RESPONSE FILE:
The comments from referees will be indicated by a C.
The author responses will be indicated by an A.

Referee #1

For final publication, the manuscript should be
accepted subject to technical corrections

Were a revised manuscript to be sent for another round of reviews:
I would not be willing to review the revised manuscript.

Suggestions for revision or reasons for rejection (will be published if the paper is accepted for final publication)
The paper now reads much better and gets its message accross much more efficiently I feel.
I have some minor comments. Once addressed, the paper will be ready for publication in my view.

C: on several occasions: "Thermochemical material" <- primordial material
J: done.
C: L. 15: "the depth-dependent ratio should be at leat 9 times ..." <- "the depth-dependent ratio should be at leat 9." (a ratio is dimensionless)
J: done.
C: L. 33: add coma between "physical properties" and "most importantly"
J: done.
C: L. 36: "An important property that influences mantle dynamics is thermal conductivity." This statement is ou of place here. I would simply remove the sentence.
J : Paragraph was simplified in response to R3 comment. The sentence is changed to read:
"An important property, thermal conductivity, often radially parametrized, controls the heat flow between
piles and the surrounding mantle and hence mean pile temperature (i.e., thermal buoyancy)."
C: L. 50: reduce <- reduces
J: done.
C: L. 61: Add "is" between "temperature" and "fixed"
J: done.
C: L. 63: "reduce" <- "reduces"
J: done.
C: L. 62: "should" is repeated twice, you can be a bit more affirmative
J : First "should" is removed and second "should" replaced by "will"
C: L. 69: "The thermal conductivities decrease with an increasing temperature and follow ..." <- "The thermal conductivity decreases with increasing temperature and follows ..."
J: done.
C: L. 74-75: "However, these measurements ... 26 GPa for perovskite." I don't understand the point of this sentence.
J : Paragraph was simplified in response to R3 comment. The sentence is changed to read:
"Under upper mantle conditions and density dependence, Manthilake et al., (2011) finds that $n \sim 0.2$ for Fe-bearing bridgmanite and periclase."
C: L. 80: "... even for a moderate temperature dependence." This is not clear
J : Clarified this point so that the end of the paragraph reads:
"For a typical temperature dependence (n = 0.5) and a reference temperature of 300 K, a lowermost mantle
temperature of 3000 K results in a $\sim 70\%$ reduction in conductivity. Therefore, at lowermost mantle pressures,
a lowermost mantle conductivity of 7.5 W/m/K at 3000 K corresponds to a conductivity measurement of 25 \unit{W m^{-1} K^{-1}}
a room-temperature. Thus, the effect of temperature plays a significant role in making the lowermost mantle very poor at transferring heat."
C: L. 81: "Depending on the interactions between cool subducting, primordial reservoirs..." <- "Depending on their interactions with cool subducting, primordial reservoirs...."
J: done.
C: L. 97: ""A comparable" <- "Such a"
J: done.
C: L.93: "... is not an insignificant factor to neglect" That's 3 negations in 7 words, please rephrase in a less contrived way!
J : Replaced text with 'cannot be neglected'
C: L. 100: Mention that you consider a compressible fluid, you just said in the previous paragraph that it is important!
J : "use compressible thermochemical mantle convection models to" is inserted into the first sentence.
C: L. 119: "We assume primordial" <- "We assume a primordial"
J: done.
C: L. 119: "Has a rate" <- "Has a heating rate"
J: done.
C: L. 124: "... using an Arrhenius formulation": What about the composition dependence?
J : Inserted the sentence:
"A factor of 30 viscosity contrast is imposed between lower mantle material and thermochemical reservoirs because dense material enriched in
iron oxide and in bridgmanite \citep{trampert04,mosca12} may be more viscous than regular (pyrolitic) mantle aggregate \citep{yamazaki01}."
C: L. 139: "has length scale" <- "has a length scale"
J: done.
C: L. 159: "Temperature-dependence" <- "The temperature-dependence"
J: done.
C: L. 161: "results in lower" <- "decreases"
J: done.
C: L. 166: "Composition-dependence" <- "The composition-dependence"
J: done.
C: L. 169: "0.8 and 0.5": Can you motivate these values by comparing with the measurements mentioned in the intro?
J : I replace the sentence on lines 168-169 with:
"For the primordial material considered in this study we consider 20$\%$ and 50$\%$ conductivity reduction
(corresponding to $K_{C} =$ 0.8 and 0.5, respectively), which encompasses a 26\% reduction in conductivity for LLSVPs estimated by \citet{deschamps1…
C: Figure 2: In cpation: "... for cases featuring KDH" not only KDH, but all dependencies
J : Inserted 'for all cases' after (left panel).
C: L. 180: "... eventually be much lower" <- "decrease as they heat up"
J: done.
C: L. 203: "influence" <- "influences"
J: done.
C: L. 209: "3410 K), an increase" <- "3410 K) and an increase"
J: done.
C: L. 210: from (2150 K up to 2290)": explain it: "piles are better at conducting core's heat into the mantle, rather than retaining it"
J : I appended the following to Line 210: "... because the piles transfer their heat to the ambient mantle more efficiently."
C: L. 215: The mantle flow is NEVER turbulent. Please change word
J : The word has been replaced with 'vigourous' which is more appropriate.
C: L. 217: "The conductivity fields...": it seems to be a result, but is an assumption: "When only pressure-dependence is considered, the thermcal conductivity field..."
J: done.
C: Section 3.2: Break the first paragraph in several (one for each alue of K_D)
J : Separated the paragraph and added a short paragraph for $K_{D} = 5$.
C: L. 242: "was increased, could be compensated" <- "was increased is compensated"
J: done.
C: L. 243: "larger than 1 was established <- "larger than 1 is established"
J: done.
C: L. 246: "Interestingly, ...", L. 250: "We observe that..." L. 252: "Interestingly, ..." try to suggest and explanation for all these
J : The first paragraph has been split into multiple parts. Added sentences to the paragraphs to explain these points.
C: L. 270: "The percentage conductivity difference" <- "The conductivity difference"
J: done.
C: Figure 8: the dashed and dashed-dotted lines do not show as such in the legend
J: done.
C: L. 312: "(for different ranges of composition concentration)": what is "composition concentration"?
J : I replace 'composition concentration' with 'C', which has been defined at the beginning of the thermal conductivity methods section.
C: L. 313: "illustrate" <- "illustrates"
J: done.
C: L. 315: "departures from reference" <- "departures from the reference"
J: done.
C: L. 318: "observed following the transient period": name the time
J : The transient periods depend on each case, but we can probably read this from the density anomaly plot from the
transition from white to red in the lowermost mantle. Also, this can be read from the heat flow timeseries where there
are departures from the initial decrease.

