RESPONSE FILE:
The comments from referees will be indicated by a C.
The author responses will be indicated by an A.

Referee #1

1) C: This study uses mantle convection simulations to address the influence of pressure-, temperature- and composition-dependence of thermal conductivity on the fate of a dense primitive layer sitting at the bottom of the mantle, mimicking the LL(S)VPs on the Earth. While previous studies have investigated the effect of those various dependencies of thermal conductivity have been investigated individually, the novelty of this work is to consider consider the interplay between them in the context of the Earth’s lower mantle structure. While I think that the results are worth being published, their presentation in the current manuscript is hard to follow and lacks a clear narrative thread, and more importantly, lacks in interpretation. Therefore, I think the paper would benefit from a thorough re-organization of the results section.

A: We have decided to expand the introduction and rearrange the results section so that the manuscript is easier to follow.
CHANGE: Paragraphs were added discussing the dependencies of thermal conductivity.

General

2) C: The study adopts a rather complex baseline set up (mixed basal + internal heating, phase transition, yield stress) to study the effect of pressure, temperature and composition on thermal conductivity. While my preference goes to simpler models when trying to unravel such systematic trends, one can argue that the current set-up is relevant for an Earth-like case, and I think it is fair.

A: Yes, the mantle convection model set-up we consider is intended to be Earth-like. However, we do not intend on reproducing the Earth mantle's history. Within this framework we have been able to examine the effect of the thermal conductivity model on the evolution of the primordial layer at the bottom of the mantle.
CHANGE: no change necessary

3) C: I find the introduction a bit too light. I would expect some more background on the physics of the various dependencies of thermal conductivity, and in particular that the trends be clearly announced: increase of pressure results in increase of thermal conductivity, increase in temperature results in decrease in thermal conductivity, increase in iron content results in decrease in thermal conductivity (it seems). Also, it seems that considering a variable thermal conductivity is particularly relevant for compressible convection. If this is indeed the case, I would expect more time to be spent discussing why.

A: We agree with R1 that the introduction can be expanded. Regarding the thermal conductivity, each of the component (dependency) of the conductivity model will be discussed. In addition, we will now address why conductivity is relevant for compressible mantle convection.
CHANGE: changes outlined in response to point 1)

4) C: In my opinion, the main weakness of the paper in its current state is the lack of apparent organization in its exploration of the parameter space, which makes it pretty confused and fails at highlighting general tendencies in the effect of the different dependences of the thermal conductivity. This is probably due in part to the simultaneous changes in investigated parameters and diagnostic quantities from one section to the other (first time-evolution of the heat flux varying KC and n, then instantaneous mantle structures varying everything, and finally time-evolution of entrainment again varying KC and n). While I seem to get the idea in the progression, I had a really hard time getting a clear picture out of this section. Maybe starting by isolating the effect of each dependencies, and then considering their correlations could make things simpler. Maybe also distinguishing more systematically between the effect on mantle flow (pile structure) and on the time-evolution (e.g. of the CMB and surface heat fluxes) could help. Another possibility would be to add dependencies successively to a fiducial model, for which a preferred value of KD would be selected before considering the effect of n, and in turn a preferred value of n would be selected before adding a third layer with KC. That could spare some cases of the parameter space if there are reason to think they are less relevant. Thse are suggestion, hopefully that can help. Anyway, one important thing which is lacking in my opinion is also some theoretical speculation on the expected effect of the parameters that should come before presenting the results, and would help their interpretation (e.g. we expect the effect of increaing KD to increase the thermal conductivity in the lower mantle and thus to make it more stable to convection (it decreses Ra) by homogenizing its temperature, potentially building heat… etc.). I think it is better for the reader to be incited to think in advance of seeing the results.

