Author’s Response to Peer Review Referee Comments

Response to RC1 “Tesseract – A High Stability, Low-Noise Fluxgate Sensor Designed for Constellation Applications” by Kenton Greene et al. by Mark B. Moldwin on May 10, 2022:

We thank the referee for the constructive comments which we have incorporated into the manuscript. Mark B. Moldwin raised an important issue, which we address below. Referee comments are in plain text our responses in italics and any content added to or changed in the manuscript are in “quoted italics”.

Technical Changes:

Line 26: Change made. Text now reads: “Constellation satellite missions to have an important role in the future of space plasma science.”

Line 27: Change made. Text now reads: “The NASA Heliophysics Science and Technology Roadmap for 2014–2033, states a driver of scientific discovery will come from a constellation mission of 30 or more spacecraft (Heliophysics Roadmap 2014).”

Line 29: Change made. Text now reads: “Space Technology 5 (ST-5) (Slavin et al., 2008), Time History of Events and Macroscale Interactions during Substorms (THEMIS) *(Auster et al., 2008) and The Magnetospheric Multiscale Mission (MMS) (Torbert et al., 2016)”*

Line 276: Change made. Text now reads: “Prototype #1 was lowest power consumption configuration; Prototype #2 was the lowest inhomogeneity and Prototype #3 was chosen for a balance of low inhomogeneity and low power consumption.”

Specific Comments

1. Define “Stability” here. In Abstract the 2nd half discusses thermal stability. Also differentiate between what is examined here and long-term gain and offset stability. The other issue of thermal stability is assumed here is that the gain is linear with temp and that the ‘test’ field is Earth field of 60000 nT to get units of ppm/C. Can (nT)/C also be given?

The reviewer raises the important point that there are different ways to quantify stability. Fluxgate Stability is often described in terms of the stability of its individual calibration parameters: offset, orthogonality, and sensitivity. These parameters can change over temperature but can also drift over time (with a constant temperature). In the context of magnetospheric applications, stability is sometimes defined in part per million (ppm) of the total measured field.

In this paper, we focus on the stability of the sensor itself. The stability of a fluxgate instrument’s sensitivity and orthogonality is determined primarily by the structural stability of the sensor (Acuna et al., 1978; Miles et al., 2017).

The stability of the instrumental zeros or offsets (nT/C) is also important to evaluating the overall stability of a fluxgate; however, the changes in offsets are thought to come primarily from the changes in cores and driving electronics over temperature rather than changes in the sensor itself (Ripka et al., 2014). We are *currently in the process of integrating the electronics and cores with the sensor base and will address the offset stability of the Tesseract Magnetometer in a future manuscript.*

We have added context to clarify this on line 44: "Fluxgate magnetometers do not measure the magnetic field absolutely and therefore must be calibrated in order to make absolute measurements. However, the calibration parameters, sensitivity, orthogonality and offset, vary with changes in sensor temperature or over time. Fluxgate stability is the degree to which these calibration parameters remain constant."

In line 46 we have added a distinction between Sensor stability and core/electronics stability: "Fluxgate offsets are thought to originate primarily from the cores and driving electronics (Ripka et al. 2014), while changes in sensitivity and orthogonality are caused predominately by changes in the geometry of the sensor. In this paper, we are concerned with the stability of the sensor."

2. Is the Ripka result with respect to thermal stability of gain or long-term stability of the gain at a given temp or both? I couldn't find in a quick read of the paper.

The Ripka 1992 review article states that inhomogeneity of the feedback magnetic field could be one of the causes offset drift. Here is the link to a quick read of the paper: https://doi.org/10.1016/0924-4247(92)80159-Z

In addition to offsets, inhomogeneities in the feedback magnetic field has been shown to effect the stability of sensitivity or coil constant (Korepanov and Marusenkov 2012) and orthogonality (Petrucha et al 2015)

We have also made the following changes to clarify this at line 112: "An inhomogeneous magnetic null at the cores is thought to contribute to degrading the stability of a fluxgate's offset (Ripka, 1992) sensitivity (Korepanov and Marusenkov 2012) and orthogonality (Petrucha et al 2015)."

3. Are there other examples that used a Merritt coil for the nulling with fluxgates or is this the first use?

The only other example of a Merritt coil feedback topology used for a fluxgate that the authors are aware was by Petrucha et al., (2015). They built a bench top prototype of sensor laboratory prototype that used a Merritt coil feedback winding and two ring geometry cores. However, limited by the ringcore geometry, Petrucha et al., (2015) places the ringcores asymmetrically inside the Merritt coil. As a result, the cores immersed in a field that is inhomogeneous by as much as 8%.

