Transporting CRM Variance in a Multiscale Modelling Framework

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Abstract. An unphysical checkerboard pattern has recently been identified in E3SM-MMF that is hypothesized to be associated with the inability of the large-scale dynamics to transport fluctuations within the embedded CRM on the global grid. To address this issue, a method is presented to facilitate large-scale transport of CRM variance in E3SM-MMF. Simulation results show that the method is effective at reducing the occurrence of unphysical checkerboard patterns on a range of time scales, from days to years. This result is confirmed both subjectively through visual inspection and quantitatively with a previously developed pattern categorization technique. The CRM variance transport does not significantly alter the model climate, although is does tend to reduce temporal variance on fields associated with convection on the global grid.

1 Introduction

The multi-scale modelling framework (MMF) is an innovative method to explicitly represent clouds in an atmospheric general circulation model (GCM) that cannot otherwise resolve convective scale circulations by coupling each GCM column to a cloud resolving model (CRM) (Grabowski and Smolarkiewicz, 1999; Grabowski, 2001; Randall et al., 2003; Khairoutdinov et al., 2005). The two models cover vastly different scale ranges and are coupled via forcing and feedback tendencies that are formulated such that the domain mean state of the CRM cannot drift away from the state of the parent GCM column (Grabowski, 2004; Randall et al., 2016). This coupling approach leaves a "gap" in the total range of scales simulated by the coupled system where neither model can provide an explicit representation. Typically, this gap occurs at the scale of mesoscale convective systems. Despite filtering out potentially important weather systems, the scale gap can be leveraged to improve computational performance through hardware acceleration with GPUs (Hannah et al., 2020) and algorithmic CRM mean state acceleration (Jones et al., 2015).

The MMF scale gap has also been implicated in causing a "checkerboard" signal on the grid of the parent GCM by Hannah et al. (2022), in which neighboring grid cells systematically exhibit more or less convective activity. A pattern detection method was developed to quantify the extent and persistence of the checkerboard pattern in E3SM-MMF. This analysis also revealed that E3SMv2 exhibits a checkerboard pattern, which seems to operate though a different mechanism that is directly linked to the revised trigger for deep convection that relies on CAPE generation by dynamics (DCAPE).

Another consequence of the MMF scale gap is that small-scale fluctuations associated with convective circulations within the CRM cannot be advected by the large-scale flow represented by the GCM. This is because the CRM only communicates to larger scales through the coupling of the CRM domain mean (Pritchard et al., 2011). Thus, variance at the scale of convective

elements can effectively become "trapped" within the CRM. This might not seem like a problematic feature of the MMF given that cloud life cycles are typically short compared to the GCM time step on the order of 10-30 minutes. However, the checkerboard problem in E3SM-MMF suggests that this peculiarity of the MMF approach can lead to an unphysical feedback

30   cycle that allows the pattern persist.

Another way to think about the variance trapping problem is to consider the CRM simply as a GCM parameterization of sub-grid scale (SGS) processes with a higher order closure. The CRM is exactly solving for the GCM-SGS variance and ignoring horizontal transport, which forces the system to produce a local balance between the production and dissipation of GCM-SGS variance. In the case of water vapor, a significant pathway of variance dissipation is precipitation, but because precipitation

35   can only dry the system it leads to an anomalous effect on the CRM mean, which can in fact be transported, and sets up the checkerboard.

An ideal solution to this issue is to allow the CRM fluctuations to be advected by the flow of the large-scale model, since this interaction between scales naturally happens in the real atmosphere. The quasi-3D framework of Jung and Arakawa (2014) is one way to enable this type of scale interaction, but this method is still in an experimental phase and not well suited to being

40   implemented into the current E3SM infrastructure. Another potential solution that has been proposed in our discussions is to relax the requirement of cyclic horizontal boundary conditions in the CRM. Several versions of how this could be achieved have been discussed, and we have conducted experiments with one approach (not shown), but all of these involve problematic compromises that discourage a detailed exploration.

The most tractable solution for transporting CRM fluctuations, or variance, is to encode these fluctuations into a set of

45   massless tracer fields that can be advected by the large-scale dynamics. Developing this method requires defining how the variance will be encoded, as well as how the variance tracers on the GCM grid will be coupled to the actual variance within the CRMs. In this study we present such a method for transporting cloud scale variance in E3SM-MMF. The general variance transport method is detailed in Section 2 along with a discussion of possible variations on the method. This is followed by a description of model experiments, satellite data, and checkerboard detection method used to validate the variance transport in

50   Section 3. Discussion of the results and conclusions are presented in Sections 4 and 5.

2   MMF Variance Transport Method

2.1   MMF Coupling Review

The canonical MMF coupling involves forcing and feedback tendencies of some quantity q between a large-scale model (GCM) and small scale model (CRM, Grabowski, 2004). The values of q are defined independently for each model, but the tendencies

55   must be formulated such that the q fields of each model maintain equality at the end of each time step. This coupling procedure starts with a provisional time step where the GCM updates the GCM variable denoted q_G with a tendency B_G, which includes things like advection, over the GCM time step Δt_G. So we write,

$$\widetilde{q_G^{n+1}} = q_G^n + B_G \Delta t_G \tag{1}$$

so that

$$B_g = \frac{\widetilde{q_G^{n+1}} - q_G^n}{\Delta t_G}. \tag{2}$$

The tilde denotes a provisional value, and (1) represents a partial time step. The corresponding CRM variable q_C is updated using the CRM time step Δt_C:

$$\frac{q_C^{m+1} - q_C^m}{\Delta t_C} = B_C + \left(\frac{\widetilde{q_G^{n+1}} - \langle q_C \rangle^n}{\Delta t_G} \right). \tag{3}$$

The subscripts m and $m+1$ denote successive CRM time steps. The term B_C represents the processes calculated within the CRM, such as microphysics. The quantity $\langle q_C^n \rangle^n$ is the CRM domain average of q_C at the beginning of the GCM time step. The last term of (3) is the mechanism that communicates the tendencies of the GCM to the CRM. In other words, it is the forcing of the CRM by the GCM.

After taking enough CRM time steps to span the length of the GCM time step, the GCM variables are updated with a feedback tendency,

$$\frac{q_G^{n+1} - q^n}{\Delta t_G} = B_G + \frac{\langle q_C \rangle^{n+1} - \widetilde{q_G^{n+1}}}{\Delta t_G} \tag{4}$$

$$= B_G + \frac{\langle q_C \rangle^{n+1} - q_G^n + B_G \Delta t_G}{\Delta t_G} \tag{5}$$

$$= \frac{\langle q_C \rangle^{n+1} - q_G^n}{\Delta t_G} \tag{6}$$

Here $\langle q_C \rangle^{n+1}$ represents the horizontal mean q_C at the end of the CRM integration, or at GCM time $n+1$.

Comparing the two sides of (6), we see that

$$q_G^{n+1} = \langle q_C \rangle^{n+1},$$

which indicates that the two model states are identical at the end of the GCM time step, or, equivalently, at the beginning, and guarantees that the two model states cannot drift apart.