A: We consider the suggestions that R1 outlines for reorganizing the results section. Introducing the reference case $K_D = 2.5$ first is too much too quickly for the reader. We now separate all the effects so that the progression of the results is easier to digest.
CHANGE: The subsections of the results are now:
3.1) Effect of a purely depth- dependent conductivity
3.2) Effect of a temperature- and depth- dependent conductivity
3.3) Including the effect of composition- dependent conductivity
3.4) Long-term stability of thermochemical reservoirs featuring mineral physics derived conductivities

Figure 3 in 3.1) now features field snapshots for purely depth- dependent cases.
Figure 4 in 3.2) and Figure 5 in 3.3) now highlight field snapshots for the non-zero end-member linear depth- dependent cases.
Figure 6 in 3.4) now features field snapshots for different thermal conductivity models featuring a mineral conductivity derived depth- dependence K_{DH}.
Figure 7 in 3.4) shows the corresponding timeseries for cases in Figure 6.
Old Figure 7 is moved to Figure 8 and the corresponding timeseries is included in Figure 9.

Specific

5) C: This article will probably mainly be read by people familiar with the equations of mantle convections. Nevertheless, I think it would be beneficial to write down the conservation equations (they are not even in the supplementary materials!), at least the heat conservation where the thermal conductivity appears, as it would make clear where the supplementary mechanism induced by varying thermal conductivity operate.

A: The heat conservation equation was added to the Methods section to help the reader understand where the conductivity model influences mantle dynamics.
CHANGE: as above.

6) C: A plot of the thermal conductivity profile corresponding to the reference state for the various KD, KC and n (not all of them but the few most relevant combinations, e.g. the cases presented in Figure 2) would be insightful. It could be a supplementary panel in Figure 1, or a stand-alone figure.

A: A new figure with a plot of the initial thermal conductivity profiles for cases featuring mineral physics defined conductivities is now included so that the conductivity reductions from different temperature- and composition- dependencies is clarified.
CHANGE: A new Figure 2 is inserted into the Methods section to highlight initial conductivity

profiles. A paragraph describing these variations is also included.

7) C: I don’t really understand why simulations run until 11 Gyr, but some values are avergaed aroung 4.5 Gyr. Anyway, the averaged values reported in Table 1 do not seem to be used.

A: The simulation time is long (~11 Gyr) to allow the system to reach a statistically steady state and to investigate how the heterogeneous conductivity will affect the long-term evolution of the primordial layer. The simulation averages are taken towards the start of simulations' statistically steady state (approximately 4.5 Gyr). Again, we do not intend to model the exact evolution of the Earth's mantle. The conditions for simulating Earth's history (i.e., initial conditions, decaying heat sources, and cooling bottom boundary) are not included in our model setup. In the current version of the manuscript, the averaged values were inset within the annulus snapshots and their trends were discussed within the results subsections. Specific values were not used often so that the text would not be inundated with numbers. We now incorporate more references to the Table 1 averages into the manuscript.
CHANGE: An updated Table 1 (including supplemental cases) is referred to in the updated results section.

8) C: Dimensional and non-dimensional quantities are often mixed, which lacks a bit of rigor.

A: The values presented in the manuscript will be presented as dimensional.
CHANGE: as above. Colour bar for conductivty cases is now modified so that one can also interpret the field data as non-dimensional.

Technical
L. 48: “compositional- dependent” ← “composition-dependent”
A, CHANGE: for clarity, "composition- dependent" will now be used throughout the manuscript.

L. 84: “a quadratic that smoothly” ← “a quadratic curve that smoothly”
A, CHANGE: inserted the word 'curve'.

Figure 1: the markers for “<200 ppm water” are hard to see, please change for a more contrasting colour.
A, CHANGE: markers for "< 200 ppm water" are changed to bright green to be more contrasting with the background.