Tesseract is the first to use of a Merritt coil feedback windings that uses the racetrack core configuration (shown in Figure 3). This combination of Racetrack geometry cores and Merritt Coil feedback topology has the advantage of keeping the cores in a very homogenous region (shown in Figure 4b) and ease of manufacturing. Tesseract's design draws inspiration from the aforementioned SMILE sensor (Forslund et al., 2008), which used three equally spaced square feedback coils and a parallel rod sensor in each axis to create a miniature cube-shaped, three axis nulled sensor.

We have added a more detailed background of previous feedback winding designs on line 108: "The idea of more complex feedback winding is not new. Designs such as Primdahl and Jensen 1982 and Chulliat et al., 2009 used stacked circular coils to create a three-axis null. A study by Petrucha et al., (2015) experimented with a laboratory prototype sensor that used a Merritt coil feedback topology with ring *geometry cores. Tesseract's design draws inspiration from the aforementioned SMILE sensor (Forslund et al., 2008), which used three equally spaced square feedback coils and a parallel rod sensor in each axis to create a cube-shaped, three axis nulled sensor."*

4. Have you ran a test to verify or is the surmise really based on previous work suggesting homogeneity is important? Does small gradient have any impact on sensitivity/noise or only temp stability?

We base this hypothesis off of past experimental studies (i.e Korepanov and Marusenkov (2012); Ripka (2014); Marusenkov (2006)) that have demonstrated that sensors with more homogeneous winding topology have less dependence on changes in the sensor's excitation behavior over temperature, which contributes to temperature instability.

Felch and Potter (1953) demonstrated that an inhomogeneous feedback field causes a residual out-of-phase cosine signal to appear at the output, that in turn causes baseline to drift. This signal can also leak into the 2f, which in addition to causing temperature instability, could raise the instrumental noise floor.

An inhomogeneous feedback field can negatively affect temperature stability of orthogonality and instrumental linearity. A study by Petrucha et al (2015) determined that in a three-axis null sensor, changing the inhomogeneity of the magnetic feedback along the core by as little as 1.3%, can alter the measured alignment of the sensor's axes by +/- 0.12 degrees. Brauer et al (1997) demonstrated that inhomogeneities as little as 1% causes large 25 nT deviations from linearity when an uncompensated earth field is applied in a transverse axis.

We have cited previous work that highlights the importance of feedback homogeneity by adding the following text on line 190: "A homogeneous magnetic null around the cores improves stability over temperature by reducing in dependance of the sensitivity on the temperature dependance of the excitation current (Korepanov and Marusenkov 2012). A study by Petrucha et al (2015) determined that in a three-axis null sensor, changing the inhomogeneity of the magnetic feedback along the core by 1.3%, can alter the measured alignment of the sensor's axes by as much as +/- 0.12 degrees. An inhomogeneous feedback field can also degrade instrumental linearity. Brauer et al (1997) demonstrated that inhomogeneities as little as 1% causes 25 nT deviations from linearity when an uncompensated earth field is applied in a transverse axis."

5. Have you demonstrated that the configuration is better than just three cores? (or is 'expected based on previous research that can be cited?

The authors have not demonstrated this experimentally. Cores with shape anisotropies that have opposite polarities like that of the rod core fluxgates have been shown to produce smaller stray fields, than ring-core geometry fluxgates (Ripka and Billingsley 2000). The authors hypothesize that shape anisotropies and symmetry of the Tesseract's two cores per axis with opposite polarities will further suppress stray fields that can cause cross-axis contamination.

We have changed the language to clarify in line 150-151: "We hypothesize that pairing of identical cores with opposite polarities in each axis may further reduce the tendency for cross-axis contamination due to mutual cancellation of their stray fields (Ripka and Billingsley 2000)."

6. What is the notional average magnetic field created by the 20-mA current?

For the engineering model Tesseract sensor (Prototype #3 in section 3.1.2) the field generated by the 20-mA current was ~190,000 nT. The field was intentionally large so that small differences (Inhomogeneities) would be easily resolved.