2.2 CRM Variance Coupling

To implement the coupling of variance between CRMs in the MMF we can largely follow the coupling scheme for an arbitrary tracer as described above. However, since we are only concerned with fluctuations about the CRM mean state, we do not want the forcing tendencies to have any impact on the CRM mean state. Thus, the CRM forcing must only operate on local perturbations rather than the horizontal mean, while the coupling with the GCM will need to be a bulk measure of the variance within the CRM. The tracer used for variance transport on the GCM grid does not have mass, so it does not affect anything outside of the CRM. The GCM is only used to move the CRM variance around, analogous to how clouds and small-scale

turbulence can be advected by large-scale flows in the real world. The cloud scale processes can still provide a representation of "cumulus friction" via CRM momentum feedback, including pressure gradient effects, which can be simulated with a 2D CRM using the method of Tulich (2015).

The CRM variance feedback tendency of some quantity q at a given vertical level of the CRM that is applied to the GCM can be formulated analogous to (4):

$$\frac{Q^{n+1} - Q^n}{\Delta t_G} = \frac{\overline{q'q'}^{n+1} - \widetilde{Q^{n+1}}}{\Delta t_G}, \tag{7}$$

where Q represents the massless variance tracer on the GCM grid, and q' is the perturbation from horizontal mean on the CRM grid. The quantity $\overline{q'q'}^{n+1}$ is the total variance of q at the end of the CRM time integration.

Since the CRM columns do have any specific location within the parent GCM column, the forcing or "injection" of variance into the CRM cannot be constrained to any particular spatial pattern. Thus, we rely on scaling the existing perturbations from the horizontal mean at each time step so that the existing spatial structure of variance within the CRM is preserved. Thus, we will define a scaling factor α, that will be used to scale the existing perturbations in order to increase or decrease the variance to match the variance forcing from the GCM.

In order to constrain the scaling factor α, the total variance forcing applied to the CRM can be formulated analogous to (3):

$$\frac{\overline{q'q'}^{m+1} - \overline{q'q'}^m}{\Delta t_C} = \left(\frac{\widetilde{Q^{n+1}} - \overline{q'q'}^n}{\Delta t_G} \right) \tag{8}$$

so that

$$\overline{q'q'}^{m+1} = \overline{q'q'}^m + \Delta t_C \left(\frac{\widetilde{Q^{n+1}} - \overline{q'q'}^n}{\Delta t_G} \right) \tag{9}$$

Note that we are ignoring the internal CRM processes that will affect the variance, but this is not important for formulating α.

At each time step the adjusted variance can be formulated in terms of the variance of the scaled perturbations, or by scaling the total variance by α^2,

$$\overline{q'q'}^{m+1} = \overline{\alpha q' \alpha q'}^m = \alpha^2 \overline{q'q'}^m \tag{10}$$

By combining (9) and (10) we can solve for α as,

$$\alpha = 1 + \frac{F \Delta t_C}{\overline{q'q'}^m} \tag{11}$$

In practice, the variance forcing is applied at each CRM time step by calculating the perturbations of q and value of α, then scaling the q perturbations by α and calculating a tendency to be applied back to the full state. This allows alternate definitions of perturbations used for the variance calculation (see discussion below). In certain rare circumstances the variance forcing can be too strong and lead to instability within the CRM. Since we are not concerned about preserving the cloud scale variance of thermodynamic tracers globally we can address this issue with a simple limiter. We find that restricting α to be between 0.90 and 1.10 works well without qualitatively affecting the model behavior.

Another important step for implementing the variance transport method is to decide which quantities will be affected. Currently, the CRM used in E3SM-MMF is based on the system for atmospheric modelling (SAM, Khairoutdinov and Randall, 2003) and uses liquid-ice static energy and total non-precipitating water as the main prognostic thermodynamic quantities. These are the only two thermodynamic quantities directly affected by variance transport in E3SM-MMF. The single moment microphysics scheme employed by the CRM allows cloud liquid and ice to be diagnosed from the total water, so there is no need to consider the variance of cloud condensate. We initially avoided transporting variance of the horizontal momentum field due to concerns about complications arising from the use of the anelastic approximation. However, we have since enabled momentum variance transport and found that it results in a notable improvement in suppressing the checkerboard pattern, so the simulations shown here include horizontal momentum variance transport.

2.3 Alternative Approaches

We refer to variance transport method described above as "bulk variance transport" since we lump the CRM fluctuations of all scales together by calculating the total variance at each vertical level. The bulk approach makes no distinction as to what scales are most represented in the variance tracers, and because we rely on the existing perturbations in a CRM when applying the variance forcing it is possible to transport relatively large scale variance in one CRM and inject into the smallest scales of another CRM. During the course of this work we have considered a few alternatives approaches to address this inter-scale variance transport that allow some flexibility in choosing which scales are represented in the variance that is being transported.

One of the earlier ideas was to transport multiple scalars for each prognostic CRM variable corresponding to the Fourier transform coefficients obtained from the horizontal dimension. This idea was appealing because it allowed the variance at each scale to be transported separately, so there was no chance of inter-scale variance exchange. However, experiments with this approach exhibited some odd behavior that we could not fully explain, and it also did not reduce the occurrence of the checkerboard pattern. We suspect that this method is fundamentally flawed because it ignores the important cancellations between Fourier modes that give rise to a realistic atmospheric state. It was also a more expensive method due to the larger number of tracers.

Another alternative that we have explored is to use a spatial low pass filter to isolate a certain scale range for the perturbations used to define the variance tracers. By using a filtered CRM state we can restrict the inter-scale variance exchange to occur only within the bounds defined by the filter. In our experiments we use a fast Fourier transform (FFT) to serve as a low pass filter in the horizontal dimension of the CRM to isolate the largest few wavenumbers. The resulting filtered perturbations can be used in the method for variance forcing and feedback calculations outlined above without modification. Experiments using this "filtered variance transport" technique were consistent with the bulk variance transport experiments, with some subtle, but unique, changes to certain climatological features. Further sensitivity experiments show that the filtered method converges to the bulk method as the cutoff wavenumber is increased (not shown). Ultimately, our analysis suggests that the potential inter-scale variance transport is not problematic, so it preferable to use the bulk method, but we have included a case with the filtered method in our analysis below.

3   Data and methods

3.1   Checkerboard Detection Method

For a quantitative assessment of the checkerboard pattern we use the pattern detection method used by Hannah et al. (2022),
which is briefly described here. The goal of this method is to catalog all possible patterns of anomalous values in a local
neighborhood of adjacent points for each point on the grid and then objectively determine if any patterns are occurring too
frequently when compared to satellite observations.

To catalogue each neighborhood state, the eight adjacent values around a central point are encoded as 0 for values less
than or equal to the neighborhood mean (including the central point), and 1 otherwise. We are left with a binary sequence
corresponding to the eight adjacent neighbors ordered in a clockwise fashion that describes the neighborhood state. A pure
checkerboard pattern shows up as an alternating sequence of 1's and 0's. The smoothness of a pattern can also be inferred from
the variations of this sequence.

The pure checkerboard pattern is relatively rare compared to other patterns, which is somewhat expected due to ubiquitous
synoptic scale gradients that can coexist with the pure checkerboard on short time scales. There is no clear way to separate
these synoptic signals from the underlying checkerboard, so it is useful to focus on patterns that contain only a portion of
the full checkerboard pattern. To do this we identify neighborhood state patterns that contain a continuous alternating binary
sequence of length four or more and consider these to be "partial checkerboard" cases. Using alternate definitions of partial
checkerboard does not qualitatively change our results (not shown).

