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Response to Referee #1

We are grateful for the comments of the referee. We appreciate very much the time and effort which the referee has devoted to our manuscript. The report will enable us to improve the presentation of the material and to put it into the right context. Here we list all the critical remarks (in italics) and give an explanation of our points.

General evaluation:

***RC1**: .... I recommend their manuscript for publication with only minor comments listed below.*

Response:
Many thanks for the supporting opinion.

Minor comments:

***RC1**: Sections 2.3 and 3.1: It would be interesting to have a physics-based discussion on why one would expect a temporal correlation between the area-integrated eddy kinetic energy and enstrophy.*

Response: As described by Hopfinger and van Heijst (1993, page 244), for two-dimensional flow it is convenient to describe the motion in terms of the vorticity equation, which then takes a scalar form. In the case of flow in a horizontal plane with velocity $\mathbf{v} = (u, v, 0)$ the vorticity $\omega = \omega\mathbf{k}$ is given by:

$$\frac{\partial \omega}{\partial t} + J(\omega, \Psi) = \nu \nabla^2 \omega \ , \quad (1)$$

where J is the Jacobian operator and Ψ denotes the stream function, defined according to $\mathbf{v} = -\mathbf{k} \times \nabla\Psi$. By definition Ψ satisfies $\nabla^2\Psi = -\omega$. Note that ω represents here the absolute vorticity, which is equal to the sum of the relative vorticity and the planetary vorticity: $\omega = \omega_{rel} + f$. From the vorticity equation (1) it is inferred that steady inviscid flows are described by $J(\omega, \Psi) = 0$, and this implies that $\omega = F(\Psi)$, with F *any* integrable function.

Hopfinger and van Heijst (1993) then discuss several known models for discrete vortex flows, such as Rankine, Oseen, Lamb, and Burgers vortices. A relatively simple model describing isolated vortex structures with continuous vorticity and velocity distributions was used by Carton and Mcwilliams (1989). The expressions for nondimensional tangentional velocities $v(r)$ and vorticities $\omega(r)$ (where normalizations are performed by appropriate length and velocity scales R and V):

$$v(r) = \tfrac{1}{2} r \ \exp(-r^q) \ , \quad (2)$$

$$\omega(r) = \left(1 - \tfrac{1}{2} q r^q\right) \exp(-r^q) \ . \quad (3)$$

Here q is the so-called steepness parameter that controls the shape of the profiles. An appealing property of forms (2) and (3) that the squares of both have finite values of the spatial integrals over infinite domains for any $q > 0$ integer or non-integer values, usually determined by the Γ (Gamma) function. Consequently, the ratios of the spatial integrals are also finite rational numbers. The particular value of $q = 2$ defines the Gaussian vortices.

As for the real ocean, mesoscale eddies are close to Gaussian vortices, according to many analyses cited in the text. A finite area of integration is not closed, eddies are coming and going, emerging and decaying, still when the Gaussian hypothesis holds, then a strong correlation is expected between integrated kinetic energy and enstrophy.

Since all these information is described e.g. in Hopfinger and van Heijst (1993), we inserted a related remark (a short version of this explanation) into the text of Subsection 2.2 ("Gaussian mesoscale eddies").

***RC1**: Equations 6 and 8: The notation of the mean of IEKE and IZ are denoted with an overbar while the temporal mean of the nominator is in angle brackets. I would suggest unifying the notation one way or the other for representing the mean.*

Response: Thanks, we followed your recommendation and denote all temporal mean values by angle brackets.

***RC1**: Line 243: In the later -> latter.*

Response: Thanks, we preformed the correction.

***RC1**: Lines 322-323: The notation Eec is EKEec in Figure 6. Please unify the notation.*

Response: Thanks, we preformed the correction.

References

Carton, X. and Mcwilliams, J.: Barotropic and baroclinic instabilities of axisymmetric vortices in a quasigeostrophic model, in: Mesoscale/Synoptic Coherent structures in Geophysical Turbulence, edited by Nihoul, J. and Jamart, B., vol. 50 of *Elsevier Oceanography Series*, pp. 225–244, Elsevier, https://doi.org/10.1016/S0422-9894(08)70188-0, 1989.

Hopfinger, E. J. and van Heijst, G. J. F.: Vortices in rotating fluids, Annu. Rev. Fluid Mech., 25, 241–289, https://doi.org/10.1146/annurev.fl.25.010193.001325, 1993.