1$. Is this correct? Does that mean that the data presented suggests that sea ice is, in general, slowly varying, but not strictly so? Can you comment on the cases where $|vt|/|a(t)| > 1$, when do such cases occur?

Your understanding of the figure is correct. Most of the ratio values are below one, but not strictly speaking. We have added a comment on when the magnitude of $v(t)/a(t)$ is greater than unity.

L 394 A summary sentence for this example, repeating, and expanding upon the introduction to the appendix would be helpful here.

Thank you. Our appendix section has been significantly restructured and this frame-indifference violation section has been removed.