I am one of the reviewers for this study.

The current study is a replication of a previous study by Ablain et al., (2019) using additional satellite altimetry (SA) data acquired since 2017. Both studies first construct a full variance-covariance (V/C) matrix of SA measurement errors. They then use a quadratic model to represent the systematic components of the observed GMSL anomalies. They identify coefficients of the quadratic as trend and acceleration during the SA period (1993-2022). Both studies used the ordinary least squares (OLS) to estimate the parameters of the quadratic representation. An elaborate scheme is used to assess the omission of the V/C matrix in the OLS solution on the uncertainties of the estimated trend and acceleration.

1 – If a proper V/C matrix is constructed, the simplest approach to quantify its impact on the model parameters would be to carry out a generalized least squares (GLS) solution, which makes use of the V/C matrix together with the OLS solution. Various solution statistics can then be compared to assess the impact of the V/C matrix on the adjusted GMSL anomalies and the model parameters. I wonder why the investigators have decided to use a convoluted scheme and ignored this obvious and direct assessment?

2 – Both studies state that the estimates for the parameters are unbiased if OLS is used. The unbiasedness of the estimated parameters using OLS as well as GLS are both conditional on the "expected values" of the disturbances being zero. However, this assumption cannot be established *a priori* since the purpose of modelling SA born measurement errors after all is to construct a realistic V/C matrix to achieve this end.

3 – An implicit assumption was also made that the quadratic model captures all the systematic variations in the observed GMSL anomalies. Again, there is no *a priori* evidence to support this assumption (I will discuss later that this is a faulty premise). Hence, the residuals, as estimates of the random disturbances, may biased unless shown otherwise.

4 – The unbiasedness assumption of the estimated quadratic model parameters using OLS or GLS does not guarantee that these estimates are comparable. If some of the GMSL anomalies with high leverages are not of good quality, they will *bias* the quadratic model parameters using OLS. These low-quality high leverage data will not impact the estimates if the GLS is used

provided that their uncertainties are properly reflected in the V/C matrix (i.e., they are down weighted). Consequently, assuming equivalency between the estimated parameters using OLS and GLS is not always warranted with certainty in practice. Therefore, using estimated parameters via OLS solution for the assessment of the V/C matrix may be faulty.

5 – If the unbiasedness of the estimated parameters is demonstrated to be valid despite the pitfalls discussed earlier, their uncertainties are still *biased* if OLS is used because *the standard error (SE) of the solution – the a priori variance of unit weight ~weighted root mean square error of the residuals* is biased (Toutenburg 1984, eqn. 2.2.45 pg. 31). There is no mention about this bias in both studies, whether it was properly accounted for in quantifying the uncertainties of the model parameters and its magnitude. This issue is important because both studies are about the uncertainties. Consequently, it is likely that the major syntheses of this study "*the stability performances of ±0.3mm/yr at the 90% confidence level (C. L.) for its trend and ±0.05mm/yr2 for its acceleration over the 29-years of the altimetry record*" were mis quantified.

6 – Another problem is modelling the effect first order serial correlation of the disturbances, AR(1) of the GMSL anomalies during 1993 – 2022. This effect is different than the bias in the SE caused using OLS discussed previously. Their presence in the observed GMSL anomalies is not recognized in both studies. Modelling the full V/C matrix does not make this effect to disappear. GMSL anomies averaged yearly, monthly, 10-day intervals (and smoothed) exhibit extremely high serial correlation with correlation coefficient, as high as 0.94 (Iz and Shum, 2020). Similar effects are also observed frequently in tide gauge records that may have AR(1) correlation coefficient as large as 0.7. Ignoring their presence will lead Type II errors in null hypothesis testing the statistical significance of the estimated model parameters. Only a handful of past studies on sea level have recognized this phenomenon. In the case of GMSL, the study by Nerem et al. (2018) corrected its effect by discounting the pertinent uncertainties using a relationship in their study (surprisingly, the magnitude of the estimated AR(1) correlation coefficient was not published!). Note that the formula for the *sample reduction* coefficient used in their study was derived using a relationship derived for univariate cases (WMO, 1966) and not necessarily valid for multiple regressions (luckily the outcome was not fatal). In any case, there

is no indication if the V/C matrix of the GMSL records incorporated the AR(1) effect in this manuscript (I am not referencing to the analyses of their tandem phases in the manuscript).