A change was made on line 282. The text now reads: "Each prototype sensor was placed within a single axis solenoid within a three-layer mumetal magnetic shield and constant current of 20 mA was applied to the feedback windings creating a magnetic field of about 190,000 nT"

7. It should be clearly stated that the temperature dependence you are looking at is on the "structure" and not the cores and electronics in the conclusions.

The reviewer makes a very important point. This paper is concerned with the stability of the sensor's structure alone independent of the effects of cores and electronics. Changes in the sensor structure are thought to be a primarily cause of instability of sensor orthogonality (Acuna et al., 1978) and sensitivity (Miles et al., 2017) over temperature. Therefore, in this paper, we focus on temperature dependence of the geometry of the sensor (sensitivity and orthogonality over temperature).

We are currently in the process of integrating the electronics and cores with the sensor base and will characterize their effect on the stability of the Tesseract Magnetometer in a future manuscript.

We have added this clarification in several places to highlight the importance of this distinction:

In the introduction; lines 59-63: "Fluxgate offsets are thought to originate primarily from the cores and driving electronics (Ripka et al., 2014), while changes in sensitivity and orthogonality are caused predominately by changes in the geometry of the sensor (Acuna et al., 1978; Miles et al., 2017). In this paper, we are concerned with the stability of the sensor."

At the end of line 335: "Here, we describe a test to measure the stability of the Tesseract sensor's feedback windings over temperature by temporarily configuring it as an air-core search coil magnetometer to directly access the attributes of the sensor base and feedback windings without any dependence on cores or electronics."

In the conclusion (line 425) we have changed the text to "We used a low-cost method to analyze the gain and orthogonality of the sensor over temperature without any dependance on cores or electronics."

Response to RC2 comments on "Tesseract – A High Stability, Low-Noise Fluxgate Sensor Designed for Constellation Applications" by Greene et al. made by Hans-Ulrich Auster on May 10, 2022:

We thank the referee for the constructive comments which we have incorporated into the manuscript. Hans-Ulrich Auster raised a few important issues about comparison to other sensors in the literature, which we address below. Referee comments are in plain text our responses in italics and any content added to or changed in the manuscript are in "*quoted italics*"

1. Features of magnetic field experiments on constellation mission are discussed in the introduction. Please add a brief statement, why particularly these applications are used as reference for the new developed magnetometer. An argument could be, that these missions are representative for almost all space born magnetometers; wide range is required (low field at apogee, high field at perigee), exposed to radiation, temperature changes due to eclipse crossings.

The Referee raises an important point about the instruments discussed in the introduction. The constellation missions mentioned in the paper have exposure to a wide range of environmental conditions such as a large range magnetic field strength, abrupt temperature changes and exposure to radiation. These one or more of conditions are characteristic of the environmental considerations that affect magnetometers in most applications in space.

We have added a brief statement to highlight this point on line 31-33 which reads: "Magnetospheric missions are representative of the conditions experienced by many space born magnetometers; they are subject to a wide range of magnetic fields (low field at apogee, high field at perigee), as well as radiation exposure, and temperature changes due to eclipse crossings."

2. The comparison is made disordered, some parameters are listed for the one, others for the second type of magnetometer. This should be harmonized, may be supported in a table format. Noise, mass, scale value and axis stability vs. temperature shall be given for all of them. All constellation missions mentioned in this paper (Themis, MMS, SWARM) are since many years in space. Long period data for offset and axis stability should be available from inflight calibration. Contact magnetometer PI's for these data (if not published) and include the inflight measured drifts into your comparison.

We have rearranged in section 1.2 so that the sensor parameters are harmonized: Size in mm^3, mass in g, sensitivity over temperature in ppm/℃, orthogonality stability in degrees, and noise in pT/√Hz are listed in order so that comparisons can be more easily made.

Lines 80 – 94 now reads: "Potentially the most stable magnetospheric field fluxgate measurements to date were taken with the Compact Spherical Coil (CSC) Sensor aboard Swarm which implements a nested three axis feedback coil wound on a MACOR shell to create a very homogenous a three-axis null at the location of the cores. From Primdahl and Jenson (1982), we estimate that the CSC's feedback coils hold their ring-cores in a field that deviates from uniformity by 1.5%. The 100x100x50 mm, 500g CSC sensor has maintained a sensitivity stability over temperature of 10 ppm/°C and very high axis stability of 0.002 degrees from -20°C to 40°C while achieving a noise floor of 6.6 pT/√Hz at 1 Hz (Merayo et al., 2008).