Lines 317-318 now read:
"However, the influence of downwellings can be observed following the transient period
(on the density anomaly timeseries, the initial transient period is characterized by the flat average height curves overlying the white
layer below 200 \unit{km}).
For cases \#16 and \#17, the transient period lasts approximately 1 Gyr and for case \#18 approximately 1.5 Gyr."
C: L. 320 "over the CMB but not rapidly increasing the pile" <- "over the CMB but do not increase rapidly the pile"
J: done.
C: L. 325: "their total height." do you mean their maximum heaight?
J : I mean their maximum pile height. The maximum height of primordial material also refers to material that has been ejected
and is floating near the transition zone.
C: L. 327: Does your composition advecetion scheme account for diffusion?
J : I just wanted to refer to the layer where the tracers are less concentrated. I replace 'chemical diffusion' with 'mixing'.
C: L. 328: "Thus, the average height of the veneer": describe how to visualize it in Figure 8
J : The next sentence is now:
"In Figure \ref{fig:figure8}, $h_{C = 0.02 - 0.90}$ is indicated in the timeseries by dot-dashed green curves and on the primordial field snapshots
by the solid green contours."
C: L. 329: "This is a good quantity to analyse for temporal variations," Not clear
J : Change "This is a good quantity to analyse for temporal variations,
since this veneer surrounds buoyant primordial material structures (i.e., columnar plumes or ejected blobs)."
to:
"The average heights are weighted with respect to the primordial material field C, when buoyant primordial structures
(i.e., columnar plumes or ejected blobs) are present, their average heights cannot be distinguished compared to the
lower-lying piles if only material with $C > 0.90$ is considered. Because the volume of material in the veneer is much
lower compared to piles, its average height can be differentiated from the piles and the onset of thermal instability is easier to
analyse using this metric."
C: L. 330: "For this conductivity model": which one?
J : "For the reference case,"
C: L. 332: "we find that thermal instability appears to manifest very late" relate to Figure 9?
J : Sentence is changed to "From the temporal variations in average heights, we find that thermal instability
appears to manifest very late in the simulation run time (after approximately 11 Gyr),
which is consistent with the gradual accumulation of heat within the piles
(observed in the slowly increasing mean pile temperature (see Figure \ref{fig:figure9}(c)))."
C: L. 345: "similarly with" <- "similarly to"
J: done.
C: L. 380: "the depth- conductivity ratio" Not clear
J : "If the conductivity in the lowermost mantle (determined by the depth- dependence) is insufficient ..."
C: L. 380: "a negative feedback": to me, it seems to a be a positive feedback
J : Changed to positive feedback
C: L. 389: "but take a longer period of time to manifest": because piles need to form?
J : The paragraph now reads:
"... If depth- dependence is insufficient to compensate for this conductivity reduction, a positive feedback loop forms whereby a poorly
conducting pile cannot evacuate heat, becomes hotter, and further reduces its conductivity. The imparted thermal buoyancy destabilizes
the reservoirs and influences CMB coverage configuration, erosion rate, and the onset of entrainment. The effect of composition- dependence
is secondary to the thermal effect and quickens the onset of instability (e.g., by approximately 2 \unit{Gyr},
comparing cases \#16 and \#17; Figure \ref{fig:figure8}). Moreover, the positive feedback loop persists over a prolonged period for
conductivity models with greater depth- dependence (e.g., by approximately 4 Gyr, comparing cases \#S6 and \#17)."
C: L. 392: "may be masked by their first order influences": Not clear
J : "Given the scope of this study, isolating the effect of conductivity on stability may be masked by their first order influences." is replaced by
"Given the scope of this study, the effect of heterogeneous thermal conductivity on pile stability
is difficult to differentiate from variable conditions that affect piles' chemical and thermal buoyancy."