L. 118: “We first defined a purely depth- dependent reference case characterized by depth- dependence, $K_D = 2.5$, with lower mantle conductivities comparable to current estimates” in Figure 1, it seems that $K_D = 10$ is the closest match to lower mantle estimates.
A, CHANGE: The case with a purely depth- dependent thermal conductivity and $K_D = 2.5$ is considered as an analogue for models that already account for the combined depth- and temperature- dependences. In the latter (hypothetical) model of thermal conductivity, the depth- dependent component would have conductivity values that are much greater than those defined by $K_D = 2.5$. HOWEVER, this section has changed and the sentence is no longer relevant.
No change necessary.

L. 121: “by approximately 75%” it seems less than that in Figure 2.
A, CHANGE: "by approximately 75%", this refers to a comparison of heat flow values averaged about 4.5 Gyr for cases #4 and #8. This comparison is between a purely depth- dependent conductivity case (#4) and heterogeneous conductivity case (#8).
HOWEVER, this section has changed and this sentence no longer refers to cases with $K_{D} = 2.5$. No change necessary.

L. 122: “in agreement with Li et al. (2022) findings” ← “in agreement with Li et al. (2022)’s findings”
A, CHANGE: fixed the citation to read "... Li et al., (2022)'s findings".

L. 150: “We observed that T_{prim} increased with greater temperature dependence (top-to-bottom rows in Figure 3).” I don’t see it.
A: The mean temperature of primordial material is indicated by T_{prim} (not to be confused with the global mean temperature of the system <T>). T_{prim} is inset within the annulus and is indicated for each case. T_{prim} can be seen decreasing for cases with the same K_D value as n is increased. In addition, the temperature field is offset with respect to the CMB temperature so that the differences in hotter temperatures within the piles is easier to see. The boundary between regular mantle material and primordial material is indicated by a green contour. When n is increased, the red colour within these contours becomes more saturated.
CHANGE: No change necessary. This trend is still visible in the new Figure 4.

For the snapshot figures, when a parameter is held constant, write it in the caption rather than for each snapshot. That will lighten a bit the figures.
A, CHANGE: Snapshot figures have been revised so that superfluous conductivity model labels are omitted.

Figure 5: Please alternate colormaps between the different quantities plotted. In particular, don’t take a “divergent” colormap for the primordial mantle one which only has two extremal values, which are both very dark and hard to distinguish in the current plot.
A, CHANGE: The colormaps for each different field are now alternated. Conductivity field snapshots now have a colormap that can also be interpretted in a non-dimensional values (i.e., the conductivity ratio).

L. 172: “and is equivalent to” ← “and which is equivalent to” ?
A, CHANGE: inserted the word 'which'.

Figure 6: Same as figure 5: it is very hard to know which colormap corresponds to thesnapshot and which corresponds to the time-evolution plots. Also please change colours for the onset of instability (the magenta is pretty hard to see).
A, CHANGE: The colormap is changed. The onset of instability is now indicated by a dashed cyan line.

L. 189: “may not compensate (or be exceeded) by” ← “may not compensate (or be exceeded by)”
A, CHANGE: The parantheses have been moved.

Supplementary Materials
L. 43: “and the depth variation of thermal expansivity imply a depth average” ← “and the depth variation of thermal expansivity implies a depth average”
A, CHANGE: "imply" -> "implies".

L. 176: “The density anomalies [...] is calculated” ← “The density anomalies [...] are calculated”
A, CHANGE: "is" -> "are".

Figure S1: the y-axis is non-dimensional height, not depth.
A, CHANGE: All vertical axes in radial profiles now correspond to depth values.

Figure S4: What do line styles correspond to?
A, CHANGE: The case numbers have been added to the legend so that the line styles are now clear. Line style descriptions are also included in the captions.

Referee #2

C: In this paper the authors carried out numerical experiments of thermochemical mantle convection by varying the spatial changes in thermal conductivity to investigate the temporal changes in the distributions of dense "primordial" materials initially imposed above the bottom boundary. My honest impression is, unfortunately, that the manuscript is very hard to follow because of its poor organization and description from the reasons summarized below. I therefore strongly suggest that the authors should thoroughly revise the manuscript before the reconsideration.

