To my knowledge the paper by Cao et al. is the first time that balloon borne GNSS radio occultations are used to investigate equatorial waves.

Although the paper contains a major technical part it is well suited for ACP and its readership, given the novelty of the presented results and its potential impact on our understanding of atmospheric dynamics in the tropics.

The paper is well written. Some technical parts were already moved into the appendix, and the technical parts remaining in the main text are needed for understanding the methodology.

The authors extract the properties of equatorial Kelvin waves in a Lagrangian frame, and the importance of a better determination of Kelvin wave momentum fluxes is discussed for a better estimation of the wave driving of the quasi-biennial oscillation. Therefore the paper is of broad interest for the readership of ACP.

Overall, the paper is recommended for publication in ACP after addressing my mainly minor comments.

My main comments are that the importance of observations in a Lagrangian frame is overemphasized, and in the introduction the credit to previous work is a bit thin.

We appreciate the reviewer's comments and suggestions, especially the recommended references. We added extra lines to address the previous work in the introduction section.

Regarding the importance of the Lagrangian framework, we agree that the difference between reference frames should be neither exaggerated nor neglected.

Specific Comments:

l.12:

With a vertical resolution of 200-500m you should be able to identify Kelvin waves and gravity waves of vertical wavelengths shorter than 2-3 km.

Identification of 2-3km waves should be possible already by space-borne GNSS RO and space-borne infrared limb sounders that can have vertical resolution as good as 1km, see Wright et al. (2011).

Wright, C. J., Rivas, M. B., and Gille, J. C.:

Intercomparisons of HIRDLS, COSMIC and SABER for the detection of stratospheric gravity waves,

Atmos. Meas. Tech., 4, 1581-1591, 2011.

With the current analysis, we have not yet observed waves with wavelengths shorter than 2-km although our resolution analysis determines that it should be possible. We modify the wording to attempt to make that distinction:

L 11:

The 200--500 m vertical resolution and the spatial and temporal continuity of sampling make it possible to extract with confidence properties of Kelvin waves and gravity waves with vertical wavelengths as short as 2--3 km.

L 289: We added Wright et al, 2011 and Tsuda et al 2011 for reference later in the resolution section.

Using the Full Spectral inversion technique the SRO observations can have improved vertical resolution as small as 500 m, depending on the refractivity gradient (Tsuda2011) though more generally limited to ~1 km for wave perturbations (Scherllin-Pirscher et al. 2021). For comparison, the HIRDLS and SABER limb-sounding satellites have a reported vertical resolution of 1 km and 2 km, respectively (Wright2011}.

l.42: You should cite some of the few existing examples as detailed below:

Antonita, T. M., Ramkumar, G., Kishore Kumar, K., and Sunil Kumar, S. V.:

Quantification of gravity wave forcing in driving the stratospheric Quasi-Biennial Oscillation,

Geophys. Res. Lett., 35, L09805, doi:10.1029/2008GL033960, 2008.

Ern, M., and Preusse P.:

Quantification of the contribution of equatorial Kelvin waves to the QBO wind reversal in the stratosphere,

Geophys. Res. Lett., 36, L21801, doi:10.1029/2009GL040493, 2009.

Alexander, M. J., and Ortland, D. A.:

Equatorial waves in High Resolution Dynamics Limb Sounder (HIRDLS) data,

J. Geophys. Res., 115, D24111, doi:10.1029/2010JD014782, 2010.

Ern, M., Ploeger, F., Preusse, P., Gille, J. C., Gray, L. J., Kalisch, S.,

Mlynczak, M. G., Russell III, J. M., and Riese, M.:

Interaction of gravity waves with the QBO: A satellite perspective,

J. Geophys. Res. Atmos., 119, 2329-2355, doi:10.1002/2013JD020731, 2014.

L 40: Thank you for your time to provide the extended list of references. We added these to the introduction.

However, current climate models with limited resolution have difficulties producing a realistic QBO in the lower stratosphere due to unresolved small-scale waves (Antonita et al., 2008; Ern and Preusse, 2009; Alexander and Ortland, 2010; Ern et al., 2014; Richter et al., 2020), leading to uncertainty in the evolution of the QBO period and amplitude in a changing climate

l.53: This is not entirely correct!

The understanding of equatorial wave properties was advanced by limb sounding observations from satellite in general.

This includes not only GNSS RO, but also several other kinds of limb observations from satellite.

In particular, infrared limb observations from satellite can achieve vertical resolutions similar to GNSS RO from satellite (see Wright et al., 2011)!