The THEMIS mission incorporated a small 70×70×45 mm, 75g fluxgate sensors that achieved a sensitivity stability of 22 ppm/°C, an axes' stability within 0.017 degrees from -100°C to 60°C and a noise of 10 pT/Hz at 1 Hz (Auster et al., 2008). The 42.4×44.3×48.7 mm, 88g Magnetospheric Multiscale Mission DFG instrument achieved a sensitivity stability over temperature of 30 ppm/°C, an an axis stability of about 0.03 degrees between -50°C and 30°C a noise floor of 8pT/√Hz at 1 Hz, (Russell et al., 2016)."

Lines 110 -113 now reads: "One of the best small sensors to date appears to be the Small Magnetometer in Low-mass Experiment (SMILE): a 20×20×20 mm, 40 g cubic sensor based on three rod cores within a three-axis feedback winding, which achieved a thermal sensitivity stability of 11 ppm/°C, an axis stability better than 0.02 degrees from -30 to 45°C, and 30 pT/√Hz at 1Hz (Forslund et al., 2008)."

3. The presented sensor design is impressive. In contrast to the straightforward OERSTEDT/SWARM design (feedback system over three single ringcores) and the more compact THEMIS design (feedback system over crossed ringcores) the cores (racetracks) are accommodated symmetric and identical for all three components. It is made similar to the very innovative Xavier Lalanne design from the 1990th. He placed six ringcores at the six planes of a cube. Please refer to it.

The Authors would like to thank the Referee for bringing this work to our attention. The VM391 magnetometer developed by Lalanne and Chulliat (Chulliat et al., 2009) is a clever sensor design that uses circular, concentric feedback coil geometry with several Ringcores placed symmetrically in order to create a sensor with three axis compensation. This design is still used in the ground-based magnetometer community including at many INTERMAGNET ground observatories.

We have added a reference to this work where we discuss previous feedback winding designs. Line 145 now reads: "The idea of more complex feedback winding is not new. Designs such as Primdahl and Jensen (1982), Auster et al., (2008) and Chulliat et al (2009) have used different methods of stacking circular coils to create a three-axis null sensor."

4. Promoting the presented sensor is ok, however, the comparison with a user defined ringcore sensor, which should imply that the presented sensor is much better than ringcore sensors in general is not acceptable. The comparison has to be made with the vector compensated ringcore sensors you have studied in the introduction.

The Authors include this comparison because it illustrates what we think are interesting features such as the effects of sensor asymmetry on sensitivity and orthogonality (Shown in Figure 11). It was not the intention of the Authors to infer that the ringcore sensor that is tested in this paper is representative of a state-of-the-art instrument. To avoid misrepresenting or overstating the comparison between the two sensors, we have rewritten several lines in the text:

In the abstract, lines 16 and 20, we have taken out comparisons to the 'ringcore' sensor. The section now reads: "The thermal stability of the sensor's feedback windings is measured using an insulated container filled with dry ice inside a coil system. The sensitivity over temperature of the feedback windings is found to be between 13 ppm/°C and 17 ppm/°C. The sensor's three axes maintain orthogonality to within at most 0.015 degrees over a temperature range of -45 °C to 20 °C. Tesseract's cores achieve a magnetic noise floor of 5 pT/√Hz at one Hz."

We concur that referring to it as 'the ringcore sensor' creates unintended confusion with sensors discussed in section 1 that use ring-cores. The ringcore sensor tested in this manuscript is the same design from Miles et al., (2013). To avoid this confusion, we now call this sensor the 'Miles (2013) ringcore sensor design' everywhere in the text:

Line 141- 142 now reads: A two-axis null ring-core sensor design based on (Miles et al., 2013) was used in the testing described section 3 and is shown for comparison.

Line 159 now reads: "The Tesseract Sensor's specifications as measured in the laboratory compared with the specifications of a more traditional ring-core sensor design described in Miles et al., 2013. Tesseract is marginally larger than the Miles et al. 2013 ring-core sensor design"

Line 217 now reads: "*Torlon is lighter and much easier to machine than the traditional Inconel ring-core bobbin or MACOR used in Miles (2013)"*

Line 367 now reads: "We performed the same test again, this time using our prototype feedback winding for the Miles (2013) 1" ring-core sensor (Figure 1b)"

Line 374-375 now reads: "The maximum that the measured feedback field deviates from uniformity along the x axis of the Miles (2013) ring-core sensor is 5.62%. The Miles (2013) sensor's feedback windings generate a field the along the center axis of the ring-core that is ten times more inhomogeneous than that generated by the Tesseract sensor."