3.2   E3SM-MMF Description

E3SM was originally forked from the NCAR CESM (Hurrell et al., 2013), but has undergone significant development since
then (Golaz et al., 2019; Xie et al., 2018). The dynamical core uses a spectral element method on a cubed-sphere geometry
(Ronchi et al., 1996; Taylor et al., 2007). Physics calculations are done on a finite volume grid that is slightly coarser than the
dynamics grid, but more closely matches the effective resolution of the dynamics grid (Hannah et al., 2021).

The MMF configuration of E3SM (E3SM-MMF) was originally adapted from the super-parameterized CAM (SP-CAM;
Khairoutdinov et al., 2005). E3SM-MMF has also undergone significant development, but the model qualitatively reproduces
the general results previously published studies (Hannah et al., 2020). The embedded CRM in E3SM-MMF is adapted from the
System for Atmospheric Modeling (SAM) (Khairoutdinov and Randall, 2003). Microphysical processes are parameterized with
a single moment scheme, and sub-grid scale turbulent fluxes are parameterized using a diagnostic Smagorinsky-type closure.
Aerosol concentrations are prescribed with present day values. The embedded CRM in E3SM-MMF uses a two dimensional
domain with 64 CRM columns in a north-south orientation and $2\,\mathrm{km}$ horizontal grid spacing.

In order to facilitate better CRM performance when running on GPU machines the CRM was completely rewritten in C++
(SAM++), which provides substantially improved GPU throughput over the original Fortran code with OpenACC directives.

The SAM++ model relies on the performance portability library of Yet Another Kernel Launcher (YAKL) [1]. In addition to GPU hardware acceleration, E3SM-MMF can also leverage algorithmic acceleration through the mean-state acceleration scheme of

180   Jones et al. (2015). E3SM-MMF also gains performance by limiting radiation to operate of even subsets of the CRM domain rather than each CRM column individually (Hannah et al., 2020). Additional refactoring of the MMF physics code has been able to enhance the hardware acceleration by minimizing data transfer overhead between CPU and GPU, as well as leverage OpenMP threading to maximize the throughput of calculations outside of the CRM.

Aside from the difference in how convection is treated, E3SM-MMF differs from the standard configurations of E3SM

185   in several important ways. The stability of E3SM-MMF is noticeably improved by reducing the global model physics time step from 30 to 20 minutes. The 72 layer vertical grid of E3SMv2 was also found to be problematic for the performance of E3SM-MMF because thin layers near the surface necessitate a 5 second CRM time step for numerical stability. Therefore, the E3SM-MMF simulation shown here uses an alternative 60 layer vertical grid that allows a longer 10 second CRM time step. A final stability concern has to do with high frequency oscillations of temperature, humidity, and wind near the surface. Both

190   models exhibit these oscillations, but they render E3SM-MMF much more susceptible to crashing. A temporal smoothing of surface fluxes with a 2 hour timescale is used to address this problem, which does not have any notable impact on the model climate (not shown).

3.3   E3SM-MMF Simulations

All simulations are run for 10 years using 128 nodes of the OCLF Summit machine (768 MPI ranks). The global cubed sphere

195   grid was set at ne30pg2 (30x30 spectral elements per cube face and 2x2 FV physics cells per element), which roughly corresponds to an effective grid spacing of $150\,\mathrm{km}$, while the average spacing of spectral nodes for dynamics in closer to $100\,\mathrm{km}$. The model input data for quantities such as solar forcing, aerosol concentrations, and land surface types, are derived from a 10-year climatology over 2005-2015 to be representative of climatological conditions around 2010. Sea surface temperatures were similarly prescribed using monthly climatological values that are temporally interpolated to give a smooth evolution (Taylor

200   et al., 2000).

3.4   Satellite Observation Data

Following Hannah et al. (2022), we use the same pattern detection method to assess the occurrence of checkerboard patterns in E3SM-MMF. We will also use satellite data remapped to the model grid to serve as a benchmark for how much noise should be tolerated in the model data. The specific time period of satellite data used for analysis is arbitrary. We choose to use daily mean

205   data over 2005-2014 for all data sets. Since the checkerboard pattern is most visible in cloud liquid water path and precipitation fields, we use comparable satellite estimates of these fields to provide a baseline of the spatial distribution of these quantities.

Satellite estimates of cloud liquid water path are provided by the Multisensor Advanced Climatology of Liquid Water Path (MAC-LWP) data product (Elsaesser et al., 2017). We use a daily-resolution version of the product (McCoy et al., 2020), with LWP estimates provided on a $1.0°\mathrm{x}1.0°$ equiangular grid that is then regridded to the ne30pg2 grid used by the model. MAC-

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[1]see https://github.com/mrnorman/YAKL for more information

210 LWP additionally provides total (cloud plus precipitating) liquid water path estimates (TLWP), and we use TLWP to create a gridded quality control mask that hashes regions for which the ratio of LWP to TLWP is less than 0.6, broadly following the recommendation in Elsaesser et al. (2017). Hashed regions envelope grid boxes for which LWP estimates exhibit substantial uncertainty (and potential systematic bias) due to errors in isolating and quantifying the cloud liquid water radiometeric signature from that of the total liquid water radiometeric signature in microwave retrievals.

215 The Global Precipitation Measurement (GPM) mission, the successor to the Tropical Rainfall Measurment Mission (TRMM), was launched in 2014 with the goal of producing accurate and reliable estimates of global precipitation with all available data TRMM and GPM eras (Hou et al., 2014). The Integrated Multi-satellite Retrievals for GPM (IMERG) combines several satellite datasets to produce an integrated rainfall data product that has proven to perform well in various regions (Anjum et al., 2018; Kim et al., 2017). Daily mean IMERG data is available on a $0.1° \times 0.1°$ grid, which is much finer than the grid used for

220 the model simulations used here. To facilitate direct comparison we regrid the IMERG data to the ne30pg2 model grid, as well as a $1.0° \times 1.0°$ equiangular grid to match the MAC-LWP data.

In addition to our checkerboard analysis and mean-state assessment, we are also interested in characterizing any change in mode of variability that might occur from enabling variance transport. For this we will focus on analyzing tropical wave variability and will use daily mean outgoing longwave radiation (OLR) from the National Oceanic and Atmospheric Adminis-

225 tration (NOAA) polar-orbiting satellites (Liebmann and Smith, 1996) as our observational benchmark.

4 Results

In this section we will compare E3SM-MMF results from simulations that employ the bulk variance transport (BVT) and filtered variance transport (FVT) methods described in section 2 to a control simulation of E3SM-MMF with no variance transport and satellite data.

230 ### 4.1 Climatology

Figure 1 shows 10-year mean maps of precipitation from IMERG and model data on the model's ne30pg2 physics grid using shaded polygons to highlight signals at the grid scale. All simulations exhibit a similar spatial distribution of precipitation with typical biases relative to observations, such as too little precip over the amazon, too much precip over the Northwest tropical Pacific and Pacific ITCZ, and a prominent double ITCZ.

235 The control simulation produces a noticeable checkerboard pattern, particularly in subtropical regions, consistent with Hannah et al. (2022). Similar results can be found for other variables related to convection (not shown). Both BVT and FVT simulations have a smoother climatology with no clear checkerboard patterns in the long-term mean, while still retaining the characteristic spatial distribution. This confirms that the variance transport method is able to address the variance trapping issue that we hypothesize is producing checkerboard patterns in E3SM-MMF.