A: In addition to the commments made by R1, we have decided to thoroughly revise the manuscript so that it is easier to follow.

CHANGE: Relevant changes have been outlined in responses to Referee #1.

C: 1. One of my major criticism is that the main issues in this study were not well described either in the abstract or introduction section. In my understanding the major intention of the authors is to reduce the thermal conductivity in the deep mantle to much lower levels than those simply expected from the dependence on temperature and pressure (or depth), in order to induce an "instability" from the initial layer of dense materials within a sufficiently short period of time. The authors should clearly indicate their ultimate motivation in earlier parts of the manuscript.

A, CHANGE: In alignment with the comments of R1, the introduction section was expanded and the intentions of our study were made clearer.

C: 2. I am not well convinced of the meaningfulness of the onset time of "instability" from the initial layer of dense materials. It seems to me that the authors had assumed that the deformation of the basal layer of dense materials occurs only in an "intrinsic" manner owing to the thermal buoyancy. However, several earlier studies had demonstrated the ultimate importance of "extrinsic" deformation induced by the cold subduction from the top surface. I do not therefore think that the onset time of "instability" can be a good measure to investigate the influence the thermal conductivity at depth on the thermal buoyancy in the basal layer of dense materials.

A: We calculate the onset time for instability from the temporal variations in the average height of dense material. It is true that these variations in average height do not discriminate between 'intrinsic' (i.e., thermal buoyancy) or 'extrinsic' (i.e., downwellings) deformation. From examining the average height timeseries and animations of the fluid flow we can see the influence of downwellings. The first downwellings impringe on the initial dense layer but do not result in a rapid uplift of material (sufficient to eject blobs of dense material). Once the initial dense layer has organized into piles, downwellings tend to move dense material laterally over the CMB but not rapidly increasing the pile height. Furthermore, we find that it is easier for downwelling currents to push primordial material that has been made lighter due to their retained heat. We agree that downwellings are important in deforming the dense primordial layer.
CHANGE: In section 3.4) this mechanism is discussed in addition to the thermal effect regarding instability.

C: 3. I felt quite odd with the authors' exaggerated claim on the overall profiles of thermal conductivity including "We find that the temperature- and depth- variations combined characterize the mean conductivity ratio from top-to-bottom" in the abstract. It is quite obvious from the assumptions made by the authors themselves.
A: Right. This statement is a bit of a tautology. The intention for this statement is that for Earth-like models that consider a variable thermal conductivity, it may be ill advised to isolate for just one dependency. On one hand, to simulate a 2-pile configuration, a purely depth- dependent conductivity may be employed, but its top-to-bottom ratio should implicitly emulate the temperature and composition effects. On the other hand, a purely temperature- dependent conductivity will result in the entrainment of a dense primordial layer. By assuming a parametrized conductivity model, it is predictable what the mean conductivity ratio from top-to-bottom will be (having specifed the temperature contrast and the dependencies of the conductivity model).
CHANGE: This sentence was omitted and changed to read:
"The mean conductivity ratio from bottom-to-top indicates the relative competition between the decreasing effect with increasing temperature and the increasing effect with increasing depth. We find that, when depth- dependence is stronger than temperature- dependence, a mean conductivity ratio > 2 will result in long-lived primordial reservoirs."

C: 4. In Section 2.1 the authors should state the reason why the phase change from perovskite (pv) to post-perovskite (ppv) is ignored in their numerical model. If they believe that the pv-ppv phase change is of little importance on the dynamic behaviors of the basal layers of dense materials, the authors should make clear the reason why.