7 – Proper statistical formalism demands that the AR(1) effect in the GMSL records to be carefully analyzed and accounted for in the V/C matrix of the observations if it is statistically significant. Previous few studies used the simplistic sample reduction approach, which assumes that the GMSL disturbances are identically and independently distributed *i.i.d*, which is false now as evidenced by this study's *full* V/C matrix. Consequently, the V/C matrix of this study requires a construct akin to a *heterogeneously first order autocorrelated* V/C matrix, abbreviated as ARH(1) or ARW(1) for the V/C matrix.

8 – The use of quadratic model to represent the systematic components of the SL and GMSL anomalies without identifying its underlying physics unambiguously is another important source of error. The non-linear coefficient of the quadratic is stated as twice the *acceleration*, and the coefficient of the linear parameter as *trend*. Both terms are colloquial and vague in the context of sea level studies with important ramifications in describing the kinematics of SL anomalies. The acceleration is a "*uniform acceleration*" with implications. The acceleration estimated through a quadratic model must be the same (uniform) for various lengths of the records. If not, then the acceleration is not uniform. It can be transient, episodic, or a variable acceleration with different physical attributes for the underlying phenomena occurring throughout the records. If the uniformity of the estimated acceleration is not demonstrated, then the model and the findings will be questionable.

9 – A statistically significant uniform acceleration demonstrably present in the GMSL anomalies will change the meaning of the *trend* parameter. In this case, the estimated coefficient of the linear component of the quadratic model is "*the initial velocity.*" *This estimate varies depending on the choice of the initial epoch of the quadratic model.* Hence, by choosing different epochs for the time variable, one will get different estimates for the so-called *trend.* There is no indication that this fact was recognized not only in this study but also in others resulting in different estimates for the initial velocities labeled as invariant trends generating a plethora of

differing values. The casual use of the term *trend* in this study in the presence of a likely uniform acceleration is therefore misleading.

10 – The underlying kinematic meaning of the quadratic model opens another source of *ambiguity*. The acceleration component of a kinematic model can also be represented equally well with a low frequency variation in GMSL (Iz and Shum, 2020). Such periodicities were already detected in TG records. A visual examination the plot of the filtered GMSL anomalies suggests that there is steady trend in GMSL rise until 2011 (w/o acceleration) followed by a change point during 2011 with a mean shift followed by another trend (note that I am not promoting my visual examination as a reliable scientific tool). If verified, another possibility is a broken trend model (Iz et al., 218) leading to an *average acceleration* in GMSL with an underlying causality to be investigated. Why does this issue matter? A kinematic model with a periodicity or with a uniform acceleration, or average acceleration all will predict similar GMSL anomalies for the near future. Yet their differences will diverge markedly for predictions extending several decades.   This would be an extremely costly blunder in climate mitigation decision making.

11 – The other issue affecting the current study and its model for predictions involves using *smoothed* GMSL records for the analyses. Smoothed records will result in loss of GMSL variability information in predicting future anomalies rendering the usefulness of the predicted anomalies. The AR(1) effect in smoothed records will also increase.

12 – Statistically significant signature of periodicities of luni-solar origin have been detected in various TG records. Recently, Iz (2022) also demonstrated their presence in GMSL anomalies in 120 yearlong yearly averaged records of the GMSL budget components compiled by Frederikse et. al., (2020).   The signature of the luni-solar forcings also present in the series generated by this study. They are statistically significant (α=0.05). Their presence suggests an incomplete quadratic model resulting in non-random residuals, reinforcing the earlier criticisms about the assumptions of the methodology (the expected values of the disturbances being zero) used in this study for assessing the impact of the V/C matrix.

The efforts to build a full V/C matrix is a worthy endeavor for optimal analyses of GMSL anomalies. However, I found the outcome of this study dubious due to the outlined methodological flaws. The estimated model uncertainties are likely to be biased, there is no evidence if the constructed V/C matrix of the disturbances accounts for the AR(1) effect, the kinematic representation of the GMSL rise through a quadratic model is ambiguously presented, incomplete, and misleading in its interpretation.

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