240 Climatological differences of precipitation and liquid water path relative to the control simulations reveal that changes induced by variance transport are not restricted to reducing the checkerboard signal (Fig. 2). Regional changes in convective

activity include a slight enhancement of precipitation in the Northwest Tropical Pacific, where there is often too much precipitation in MMF models (Khairoutdinov et al., 2005). Overall, the magnitude of climatological changes from variance transport seem relatively small, such that the net top of atmosphere radiative balance in these simulations differs by less than $1\,\mathrm{W\,m^{-2}}$

245 (not shown). Thus, we conclude that enabling variance transport does not significantly alter the model climate.

4.2 Checkerboard noise

Figure 3a,c shows the result of combining neighborhood state patterns by the number of local extrema using liquid water path and precipitation data from the Northwest Tropical and Subtropical Pacific. The difference of each fractional occurrence value relative to the satellite data on the ne30pg2 grid is shown in Figure 3b,c. This analysis highlights how the pure checkerboard (8

250 local extrema) is quite rare, and this pattern is observed too often in the control simulation (red bars). The control simulation has an even larger prevalence of patterns with 3, 4, and 5 local extrema relative to all other data sets owing to the fact that instances of partial checkerboard are more prevalent than instances of pure checkerboard. Inversely, the smoothest patterns with no local extrema are produced too infrequently in the control. Both BVT (green) and FVT (blue) cases are more similar to satellite observations in their distribution of pattern occurrence, with the BVT case being the most similar to observations.

255 Figure 4 shows a map of partial checkerboard fractional occurrence using liquid water path data. The spatial distribution of partial checkerboard occurrence is similar among all three simulations, but the BVT and FVT cases show a dramatic reduction in how often these patterns are observed. However, these cases are still producing more checkerboard instances compared to satellite data, consistent with Figure 3.

In addition to showing the more frequent occurrence of checkerboard patterns, Hannah et al. (2022) also found that checker-
260 board "events" were surprisingly persistent by identifying periods in which a neighborhood stays in a partial checkerboard state for an extended time. Figure 5 shows a histogram of these partial checkerboard event lengths in liquid water path and precipitation fields using oceanic data between 60°S-60°N. In both variables we see that E3SM-MMF show a larger number of events of any length when compared to satellite data, with some events lasting nearly 100 days. The FVT case also exhibits a tendency to produce longer partial checkerboard events than is observed in satellite data, whereas the BVT case is very similar
265 to the satellite data.

Figure 6 shows time series of partial checkerboard occurrence calculated at every time step starting from initialization for the three model configurations over ocean regions between 60°S-60°N. Black dashed lines indicate the climatological partial checkerboard occurrence from satellite data for reference. Interestingly, the checkerboard signals are detectable in the first few time steps, but the fractional occurrence continues to increase over the first 4-5 days before leveling off. The speed of the initial
270 spinup is consistent with the authors' experience that convection within the CRM develops very quickly after initialization. This further illustrates that while the CRM variance transport method can ameliorate the checkerboard pattern persistence and yield a much smoother climate, the model still produces a relatively noisy solution on short time scales relative to observations.

4.3 Variance Tracer Analysis

In this section we will look at the CRM variance tracers directly. We will characterize the climatological distribution, and how
they are related to precipitation as a measure of convective activity. We also examine the GCM-to-CRM forcing and CRM-
to-GCM feedback tendencies that are used to couple the two models in order to explore how CRM variance is produced and
transported. This discussion is meant to provide a broad sense of how the CRM variance transport works in practice.

Figure 7 shows the zonal mean climatology of all three CRM variance tracer amounts on the GCM grid, along with the CRM
feedback and GCM forcing tendencies. For this analysis we only use data from the BVT case as the control does not have these
quantities and results from the FVT case are effectively identical. The largest CRM variance tracer magnitudes occur in the
tropics, which is expected since this is where the strongest convective activity occurs. There is also a noticeable amount of
CRM variance around the upper tropospheric jet regions, which is likely associated with mid-latitude storm activity.

The GCM forcing and CRM feedback tendencies are expected to balance each other, similar to any quantity that is coupled
across the two models. However, some aspects of the zonal mean patterns are not immediately intuitive to understand. We
might naively expect the CRM to produce positive variance tracer tendencies on average, which would be balanced by negative
tendencies from the advection by the GCM, but this is not the case. Additionally, the vertical distribution of forcing and
feedback tendencies reveal a complicated picture, with the CRM producing negative feedback tendencies at low levels and
above 500 hPa.

It is not immediately clear what causes the negative zonal mean CRM feedback tendencies. Part of the explanation may be
that CRM variance produced in the boundary layer is often advected upwards into the free troposphere, leading to an apparent
low-level sink of CRM variance. A full explanation of these tendencies would require a detailed budget calculation to quantify
how variance is produced and transported in both the CRM and GCM, which is outside the scope of the present study.

An alternative way to examine the CRM variance tracers is to bin daily mean data by the daily mean precipitation rate to
get a sense of how the feedback and forcing tendencies balance across various convective regimes. This analysis is shown in
Figure 8 using 1-year of data from the BVT case. This approach reveals a more consistent picture of how the CRM variance
tracer forcing and feedback tendencies are balanced. The CRM tendency of liquid-ice static energy is positive on average when
precipitation is being produced, consistent with our expectations. The CRM tendency of total water variance shows strong
negative values aloft that get stronger with higher precipitation rates. These negative tendencies are likely explained by the fact
that precipitation is generally a sink of total water variance.

4.4 Does variance transport affect variability?

In this section we provide a cursory look at how the CRM variance transport affects variability by focusing on convectively
coupled waves in the equatorial region. Figure 9(a-d) shows un-normalized symmetric spectra of top-of-atmosphere outgo-
ing longwave radiation (OLR) over the region 15°S-15°N for NOAA satellite observations, E3SM-MMF, E3SM-MMF+BVT,
and E3SM-MMF+FVT following Wheeler and Kiladis (1999). Lower panels (e-f) show the difference between each simula-

305 tion relative to NOAA observations. Positive and negative wavenumbers indicate eastward and westward propagating modes, respectively. Theoretical dispersion lines are shown for equatorial Kelvin and Rossby waves (Matsuno, 1966).

The difference panels in Figure 9 highlight the fact that the model produces too much power at almost all regions of the spectrum. This corresponds to an overall larger temporal variance on daily time scales in quantities associated with convective activity (not shown). Enabling CRM variance transport acts to reduce this temporal variance in many areas of the equatorial
310 region, which is reflected in the spectra of the BVT and FVT cases. This is a somewhat unexpected sensitivity, but also a welcome result since it appears to make the simulated variability slightly more realistic.

The most prominent feature of the observed equatorial wave spectrum is a peak around 30-90 day periods (0.01-0.03 day^{-1}) and eastward zonal wavenumbers 1-2, which is the signature of the MJO. E3SM-MMF captures this feature to some extent, but the spectral power is slightly weaker than observations despite having stronger power in almost all other bands. Furthermore,
315 the power in the MJO band decreases slightly when using BVT, and increases slightly when using FVT relative to the control. The MJO sensitivity to CRM variance transport is somewhat surprising as we hoped that the MJO would be improved by allowing CRM states be advected by the circulation anomalies associated with the MJO. This suggests that the MJO deficiencies in E3SM-MMF are likely due to other factors, such as the lack of ocean coupling (Benedict and Randall, 2011; DeMott et al., 2014).