A, CHANGE: In the methods section, we now discuss the reasons why the phase change from perovskite (pv) to post-perovskite (ppv) is ignored in our numerical model. The effects of the pv-ppv transition properties on the stability and structure of primordial reservoirs has been investigated previously by Li et al., (2015). There are many controlling parameters for the pv-ppv transition including the temperature of the CMB, the viscosity contrast between pv and ppv, and the viscosity contrast between ppv and primordial material that can affect the stability of piles. For instance, weak ppv (i.e., low viscosity contrast between pv and ppv) and a low Tcmb (i.e., Tcmb ~ 3350 K) can result in entrainment of primordial reservoirs. Because of the model setup we consider in our study, the inclusion of a pv-ppv transition will result in the entrainment of a dense primordial reservoir. Thus, the pv-ppv phase transition will mask the effect of thermal conductivity on the stability of primordial reservoirs that we are examining.

C: 5. I was quite disgusted to see that the profile given by equation (2) is denoted by "KD=9.185". Such a denotation should be used only for the profiles given by equation (1) !!

A, CHANGE: All usage of the label "K_D = 9.185" will be replaced with "K_{DH}" to indicate that this depth profile was defined by parameterizations published in Deschamps and Hsieh, (2019).

C: 6. Near equation (3) the authors should indicate the magnitude of the reduction in thermal conductivity due to the increase in temperature within the modeled domain by the choice of n=0.5 and n=0.8.

A, CHANGE: The magnitude of the reduction in thermal conductivity depends on temperature (which changes with depth). A simple calculation was added to the methods section to show how much the conductivity is reduced at CMB temperatures for different n and K_{C} values. The reduction in conductivity will also be shown by the inclusion of a new figure which shows the initial conductivity profiles for cases featuring K_{DH} with different temperature- and composition- dependence.

C: 7. Near equation (4) and later, "compositional correction" should be rephrased with "compositional dependence". The word "CORRECTION" could imply that the numerical experiments without the compositional dependence in thermal conductivity are meaningless.

A, CHANGE: In alignment with the comments of R1, all usage of "compositional correction" was rephrased to "composition- dependence".

C: 8. In Section 3.1 "QCMB" is used without explicitly defined.

A, CHANGE: Q_{CMB} and Q_{SURF} will be clearly defined in Section 3.1.

C: 9. To show the 2-D distributions of thermal conductivity in Figures 4 and 5, the authors should show the ratio of thermal conductivity to its surface value (KS) rather than the values of conductivity itself.

A, CHANGE: As suggested by R1, we will keep consistent with dimensional values. We currently present 2D distributions of thermal conductivity in Figures 4 and 5. By plotting the conductivity fields relative to the surface value k_{S}, it would be converted to the non-dimensional conductivity field. The colour map is now adjusted so that conductivity values can be interpretted nondimensionally with colour. That is, values < 3 W/m/K (ratio < 1) tend to blue-ish hues and values > 3 W/m/K (ratio > 1) tend to brown hues.

C: 10. The paper by Marzotto et al. (2020) has not been cited anywhere in the main text.

A, CHANGE: Marzotto et al., (2020) will be removed from the references. Conductivity data points included in Figure 1 are from the references cited within the caption.

C: 11. Near equation (8) of the Supplement, the spatial coordinates should not be in Cartesian (x,y,z) but in 2-D polar (r,phi) in this study. Similarly in equation (12) of the Supplement, the coordinate is not in 3-D polar (r,theta,phi).

A, CHANGE: Coordinates for 3D geometry (x,y,z) or (r,theta,phi) will be replaced by 2D spherical annulus coordinates (r,phi).

C: 12. In Section 3.1 of the Supplement, I think that the authors can use the potential temperature instead of the "adiabatic correction" a(z).

A, CHANGE: The potential temperature definition will also be stated in addition to the adiabatic correction.

C: 13. The paper by Xu et al. (2004) has not been cited anywhere in the Supplement.

A, CHANGE: Xu et al., (2004) is removed from the references. This reference was included in the methods section discussion on the thermal conductivity model (Section 2.2) and had been moved from the supplement to the main text. The reference to Klemens,(1960) in the supplement is also be removed for the same reason.