Referee #3

Review of : « Influence of heterogeneous thermal conductivity on the long-term evolution of the lower mantle thermochemical structure:
implications for primordial reservoirs »
by Guerrero et al.

In this manuscript, the authors seek to understand the effect of laterally-variable thermal conductivity controlled by depth, temperature
and/or compositional effects on the shape
and the dynamics of deep-seated thermochemical heterogeneities.

This manuscript has gone once into review already, and I generally believe that the authors made significant efforts to reply to the previous
referees’ comments, especially on the structure of the introduction and the presentation of the results. However, I still believe that they
can still improve their manuscript structure, which I still found hard to follow. I think that the reason is that the authors often use
complex phrasing and repeat similar statements throughout the text, which makes the text too cumbersome. I therefore believe that the authors
should make significant efforts to simplify the text in general and therefore gain in conciseness and clarity. I here-below detail some of
the instances where this can be done.

I also detail a few comments and suggestions. I think that if these comments are adressed, this manuscript should be published, given that
the results shown are of general interest for the Solid Earth community.

Main comments:

C: (1)
One of the parameters used by the authors to assess thermochemical pile dynamics and instability is the spatial average of (d□/□)prim.
I wonder how this spatial average at all times is representative of individual pile behaviour (since in most models, there are 2 piles).
Do they destabilize differently (in terms of tinst for example) or rather synchronously (as suggested by some of the composition field snapshots)?
If the latter, this would add to the argument that the instabilities detected by the authors are due to intrinsic properties of the piles.

J: Inserted the following into the first paragraph following Figure 8 in Section 3.4:
"The average heights and vertical variations in the density anomaly profiles evolve synchronously,
correspond to the same measure of instability, $t_{inst.}$, and can capture changes in individual
pile behaviour (with confirmation in the primordial field snapshots)."