320 **5 Conclusions**

In this study we have presented a method to address the variance trapping problem in E3SM-MMF, and associated checkerboard patterns, by allowing cloud scale fluctuations in the CRM to be transported on the global grid. This method for "CRM variance transport" is applied to liquid-ice static energy, total water, and horizontal momentum within the embedded CRM. Two implementations of the variance transport method are implemented in E3SM-MMF that differ in how perturbations are
325 calculated to obtain the the variance tracer magnitudes. The first method uses simple differences from the horizontal mean to define a total or "bulk" variance, whereas the second uses a FFT as a low-pass filter in space to define a "filtered variance" that restricts the transported variance to only affect the lowest frequency fluctuations within the CRM.

Analysis of the model climatology shows that checkerboard patterns are no longer visible when using either bulk variance transport (BVT) or filtered variance transport (FVT). The pattern detection method of Hannah et al. (2022) is used to quantita-
330 tively compare the simulations and satellite observations. These results show that E3SM-MMF is more similar to observations when variance transport is enabled, with the BVT method giving the best performance.

Enabling variance transport generally does not have a large affect on the model climate, although it tends to reduce the temporal variance of fields associated with convection on the GCM grid, such as precipitation. A notable example of this is the spectra of equatorial outgoing longwave radiation, which shows too much power in the control simulation relative to satellite
335 observations that is reduced when variance transport is enabled.

While the results of employing CRM variance transport are encouraging, and certainly useful for obtaining a more realistic solution, E3SM-MMF still exhibits a relatively noisy solution when compared to observations. The occurrence of partial

checkerboard patterns, while significantly reduced, is still more frequent than observations. It is unclear whether the variance transport method could be modified to enhance its effect and, perhaps, "tune" the level of noise in the model, but this may be

340 an interesting line of future experimentation.

There are other open questions that we have not addressed here. The bulk variance transport method is general enough to work with a 3D CRM, but we have not explored 3D experiments in detail due to the large cost. The hardware and algorithmic acceleration techniques available to E3SM-MMF make this a possible avenue for future work. Another open question has to do with the importance of inter-scale variance transport. The comparison between bulk and filtered cases suggest that this is not

345 a problem. However, if variance is traversing the scale range via the transport scheme, we don't have a good way to quantify this and understand its downstream effects in the global model. It seems plausible that this issue might be more important as the CRM size becomes larger, or if the CRM numerics are changed to promote a smoother solution at the smallest scales. An idealized limited area MMF model might make these questions more approachable.

Code and data availability. The E3SM project, code, simulation configurations, model output, and tools to work with the output are de-
350 scribed at the website (https://e3sm.org). Instructions on how to get started running E3SM are available at the website (https://e3sm.org/model/running-e3sm/e3sm-quick-start). All code for E3SM may be accessed on the GitHub repository (https://github.com/E3SM-Project/E3SM). The raw output data from E3SM-MMF used in this study is archived at the Oak Ridge Leadership Computing Facility (OLCF) operated by Oak Ridge National Laboratory (ORNL) and the Department of Energy (DOE). The specific branch used to conduct the simulations can be found at https://github.com/E3SM-Project/E3SM/tree/whannah/mmf/vt-validation and is also archived at https://doi.
355 org/10.5281/zenodo.6578523. The analysis code and a condensed version of the data needed to reproduce our results is also archived at https://doi.org/10.5281/zenodo.6578575.

Author contributions. Walter Hannah developed and tested the variance transport implementation, ran the simulations, performed the analysis, and prepared the manuscript. Kyle Pressel contributed to the conceptual design and validation of the variance transport method, and helped prepare the manuscript.

360 *Competing interests.* No competing interests are present.