To put your results better into context, you should modify this statement and add the following references:

Salby, M. L., Hartmann, D. L., Bailey, P. L., and Gille, J. C.:

Evidence for equatorial Kelvin modes in Nimbus-7 LIMS,

J. Atmos. Sci., 41, 220-235, 1984.

Shiotani, M., Gille, J. C., and Roche, A. E.:

Kelvin waves in the equatorial lower stratosphere as revealed by cryogenic limb array etalon spectrometer temperature data,

J. Geophys. Res., 102, 26131-26140, 1997.

Srikanth, R., and Ortland, D. A.:

Analysis of Kelvin waves in High-Resolution Doppler Imager and Microwave Limb Sounder stratosphere measurements using a constrained least squares method,

J. Geophys. Res., 103, 23131-23151, 1998.

Ern, M., Preusse, P., Krebsbach, M., Mlynczak, M. G., and Russell III, J. M.:

Equatorial wave analysis from SABER and ECMWF temperatures,

Atmos. Chem. Phys., 8, 845-869, 2008.

Alexander, M. J., and Ortland, D. A.:

Equatorial waves in High Resolution Dynamics Limb Sounder (HIRDLS) data,

J. Geophys. Res., 115, D24111, doi:10.1029/2010JD014782, 2010.

L49: Once again, thank you for providing more extensive references. We added text to address the contribution of limb-sounding techniques in general, and reworded the paragraph.

Limb-sounding satellite observations have led to key advances in understanding equatorial waves, in part due to their high vertical resolution capable of resolving detailed vertical variations (Salby et al., 1984; Shiotani et al., 1997; Srikanth and Ortland, 1998; Ern et al., 2008; Alexander et al., 2010). GNSS radio occultation (RO) is one of these limb-sounding techniques that has had a particularly extensive impact because of the large and steadily increasing number of satellites now providing temperature and refractivity profiles to study atmospheric waves in the stratosphere (e.g. Tsuda et al., 2000; Randel et al., 2003; Randel et al., 2021).

l.76-78: please give a reference for this statement, give reasoning, or delete it

We added a reference but made the reasoning less specific.

For example, in QBO shear zones wave vertical wavelengths will shrink wherever the wind approaches the wave phase speed. If the wave can survive to higher altitude where density is lower and the wave has shorter vertical wavelength, the wave can impart a significantly stronger force on the QBO flow (Vincent and Alexander, 2021).

l.253: this describes only the situation of setting occultations, but not for rising ones

L 253: Yes, the statement here only applies to setting occultation. For rising occultations, the tangent point drifts towards the balloon from a further position. We changed it to:

Because the motion of the GNSS satellite is much greater than that of the balloon during an occultation, the tangent points drift horizontally away from the balloon as the ray paths from a setting satellite descend through the atmosphere, or towards the balloon for a rising satellite, as shown in Figure 5.

caption of Fig.5: To which altitude do the SRO locations (yellow circles) refer to?

Space-borne RO such as COSMIC-2 also has a slanted profile, and a reference occultation location (lat/lon) is specified such that the excess phase equals 500 m is used to geographically mark this profile location. The tangent point drift of space-borne RO over a similar altitude range (5 – 20 km)

is much smaller, so we do not specifically consider its drift and use the reference occultation location (lat/lon) to define the COSMIC profile. This location is explicitly declared in the downloaded COSMIC-2 profiles. We changed the caption to read:

yellow circles denote COSMIC-2 RO profile reference locations (neglecting the much smaller tangent point drift) over the same day.

l.354: What dropsonde products? The sondes are mentioned only here without any further explanation.
Please give some more information!

Corrected. This is a typo, it is the radiosonde. No dropsondes were deployed in the Strateole-2 project or involved in this study.

l.519-537: You are making too strong statements here! This part of the discussion should therefore be carefully revised and downtoned!

We removed some of the emphasis by deleting the following text at Line 509 and kept only the text in the Discussion section:

~~We note that theoretical dispersion curves for equatorial waves (e.g., \cite{Kiladis2009}) are calculated in the intrinsic (Lagrangian) reference frame, so BRO measurements provide a unique advantage for the interpretation of equatorial waves. This unique BRO dataset, together with data from other instruments onboard the same balloon and other Strateole-2 balloons, will provide an opportunity to study the fine vertical structure of waves in the tropical tropopause layer (TTL) and below, and to investigate their relation to tropical convection and convectively-coupled equatorial waves.~~

Around line 531-537, we also deleted some assertive wording.