C: (2)
I would also check whether two separate models with the same input parameters but a different convective history
(generated from an initial Tre field with different initial random perturbations) lead to significantly different (d□/□)prim, hc, ACMB and tins.
If so, I would tend to think that the « extrinsic » factors (such as the effect of slabs) are predominant in the dynamics of the thermochemical
piles, and that the description of Fig. 8 (From line 311) is not that meaningful to explain the role of conductivity on LLSVP dynamics.
Can the authors argue on this point ?

J: Inserted the following as the second paragraph following Figure 8 in Section 3.4:
"\indent Different initial conditions altering the thermal histories of case \#17 are examined to check the robustness of our findings.
We consider lower and higher initial mantle temperatures, T_{init}, of 1750 \unit{K} and 2250 \unit{K} and temperature perturbations,
dT, of 375 \unit{K} are added to cases with $T_{init} = 2000$ \unit{K} and 2250 \unit{K} (see Figure S15 for the radial profiles
and Table S4 for model averages in the Supplement). Overall, initial conditions on temperature control the duration of the initial
transient phase, but do not substantially alter the subsequent evolution of the system. In particular, greater dT does not
significantly alter the influence of initial downwellings or the evolution of piles and $t_{inst.}$ is approximately 0.2 \unit{Gyr}
later (Figure S16(a),(b)). In addition, $t_{inst.}$ is approximately 2 \unit{Gyr} earlier (1 \unit{Gyr} later) for systems with hotter
(cooler) T_{init} (Figure S17). A greater dT further decreases $t_{inst.}$ when $T_{init} = 2250$ \unit{K} (Figure S16(c)).
The different histories (with similar downwelling planforms) show that the onset of instability within piles is an intrinsic thermal
effect and hotter thermal conditions are also consistent with this finding."

C: (3)
I also wonder whether the authors can go further in the comparison of the dynamics of the thermochemical piles and their properties
(such as (d□/□)prim, hc, ACMB, tinst) in their different models and the observed characteristics of LLSVPs. Can the authors discuss a bit more
whether their models can help to refine the conductivity parameters for LLSVP material from the comparison of the dynamics of their modeled LLSVPs
with observations ? Can they hint at a conductivity ratio range necessary to match current observations (maybe they could also exploit the
QCMB/Qsurface ratios?) ? To what extent can they use their calculated (d□/□)prim, hc, ACMB, tinst for comparisons with the Earth ?
To me, although this is shown as a goal on line 309-310 (in the results section), such a paragraph is currently missing in the discussion section.

J:

I inserted the following two paragraphs into the discussion section:
"Our models are consistent with the 2-pile configuration found in lower resolution tomographic models. Of our simulations that attain a 2-pile configuration at approximately 4.5 \unit{Gyr}, when $n = 0.5$ (lower temperature- dependent conductivity), we find that the conductivity fields suggest a pyrolytic lower mantle conductivity between 8 and 10 \unit{W m^{-1} K^{-1}} and pile conductivity between 2 and 8 \unit{W m^{-1} K^{-1}}. When $n = 0.8$ (greater temperature- dependent conductivity), we find that the conductivity fields suggest a pyrolytic lower mantle conductivity approximately 5 \unit{W m^{-1} K^{-1}} and pile conductivity between 2 and 4 \unit{W m^{-1} K^{-1}}. These simulations suggest that a conductivity ratio greater than 1.5 and not much greater than 2 can match estimates of lowermost mantle conductivity. Furthermore, we find that in combination with temperature- dependence, the depth- dependence we propose K_{DH}, as well as linear profiles with $K_{D} = 5$ and 10, produce conductivity values between the upper and lower bounds of measurements made by\citet{geballe20}. The values of h_{C} and A_{CMB} are overall consistent with observations pointing to mostly continuous LLSVPs (as seen, for instance in \citet{houser08} seismic tomography models), but somehow disagree with the bundle models proposed by \citet{davaille20} on the basis of full waveform tomography \citep{french14}. For instance, the maximum height of our piles is approximately 500 km, which is 300 km larger than the estimates of dense basal layer in high-resolution tomographic models \citep{davaille20} and the CMB coverage we find is greater than 30\%. From our findings, we suggest that increasing the temperature dependence to an intermediate value between 0.5 and 0.8 could allow piles to attain less CMB coverage and more height; which, may help piles attain plume-like morphologies in addition to dense layering. Alternatively, lower buoyancy ratio and/or viscosity parameters may also lead to bundle-like structures \citep{deschamps18}."