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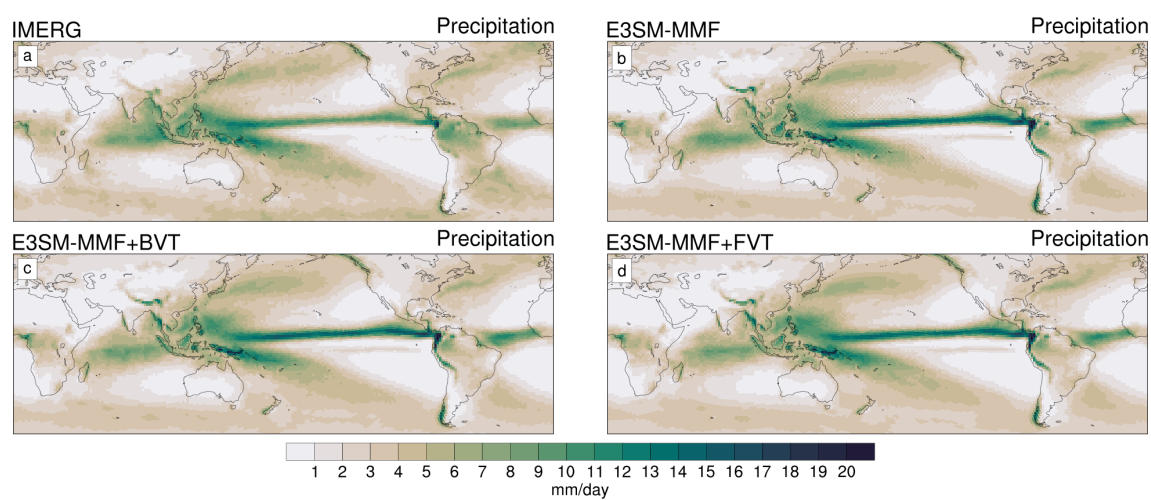
Figure 1. Maps of 10-year mean precipitation for IMERG (a), E3SM-MMF (b), E3SM-MMF with bulk variance transport (BVT, c), and E3SM-MMF with filtered variance transport (FVT, d).
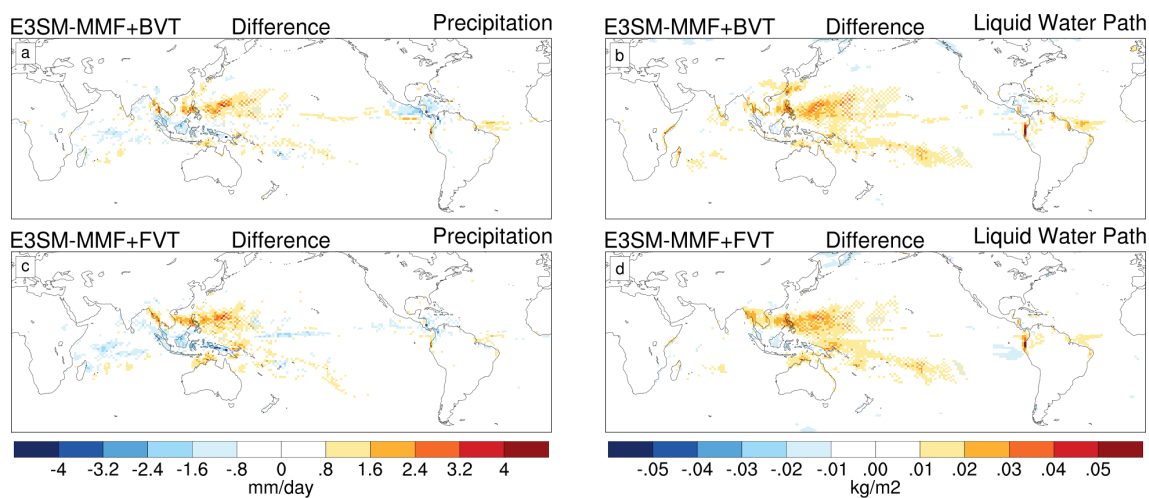
Figure 2. Maps of 10-year mean precipitation and liquid water path differences relative to the E3SM-MMF control simulations for E3SM-MMF+BVT (top row) and E3SM-MMF+FVT (bottom row).
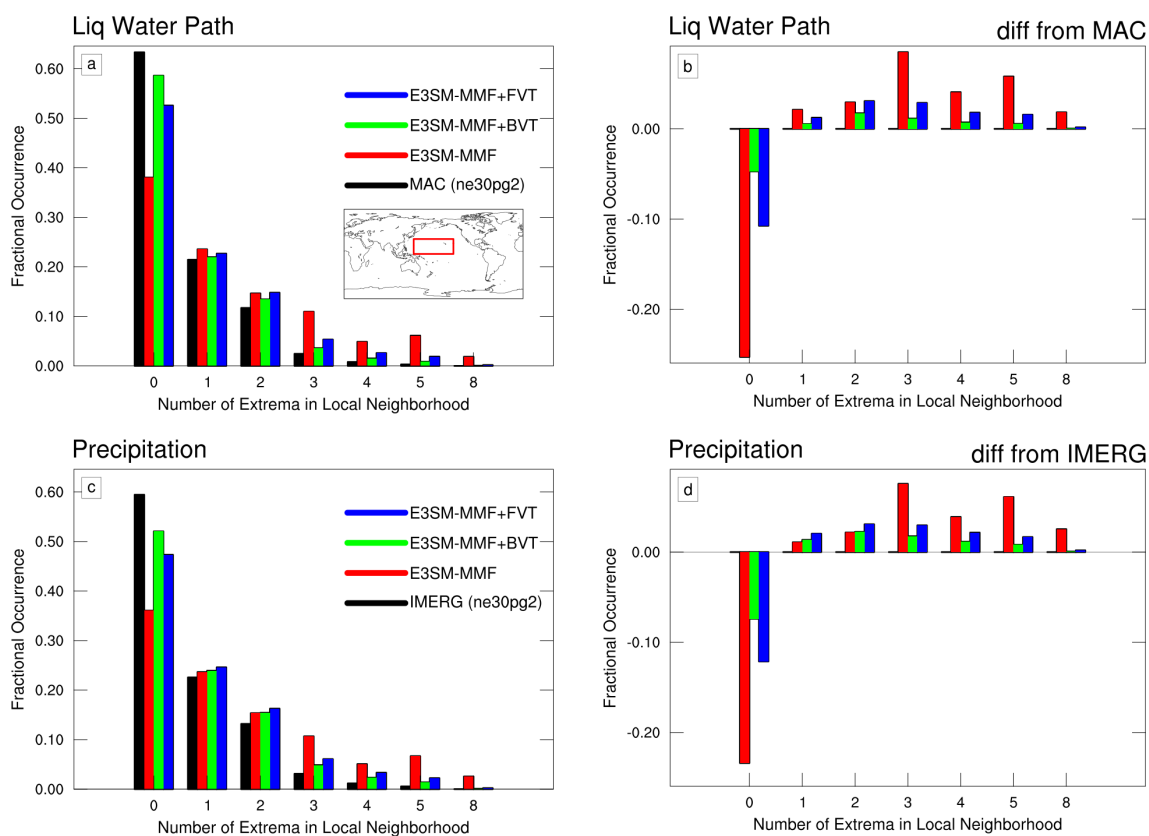
Figure 3. (a,c) Fractional occurrence of neighborhood patterns combined according on the number of local extrema (see text) over the region 0-30N and 140-220E (see inset map) for 10 years of satellite and E3SM-MMF data. Results for IMERG precipitation and MAC liquid water path were calculated from data regridded