The radiosonde data in the ground-based reference frame in Figure 13(b) may be capturing the same waves but at different intrinsic periods. This is further complicated due to the likelihood that the data likely contains many wave types. The intrinsic period and wave speed can be significantly different, also affecting any estimates of momentum flux.

Indeed, the wave dispersion relations / dispersion curves are valid for intrinsic frequencies. However, usually the background wind is known relatively well, so that calculation of intrinsic frequencies from ground based frequencies should be possible with good accuracy.

It should be possible. Perhaps this is one of the first cases where that has been demonstrated with observations. We point out the large discrepancies shown in background winds in previous balloon campaigns (Podglajen et al 2014).

Displaying dispersion curves for zero wind conditions makes also some sense because most of the tropical waves are excited in the troposphere where winds are relatively weak.

Line 482: We see the point and have deleted the sentence, "~~This is an important point to consider when estimating tropical wave signatures from satellite data and interpreting the link between convection, clouds, rain, and tropical weather.~~ "

Therefore I agree with the authors that the Lagrangian perspective offers a different view that may help to better understand details of the wave physics, but I do not see the major benefit of observing in a Lagrangian frame.

L12: We provide a neutral explanation of the benefits and leave the reader to determine whether it is major.

The results illustrate the difference in Kelvin wave period (20 vs 16 days) in the Lagrangian versus ground-fixed reference, and as much as 20% difference in amplitude compared to COSMIC-2, both of which impact estimates of momentum flux.

l.555: To me, it looks like several corrections and iterations are needed to arrive at a final product release that is trustworthy enough for operational data assimilation.

Can you comment on this issue? Do you think that providing near real time data for operational data assimilation is really feasible? How long would it take from an observation to a final data product?

There are indeed many procedures that need to be optimized and automated before the BRO data can be used for any assimilation, but the procedures are essentially the same as COSMIC-2. This part of the work is already underway as we are working on a full workflow to process the RO data from the aircraft platform for assimilation into models for hurricanes and atmosphere river forecasts. Regarding real-time processing, there are a few challenges that need to be addressed. First, the Iridium satellite communication could be improved to reduce the delay from 1-hr to a shorter period. Second, accurate GNSS satellite orbit and clock products are needed in a real-time manner. The COSMIC office at UCAR/NCAR generously shared their real-time orbit/clock products so that we can use them to do the refractivity retrieval. This part of work is also underway in the aircraft RO projects. A companion paper on the aircraft RO is in preparation with more details on these topics. When the airborne RO RT or Near-RT automation is finished and proven to be feasible, we believe it is also possible to be implemented on a balloon platform.

L563: We provided additional detail in the text:

The COSMIC-2 constellation provides profiles over the tropical oceans operationally for assimilation into numerical weather prediction models and achieves a median latency of ~30 minutes (Weiss et

al., 2022). The increased density of the Strateole-2 observations provides potential added value for models, and the processing scheme is quite similar. The observations are transmitted via Iridium satellite communication link at 1 hour intervals, so there could be as much as 30 minutes additional median delay, which would still be of potential value for NWP. Efforts are underway to develop low-latency products for aircraft RO observations. Implementing the same techniques for near real-time balloon observations could contribute to improving model initial conditions over the tropical oceans in future campaigns.

Technical Corrections:

l.370: Figures 6(d) -> Figure 6(d)

Done

l.430-431: "including Kelvin waves through shorter period gravity waves"

Please reword this phrase for clarity!

Deleted the phrase.

l.452: with the wavenumber -> with the zonal wavenumber

Done

l.724: mars -> Mars

Done

l.760: "Hoeg, P., H., A., K., G., S., S., Belloul, B.," ???

reference looks strange!

Fixed

l.774: "Kirchengast, G., J., H., and W, P.: "

Fixed

l.838: "H., Harijono, W. B., , and Kato, S.: "

Fixed

l.841: reveald -> revealed

Done

l.842: occultationi -> occultation
Done

These minor technical issues are all addressed in the context and citations. We thank the reviewer for their careful attention to detail.

[I am Editor for this paper. One of the referees who originally agreed to provide a report on this paper has failed to respond. Therefore I am acting as a referee myself.]

This is an interesting paper setting out algorithmic details of extracting temperature information from balloon-based radio occultation during one of the preliminary STRATEOLE balloon flights. It is clear that when the main STRATEOLE campaigns are under way, with radio occultation available from many balloons on long-duration flights, this technique will provide much useful new information on temperature profiles in the TTL. This paper provides much useful preliminary information.