C: (4)
I don’t understand why the authors analysed their models at 4.5 Gyr. Indeed, they state in the main text (Line 192-193) and in the supplementary material Lines 118-119 that « the longer simulation time (11Gyr) is necessary to allow the simulations’ heat flows to achieve a quasi steady state ».
However, they analyse a their so-called « derived quantities » (supplementary paragraph 3.3) « averaged over a 2 Gyr window centred about t = 4.5 Gyr ».
Therefore, it gives the feeling that they analysed their models before reaching a quasi-steady state. Even if it’s not the case, I still wonder why the authors did not analyse the derived quantities at the end of each simulation, to be sure that their quantities shown in Table 1 are derived from a system in quasi-steady state.

J: Inserted the following into the last paragraph of the methods section:
"In addition, it is possible that shorter simulation times (i.e., 4.5 \unit{Gyr}) may lead to a premature conclusion that primordial reservoirs have reached a metastable state. For systems with heterogeneous conductivity and constant heating conditions, significant changes in the evolution of thermochemical reservoirs may take longer to manifest."
AND
"As the system evolves after 4.5 \unit{Gyr}, the pile volume diminishes and the core-mantle boundary surface is further exposed. The surface and core-mantle boundary heat flows oscillate about different values in a new quasi-steady state. However, pile statistics including mean temperature and average height differ compared to its previous quasi-steady state. The averaged properties centred about 9.0 \unit{Gyr} and averaged from 3 to 11.2 \unit{Gyr} are presented in the Supplement (Table S2 and Table S3, respectively). In general, while the A_{CMB} values have decreased, the trends referred to in the figures still hold."

C (5)
To gain in clarity and conciseness, as I previously stated, I would advise the authors to rephrase some of their sentences (use short sentences, mostly with active form, use present tense for conjugated verbs (cf line 208-212), and remove all necessary expressions, such as « In light of this fact », line 30). I would advise to especially focus on the rephrasing of :
- Line 3-9.
J: done.
- Paragraph line 23-32
J: done.
- Line 36-41 : «an important property that influences mantle dynamics is thermal conductivity ». Say why directly after and simplify the rest of the paragraph.
J: done.
- Line 59-68
J: done.
- Line 69-83
J: done.

In fact, so far, the introduction follows this structure (per paragraph) :
1. LLSVPs observations and thermal/thermochemical interpretations,
2. LLSVPs characteristics and influence on stabillity,
3. Measured and estimated conductivity with depth/Tre and composition+ numerical models
4. Estimates and measured conductivity at HP/HT.
5. How does evolve the conductivity with P in measures/estimates.+ effect of composition on heat transfer through LLSVPs.
6. How does evolve the conductivity with increasing Tre (measures/estimates).
7. Effect of P and T dependence of conductivity in numerical models. + importance of considering compressibility.
8. Problem and structure of the article
Instead, I would change the structure of the introduction and synthesize it into the following paragraphs (this is just a suggestion):
1. Show that LLSVP dynamics, structure, origin and composition are uncertain.
2. Highlight the main characteristics of thermal conductivity as shown by experiments/molecular dynamics…, and the fact that its variations with depth/Tre/composition are uncertain, although significant.
3. Such variations could alter LLSVP dynamics and stability, as shown by already published numerical models.
4. Hightlight the limitations of these existing models (no fully heterogeneous thermal conductivity/ neglection of compressibility, … although these parameters can significantly alter LLSVP structure and dynamics).
5. Goal of the manuscript and structure of the article
J: We preferred to keep the structure of the introduction in tact to keep the flow of discussing thermal conductivity while adding sentences that highlight the uncertainty surrounding LLSVPs and the properties that affect their long-term evolution.

- Line 127-135 : Maybe, it's possible to streamline a bit this last paragraph and come more quickly to the point that :
"modeling the effect of pv-ppv transition would mask ..."
J: done.
- Line 374-389 : to me, these paragraphs rather belong to the conclusion, which repeats this. Consider at least reducing the size of that paragraph ?
J: done.