to the ne30pg2 grid used by the model for direct comparison. (b,d) Difference of fractional occurrence relative to satellite data.
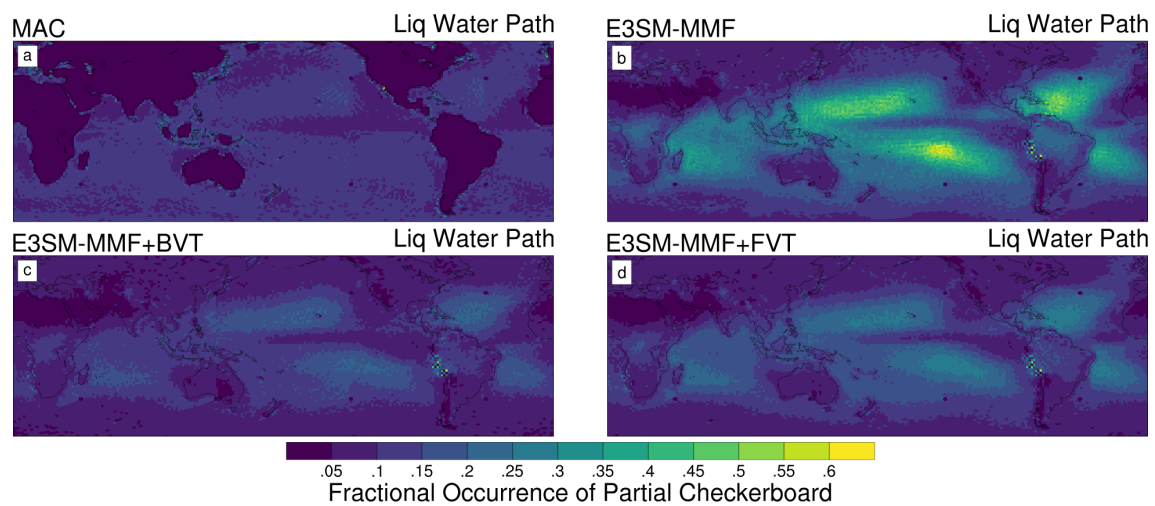
Figure 4. Maps of fractional occurrence of partial checkerboard patterns (see text) on the ne30pg2 grid for MAC-LWP (a), E3SM-MMF (b), E3SM-MMF with bulk variance transport (BVT, c), and E3SM-MMF with filtered variance transport (FVT, d).
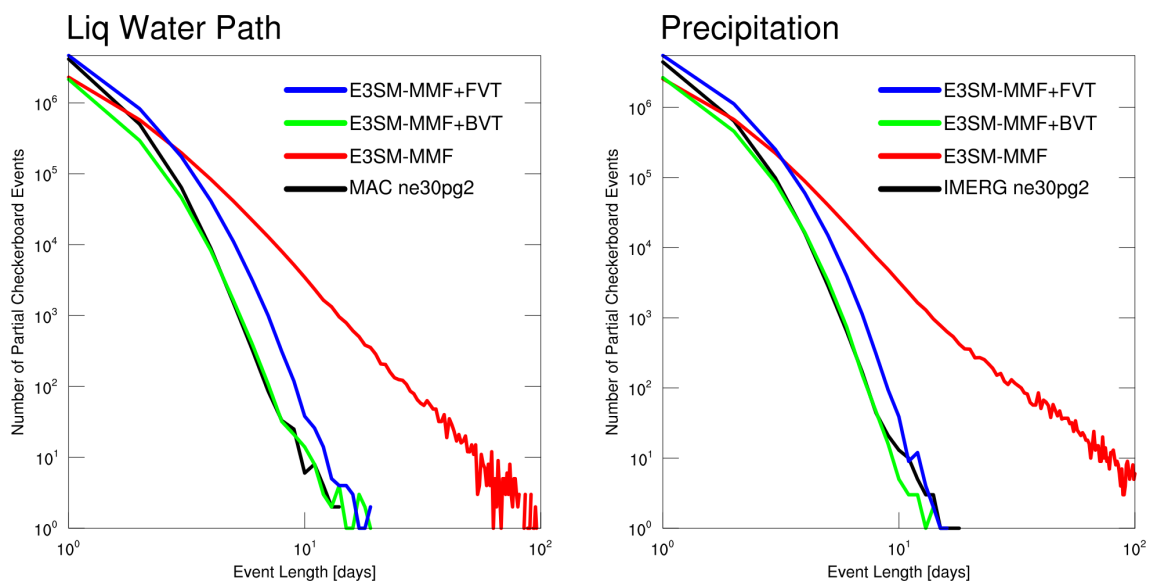
Figure 5. Histogram of the event length, with events defined as continuous periods with partial checkerboard neighborhood state. Data was restricted to oceanic points equatorward of $60°$ latitude in both hemispheres for all 5-years available.
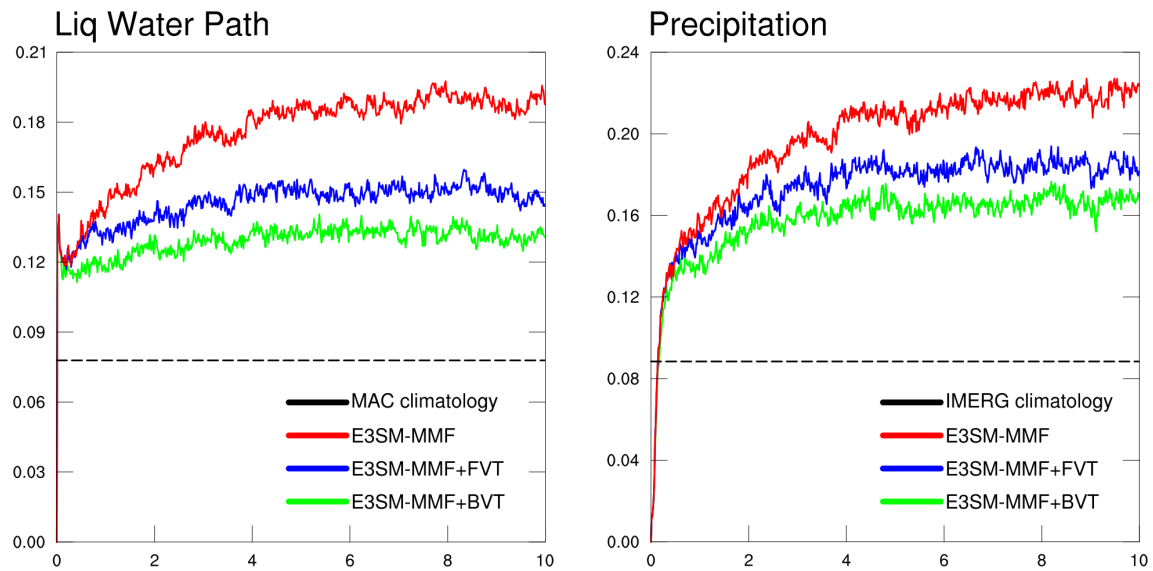
Figure 6. Time series of partial checkerboard occurrence equatorward of 60° latitude in both hemispheres using data at each model time step.
