

Following the comments of the two referees, I would like as Editor to raise a point that I think is important, and possibly critical for acceptance of the paper. It is the symmetric positive semi-definite character of the matrices that are defined in the paper for representing localized covariances and correlations.

As reminded by the authors, a covariance matrix (and in particular a correlation matrix) must be symmetric positive semi-definite (SPSD, meaning without negative eigenvalues). If that condition is not verified in an EnKF, the minimization of the variance of the estimation error that is implicit in the analysis step of the EnKF (as also in variational assimilation) may lead to negative ‘minima’ (actually saddle-points) and to absurd results.

I understand that all localized correlations determined in the paper are obtained through formulas of the type of Eq. (3), *i. e.*, through Schur-multiplication of an original covariance matrix \mathbf{P} by a localization matrix \mathbf{C} . For a given localization matrix \mathbf{C} , the localized matrix \mathbf{P}_{loc} defined by the Schur-product will be PSD for any covariance matrix \mathbf{P} , if, and only if, the localization matrix is itself PSD.

Is the condition that the obtained matrices are PSD verified in the paper ? As a precise example, do the quantities obtained through the EOL minimization Eq. (5-7) define (as it seems to be the authors’ purpose) a correlation matrix, *i.e.* an PSD matrix with 1’s on its diagonal (the remark ((ll. 374-375) ... *the EOL exhibited values larger than one when estimated after applying the SEC* suggests that all ‘correlations’ defined in the paper are not proper correlations) ?

These questions are not discussed, nor even mentioned, in the paper. I think they should be. It is possible that they have been discussed in previous papers (either by the authors of the present paper or by other authors), where responses that are relevant for the present paper can be found. If so, appropriate references and explanations must be given.

If not, I think it is necessary to check the PSD character of either the localization matrix \mathbf{C} or the localized matrix \mathbf{P}_{loc} (or both). That can be done on the basis of theoretical considerations (it is not clear to me if the correlation matrices defined by the EOL minimization Eq. (5-7) are even symmetric). Or it can be done alternatively through numerical computations. There are in the present case 4 physical variables and 20 vertical levels, so that the relevant matrices have dimension 80 x 80, of which it must be possible to determine explicitly their full spectrum of eigenvalues. And if that is too costly, it is possible to consider submatrices, for instance by reducing the number of vertical levels.

If matrices that are meant to be PSD, while being symmetric, turn out not to be PSD, but with only a small number of small negative eigenvalues, one solution may be to set those eigenvalues to 0 (or to small positive values). If the negative character of the matrices is significant, there will be a real problem, which will have to be solved or at least discussed in depth. It may be that the conclusion of the paper will be that difficulties remain, to be solved in future works.

In any case, I consider as Editor that a proper discussion of those PSD aspects is critical for acceptance of the paper.

I add one remark. The authors mention on several occasions (*e.g.* ll. 44-45) distance-dependent tapering functions with a cut-off at finite distance. A distance-dependent PSD function that is continuous at the origin (*i.e.* at distance 0) cannot have a discontinuity

elsewhere (that would be inconsistent with the requirement that the correlation between two close points must tend to 1 when the distance between those points tends to 0). It may be that people who have used such 'cut-off' functions have not run into difficulties because of the 'small' negativity of the corresponding covariances-correlations, but those functions cannot mathematically be SPSPD.