My only reservation about the paper is that it has been submitted as a Research Article, and indeed I as Editor accepted in on that basis, but having looked at the paper in more detail it does seem to me to be more a "Measurement Report' than a 'Research Article' -- see the description of different paper types at:

https://www.atmospheric-chemistry-and-physics.net/about/manuscript_types.html

In raising this I am not trying to obstruct publication of the paper. But I do ask the authors to consider whether the paper would be more appropriately published as a measurement report. (As far as I am aware, all the standard bibliographic databases make no distinction between the different manuscript types and I believe that the category of 'Measurement Report' was introduced to acknowledge the value of this kind of contribution and to avoid publication delays where, e.g., referees or Editors were reluctant to recommend publication of a potentially very valuable paper because of confusion about whether it contained sufficient new scientific results to the publishable.)

We understand the arguments for suggesting a Measurement Report, however we believe the results are new and of high potential interest, and the first reviewer showed no reluctance in recommending publication. We included an additional statement on the main results in the abstract at line 13:

"The results illustrate the difference in Kelvin wave period (20 vs 16 days) in the Lagrangian versus ground-fixed reference, and as much as 20% difference in amplitude compared to COSMIC-2, both of which impact estimates of momentum flux."

The fact that the equatorial wave results come from a new type of observation (balloon-borne radio occultation) and clearly demonstrate what new information this type of data can bring should be more than sufficient to qualify as new scientific results.

l12: 'A short dataset from the extra Galileo and GLONASS ... planned follow-on' -- the only mention of this in the text is almost a repeat of this sentence -- l588-589 -- saying that no details are given in this paper. It doesn't really seem appropriate to mention this point in the abstract.

Line 505-510: We provided more details on the Galileo, GLONASS, and Beidou observations, to better emphasize the significance of doubling the number of profiles. This increased density of

sampling provided by the additional constellations is a key factor for distinguishing the advantages of BRO relative to the quasi-random sampling of spaceborne RO.

These capabilities will be further enriched by recovering data of multiple constellations, including Galileo, GLONASS, and Beidou. 24 hours of data were retrieved from the Galileo and GLONASS constellations sufficient to demonstrate quality comparable to GPS. 12 hours of data were retrieved from Beidou. Due to data transmission limitations, however, it was not possible to transmit the additional constellation datasets from the entire flight. The data were sufficient to verify the number of occultations from each constellation. On average, there were ~2 profiles retrieved per hour from the GPS constellation. Including data from all GNSS constellations more than doubles the number of profiles.

l11-16: There is really no concrete information on results here in the abstract. That makes the status of 'Research Article' difficult to justify.

We included an additional statement on the main results in the abstract at line 13:

"The results illustrate the difference in Kelvin wave period (20 vs 16 days) in the Lagrangian versus ground-fixed reference, and as much as 20% difference in amplitude compared to COSMIC-2, both of which impact estimates of momentum flux."

l35-52: This is quite a long introduction that could be shorted significantly simply to say, with appropriate references, that the QBO is important and but aspects of it are poorly understood, one being the quantitative role of different waves in driving the QBO winds changes in the lower stratosphere.

We removed Lines 42-44, 48-53.

l41: 'uncertainty in period and amplitude'?

Line 43 changed to:

leading to uncertainty in the evolution of the QBO period and amplitude in a changing climate.

l43: 'somewhat muted in forecast models compared to observations' -- I think that you are really talking about free-running global climate models here -- not models that start from an observed state. I think that most readers will interpret forecast models as the latter.

We removed this statement when we shortened the introduction.

l76-78: 'For example, the difference ... 40 per cent difference in the force in the QBO' -- this seems to be very specific quantitative statement and I wondered what calculation exactly had been done to arrive at it.

Line 74: Changed to

"For example, in QBO shear zones wave vertical wavelengths will shrink wherever the wind approaches the wave phase speed. If the wave can survive to higher altitude where density is lower and the wave has shorter vertical wavelength, the wave can impart a significantly stronger force on the QBO flow (Vincent and Alexander, 2021)."

l83: 'tangent point … drifts' -- I suppose that this is standard terminology but it is potentially a little confusing since you have also referred to the 'drift' of the balloon itself. Perhaps consider later in the paper whenever 'drift' is used whether it is always absolutely clear -- to a non-expert reader -- whether it refers to the position of the balloon or the position of the tangent point.

l4 and throughout text:

 To avoid the potential confusion between the balloon drifting and the tangent point drifting, we choose to use 'float' for the balloon and keep 'drift' solely for the tangent point.

l87-88: I assume, on the basis of the previous statement re 500km, that short here implies say 1000 km or less and larger-scale implies say a few 1000 km.