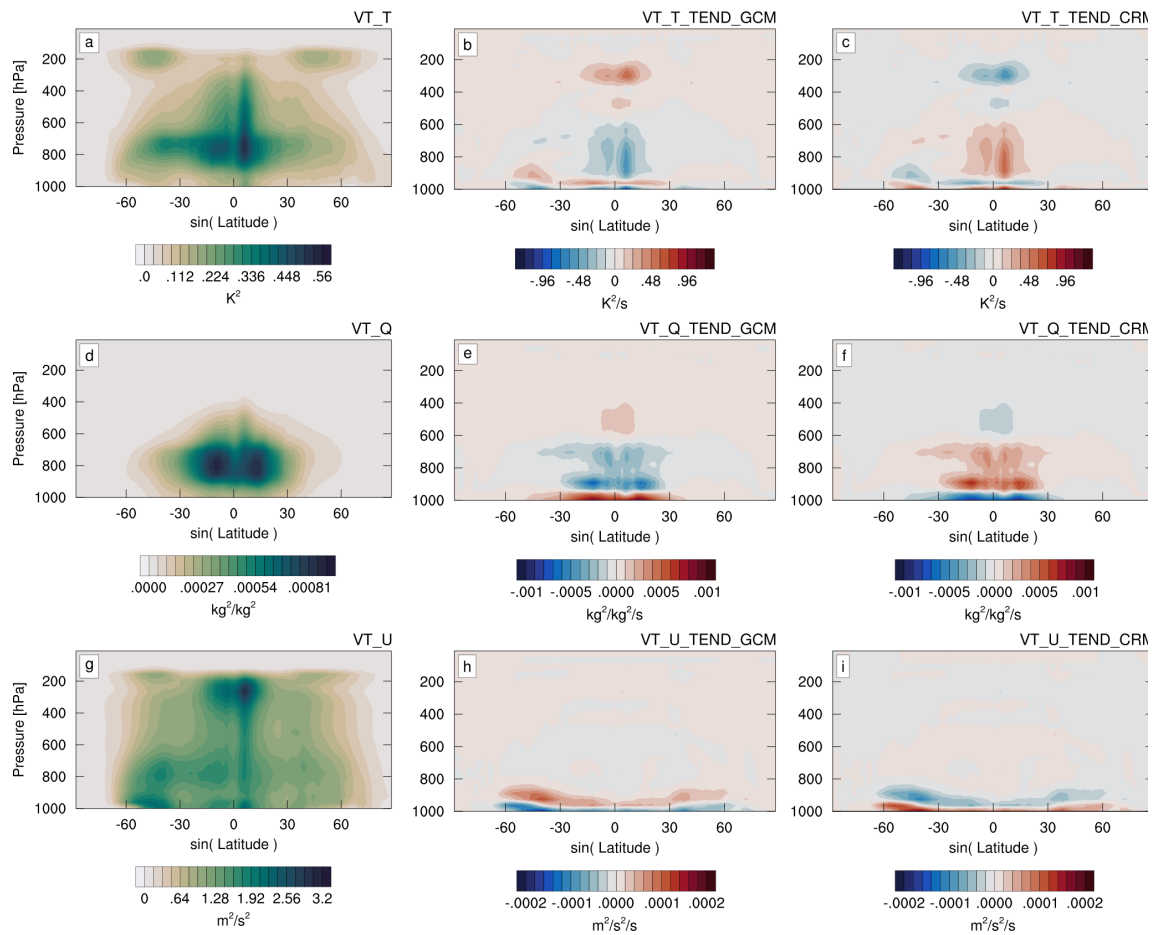
Figure 7. 10-year zonal and time averaged CRM variance transport tracer amount (left), GCM to CRM forcing tendency (middle), and CRM to GCM feedback tendency (right) for the E3SM-MMF+BVT case.
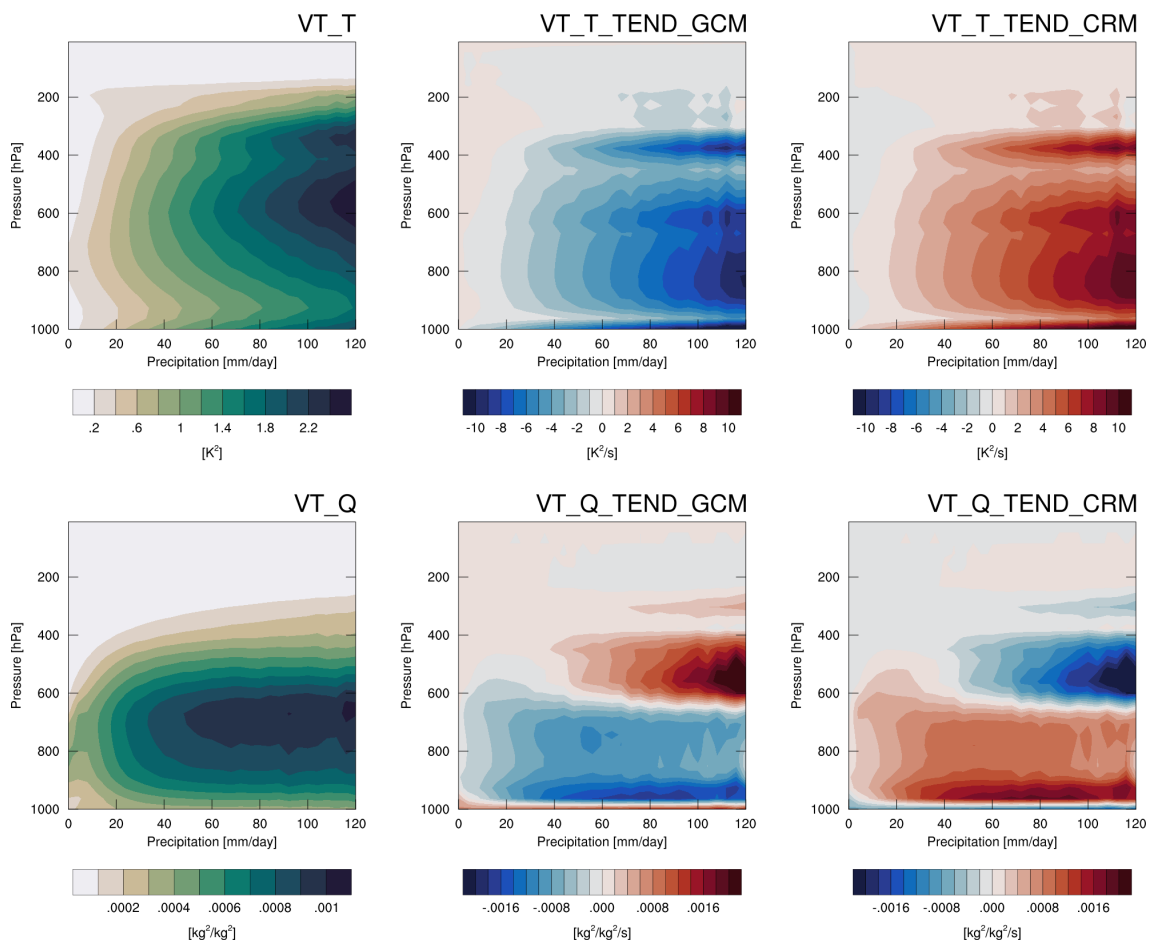
Figure 8. Vertical profiles of CRM variance transport tracer amount, CRM feedback, and GCM forcing binned by precipitation rate for liquid-ice static energy and total water using 1-year of daily mean data from the E3SM-MMF+BVT case.
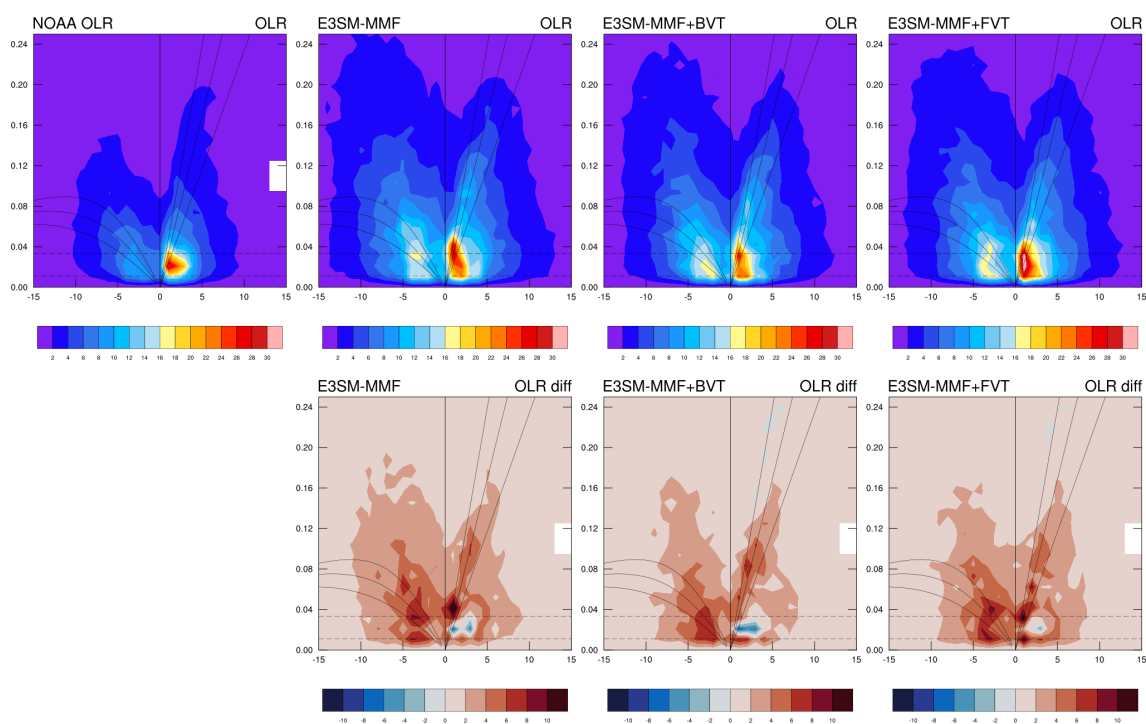
Figure 9. symmetric wavenumber-frequency spectra of Tropical outgoing longwave radiation over $15°$S-$15°$N for data from NOAA polar orbiting satellites (a), E3SM-MMF (b,e), E3SM-MMF+BVT (c,f), E3SM-MMF+FVT (d,g) including differences between each simulation and NOAA data.