Other comments:

C: - Line 15 : I’m not sure that we can say that a profile is underestimated. Does it mean that the value of conductivity is underestimated at some specific depths, or at all depths ? If the latter, maybe replace by « when conductivity is underestimated » ?
J: In particular, the reservoirs are destabilized because the conductivity of the reservoirs is reduced. The relevant conductivity values for reservoirs are obtained in the lowermost mantle. The 'underestimation' occurs either because the temperature- dependent conductivity is stronger (and reduces conductivity values at all depths) or because the composition- dependent conductivity is stronger (and reduces conductivity values only within the piles). The sentence was simply changed to "when conductivity is underestimated"
C: - Line 20 : « greater » : by how much ?
J : Inserted "twice" so that the last sentence reads:
"For the cases we examine, when the lowermost mantle's mean conductivity is greater than twice the surface conductivity, reservoirs can remain stable for very long periods of time, comparable to the age of the Earth."
C: - Line 25-26 : replace by « have been defined as purely thermal (Davies et al, 2012) or thermo-chemical (e.g...) in nature. »
J: done.
C: - Line 30-31 : add references for both end-member views on the origin of thermochemical piles.
J : Sentence now reads:
"Two end-member views on the origin of piles (and their chemical composition) have been proposed: a primordial layer (composed of anomalously dense material) \citep[e.g.,][]{labrosse07,lee10}; and a growing layer (composed of mid-ocean ridge basalt sinking in the deep mantle and accumulating at the CMB) \citep[e.g.,][]{hirose99,nakagawa09}."
C: - Line 32 : why/how do he authors choose one of the end-members ?
J : We state that we prefer the primordial origin of thermochemical material because of evidence of undegassed material found in ocean island basalts. Simulations are initiated with dense layer at the base of the mantle.
This paragraph was simplified and no longer has the line "In light of this fact, ... we adopt the former"
C: - Line 33 : something missing between « properties » and « most » ?
J: added a comma.
C: - Line 34 : I would remove the term HPE in the abstract, and instead give the full name « heat-producing elements » in the introduction on line 34 : «heat-producing elements (HPE) », and then use just HPE, for example on line 118.
J: done.
C: - Line 35 : « underestimating their density and viscosity contrasts… » : This could lead the reader to think that we know the correct range of variation of these parameters… Then, what defines what is the “correct” proportion of entrained material? Develop a bit here.
J : Paragraph was simplified in response to comment (4). And now begins with:
"Piles' unique chemical composition determines physical properties, most importantly density, viscosity, and enrichment in HPEs that affect their long-term stability. These property values remain uncertain, but numerical simulations that emulate Earth-like piles help constrain their ranges \citep[e.g.,][]{li14,li18,li19,gulcher20,citron20}."