It would be useful to have some indication.

Line 85 changed to:

The slanted nature of the profiles also presents difficulties in resolving wave properties in the presence of horizontal scale variability shorter than about 1000 km, without explicitly considering the drift.

l120: Just to be clear -- the implication here is that this technique works ONLY for quasi-hydrostatic waves. (Any comment about that re implications for estimating QBO momentum fluxes.)

Tropical waves typically have a broad variety of periods and scales, some are hydrostatic and some are not. Those waves collectively contribute to the forcing QBO. This study focuses on waves with longer periods and larger scale, which are part of the wave spectrum.

l182-186: You refer to various intervals when different radio occultation signals were recovered and note that these are indicated by dashed boxes on Figure 1, but there doesn't seem to be any information on which box corresponds to which interval.

L180: A note is added in the text to clarify the boxes and periods

The recovered dataset includes GPS only data from 6 December 2019–22 December 2019 (largest box on the left on Figure 1), 1 January 2020 (box in the middle on Figure 1), 9 January 2021, and 11 January 2021–14 January 2021 (box on
the right on Figure 1).

l185: 'data' is needed only once.

The second one is removed.

l200-201: I suppose that 'epoch' is being used here in a technical sense. What does it mean.

Line 197: changed to:

The final calculation was solved for the coordinates of the balloon antenna at each time sample.

l236: 'loess' -- I suggest capitalising this -- even if it is not capitalised in program scripts.

Changed to LOESS.

l307: 'consecutive' -- should this be 'regular' (in contrast to 'random')

Line 322: The BRO profiles are not really regular, they are random as well. But they are closely aligned with the balloon position, thus making them continuous. Changed to:

Multiple irregularly spaced but temporally and spatially consecutive profiles form near-parallel transects ~400 km wide along the trajectory.

Figure 4: 'lowest tangent point further from the balloon path' -- I think that this means that for each derived profile the lowest point on the profile corresponds to the horizontal location furthest from the balloon path? Is that correct? Perhaps make slightly more explicit.

Fig. 4: Yes, this interpretation is correct. Changed to:

Red and blue lines denote rising and setting BRO profiles, respectively, that are projections of slanted profiles with the highest tangent point on the balloon path and the lowest point furthest away.

l364: 'BRO profile matches the ERA5 profile well for this case, which we attribute to the fact that they both consider the same tangent point drift' -- on first reading I was slightly confused by 'for this case' because I interpreted the sentence as implying that 'same tangent point drift' applied to this case, but perhaps not to other cases. But in fact my understanding is that construction of ERA5 profiles with the same tangent point drift as BRO is a standard feature of your calculation.

Line 361: Your interpretation is correct. 'For the case' we mean the results here only apply to the comparison for the set of profiles. We removed "for this case".

l372: add 'temperature' -- i.e. 'depress the cold point tropopause temperature by ~1K'?

Added.

l423: You give 'quadrature in zonal wind and temperature' as permitting identification as a Kelvin wave here -- then at l478-479 you essentially repeat the same point and cite Andrews et al. Remove repetition?

The second instance is removed.

l494: 'was used in the calculation' -- do you mean in the Sato and Dunkerton calculation?

L489-491, Yes, it refers to the calculation in Sato and Dunkerton 1997. We reword the sentence to make it clear.

This is near the values deduced from radiosonde data in Sato and Dunkerton (1997, Fig. (7)) during the westerly phase of the QBO, which was about 0.1–0.2 mPa at around 20 km.

l499-504: This statement seems to be more appropriate for the concluding Section.

We leave it here because this is where the intrinsic frequency of the observations are described.

l594-604: As my comment above re l35-52, there is a certain amount of general motivation here that is pretty much the same at the end of the paper as it was at the beginning -- i.e. the results in the paper, apart from showing temperature retrieval from BRO is possible, have not really focused this motivation.

Line 583-587: added text to focus on the results more:
However, the BRO observations with slightly higher vertical resolution and denser sampling retrieved as much as 20% higher amplitude temperature variation associated with the wave. The BRO observations present the advantage that the waves are naturally measured in the intrinsic reference frame. The BRO profiles show an intrinsic period of 20 days for the Kelvin wave as compared to 16 day period for the Kelvin wave in the ground-referenced COSMIC-2 dataset. Both the difference in amplitude and the difference between periods determined from the BRO vs COSMIC-2 datasets affect the calculation of momentum flux.