

To the authors,

I thank you for the revised version of your paper. You now discuss the symmetric positive semi-definite (SPSD) character of the localization matrix \mathbf{C} . You also mention in the response in the Interactive Discussion that this matrix is not SPSP (with eigenvalues ranging from -0.5 to 50, which suggests that the negative part of the matrix is in a sense ‘small’). You also mention that the matrix \mathbf{C}' obtained by setting the negative eigenvalues to 0 changes the localized values by up to 15%.

These results are interesting, and should in my mind be included (together with other possible results) in your paper. More generally, I think that the diagnostics you have performed on the basis of the matrix \mathbf{C} (starting with subsection 3.1) should be performed again with the corrected matrix \mathbf{C}' (or actually with any other appropriate SPSP matrix you might build from your original matrix \mathbf{C}). I do not think the required cost would be prohibitive, and that would greatly increase the interest and significance of your paper. Actually, the diagnostics you have performed with your matrix \mathbf{C} could be removed from the paper if they are replaced with new diagnostics obtained with an SPSP localization matrix. It may be that the new diagnostics will turn out to be similar to the previous ones. If so, that would actually be very instructive.

I make three additional remarks

- It would be appropriate to mention explicitly that the matrix \mathbf{P}_{loc} defined by the Schur product (3) is SPSP for any SPSP matrix \mathbf{P} if, and only if, the localization matrix \mathbf{C} is itself SPSP (I cannot mention an explicit reference for that, although I know there exist some, but I can provide you with a proof if necessary).

- You write *As some DA algorithms require SPSP covariance matrices* (l. 237 of your Authors'.Tracked.changes file) and *covariances or localization matrices often need to be symmetric positive semi-definite* (l. 465). Well, covariance matrices are SPSP by definition. As for DA algorithms, all algorithms that are based, explicitly or implicitly, on a Bayesian approach (and that is the case of Kalman Filter and of all its variants, particularly ensemble variants, in which probability distributions are described by finite samples) require proper covariance matrices (if that condition is not verified, the corresponding computations are meaningless and can lead to absurd results, as has actually been observed by many authors).

- Localization is necessary in the various forms of Ensemble Kalman Filter because of the relatively small size of the ensembles that are used. It might be useful, if this is possible, to give some quantitative information on how the need for localization decreases when the ensemble size increases.

I am going to ask you to make a major revision of your paper along the lines above. I will submit your new revised version to further review.