C- Line 39: “,which,”
J : Unclear. Remove the space between commas?
C: - Line 50 : « reduces » instead of « reduce »
J: done.
C: - Line 59 : instead of « new », shouldn’t it rather be « unprecise/unconstrained » ?
J : Replaced 'new' with 'unconstrained and coarsely sampled'.
C: - Merge lines 59-61 into 1 sentence.
J: done.
C: - Line 61 : « is fixed » ?
J: done.
C: - Line 84 : « using » instead of « utilizing » ?
J: done.
C: - Line 100-105 : should the authors precise somewhere that they will use compressible models ?
J: done.
C: - Line 124-126 : simplify : « …to avoid the development of a stagnant-lid » and then « nondimensional viscosity varies between 10-3 and 105».
J: done.
C: - Line 144 : « First, we consider a linear depth-dependence… »
J: done.
C: - Line 145 : Add « where KD = … »
J: done.
C: - Line 154 : add a reference to Fig.1
J : There is a reference at the very end of the section. I inserted a "see Figure 1" within the parentheses on Line 154.
C: - Line 159 : « Temperature-dependence of conductivity is … »
J: done.
C: - Line 161 : Lower compared to what ? Precise it.
J: done.
C: - Line 183 : remove « and may be used to briefly outline where heterogeneous conductivity affects the dynamics. »
J: done.
C: - Line 212 : « through » or « to » ?
J: done.
C: - Line 217-220 : t is evident that using a depth-dependent only conductivity, conductivity will have a radially symetric distribution and that piles will have an identical conductivity to the one of the surronding mantle. I would therefore remove these parts.
J : Removed.
C: - Line 218 : which simplification are we talking about. I get it but it’s not clear. Consider rephrasing.
J : Removed.
C: - Line 217-227 : there are many repetitions in this paragraph (notably on the fact that conductivity increases from surface to CMB – line 217/224-225). Consider shortening it.
J : The paragraph now reads:
"Comparing between each K_{D} case, for some arbitrary thermal gradient between piles and ambient mantle, a greater lowermost mantle conductivity will result in more efficient heat extraction; and thus a lower mean pile temperature. Furthermore, the heat flow at the core-mantle boundary is also increased (from 4.6 TW up to 14.7 TW, for K_{D} between 2.5 and 10), which encompass the lower and upper limits predicted for the Earth \citep{jaupart07}. Note that the greatest bottom-to-top conductivity ratio we consider, $K_{D} = 10$, implies a conductivity value of 30 \unit{W m^{-1} K^{-1}} at the CMB. However, the predicted thermal conductivities in the lowermost mantle do not exceed 10 \unit{W m^{-1} K^{-1}}. Therefore, temperature- dependence, which reduces the thermal conductivity, must be considered."
C: - Line 226 : remove « greatly » ?
J: done.
C: - Line 252 : replace « in a lower extent» by « to a lower extent» ?
J: done.
C: - Line 254 : replace reservoirs » by « piles » ?
J: done.
C: - Line 256 : replace « much » by « more » ?
J: done.
C: - Line 287-288 : consider reformulating (especially, I don’t understand what the authors mean by « slowly manifested thermal instability » and how it relates to « Thus, a much earlier ejection… »)
J : The final two sentences have been replaced with:
"Nevertheless, at greater depth- dependences, the compounded conductivity reduction is amplified, and thermal instability within piles is inevitable. Comparing cases when composition- dependence is neglected or is considered, the difference in $t_{inst.}$ is greater when depth- dependence is greater."
C: - Line 309 : « liberating ». Use a different word ?
J : Now "exposing".
C: - Line 315-322 : consider adding a subduction isotherm on the composition snapshots of Fig. 8 to better illustrate this discussion. And clearly identify the « transient period » on the plots ?
J : Subduction isotherms have now been added to the composition snapshots in Figure 8. Added:
"However, the influence of downwellings can be observed following the transient period (on the density anomaly timeseries, the initial transient period is characterized by the flat average height curves overlying the white layer below 200 \unit{km}). For cases \#16 and \#17, the transient period lasts approximately 1 \unit{Gyr} and for case \#18 approximately 1.5 \unit{Gyr}."
C: - Line 343 : « sooner compared to the reference case ».
J: done.

C : - Table. S1 : « Total internal heating rate » is not a non-dimensional parameter. Add spaces or line-centered dots between units in the units column. Add the non-dim value for the yield-stress gradient. Its unit should be Pa km-1 or Pa m-1.
J : The units and their format are set by the publisher. "Non-dimensional parameters" is replaced by "Governing parameters".
The ductile yield-stress is formulated using pressure. Therefore, the yield-stress gradient is has units of Pa/Pa. So that the dimensional and nondimensional units are identical.
C : - Fig 3 and next figures with similar snapshots : is the temperature field the relative total temperature difference compared to the CMB or the total temperature minus the adiabat difference with the CMB ?
J : These are the total temperature field offset by the CMB temperature.
Line 207 now reads: "In Figure 3 and all subsequent figures, the temperature fields are offset relative to the core-mantle boundary temperature ($T_{rel,CMB} = T - T_{CMB}$) to help illustrate temperature excesses (or deficits) within the piles."
C : - Figure 8 and SI 11, SI 12 and SI 13 : the legend of the bars are the same for hc, hc > 0.9, and hc = 0.02-0.9. They should be different from one another, to reflect the different linestyles of the green lines.
I would use a different colour than cyan for the onset of instabiliy because it’s not clearly visible. In the caption fo Fig 8, say explicitly that hc is the height of primordial material. And say explicitly that the height are plotted using green lines.
J : The plots are fixed so that the legend has the correct line styles. The dashed-vertical cyan line is now green since this is visible.
Subduction contours are added to aid in the discussion of the figure.