Line 523 now reads: "The Miles (2013) ring-core sensor design (Figure 1b) was also temporarily configured as an air-core search coil"

Line 529 now reads: "The u_3 *Angle (blue) changes the most, presumably because the Miles (2013) ring-core sensor base is most asymmetric between the Y and Z Axes (Figure 1b)."*

This paper is a study of the sensor itself without cores or driving electronics. The tests described are concerned with of the effect of sensor structure on stability, such as the feedback topology and axis alignment with applying active feedback The sensor is currently in the process of being integrated with cores and flight electronics. The full vector compensated, and calibrated results will be discussed in a forthcoming manuscript.

In line 53: "Fluxgate offsets are thought to originate primarily from the cores and driving electronics (Ripka et al., 2014), while changes in sensitivity and orthogonality are caused predominately by changes in the geometry of the sensor (Acuna et al., 1978; Miles et al., 2017). In this paper, we are concerned with the stability of the sensor. Factors suspected of degrading a fluxgate sensor's stability include an inhomogeneous magnetic null (Ripka, 1992) and skewing of the axes due to mechanical and thermal strain (Primdahl, 1979)."

5. Quantities are mixed up. It shall be clearly distinguished between stability of offsets, scale values and orthogonality. The vector compensation stabilizes the orientation of the magnetic axis while the offset stability depends on core properties only. Thus, for scale value and axes stability it is fully unimportant which type and geometry of magnetic material is used as core.

We agree with the reviewer that past studies (i.e. Acuna et al., 1978; Petrucha et al., 2015; Primdahl and Jensen 1982) have demonstrated that the stability of the orthogonality is determined by alignment feedback windings that provide vector compensation, while the offset stability depends on driving electronics. This paper is concerned with the sensor and therefore we do not focus on electronics, or offset stability, instead we focus on sensitivity (scale value) stability and orthogonality (axis alignment) stability.

Once the sensor and the electronics are fully tuned, we plan to extensively calibrate and characterize the full instrument including offset stability over time and offset over temperature.

We have added new text to the introduction of the manuscript to highlight this distinction between offsets, sensitivities and orthogonality in lines 59 – 63: "Fluxgate offsets are thought to originate primarily from the cores and driving electronics (e.g., Ripka et al. 2014), while changes in sensitivity and orthogonality are caused predominately by changes in the geometry of the sensor. In this paper, we are concerned with the stability of the sensor's sensitivity and orthogonality."

And on lines 120 – 122: "An inhomogeneous magnetic null at the cores is thought to contribute to degrading the stability of a fluxgate's offset (Ripka, 1992) sensitivity (Korepanov and Marusenkov 2012) and orthogonality (Petrucha et al 2015). "

We have also made clearly distinguished between sensitivity, orthogonality and offsets Section 3. In line 364: "Here, we describe a test to measure the stability of the Tesseract sensor's feedback windings over temperature by temporarily configuring it as an air-core search coil magnetometer to directly access the attributes of the sensor base and feedback windings without any dependence on cores or electronics."

In the conclusion (line 448) we have changed the text to "We used a low-cost method to analyze the gain and orthogonality of the sensor over temperature independent of cores or electronics."

6. The analysis of the uniformity of the feedback coils has been intensively discussed. No question, high homogeneity is better than low homogeneity, however in case you want to underline the importance of the uniformity, you have to quantify it. What is the impact on offset, scale value, linearity, and orthogonality behavior really? Particularly racetracks with a significant length/diameter ratio might disturb the uniformity you have hardly achieved by the sophisticated feedback coil design.

We agree with the reviewer that it is important to quantify the impacts of homogeneity on sensor stability. We base this off past experimental studies that have demonstrated that an inhomogeneous feedback field can negatively affect temperature stability of orthogonality and instrumental linearity.

A study by Petrucha et al (2015) determined that in a three-axis null sensor, changing the inhomogeneity of the magnetic feedback by that deviates from average along the core by 1.3%, can alter the measured alignment of the sensor's axes by as much as +/- 0.12 degrees.

Brauer et al (1997) demonstrated that inhomogeneities as little as 1% causes 25 nT deviations from linearity when an uncompensated earth field is applied in a transverse axis.

Several studies have demonstrated that sensors with more homogeneous winding topology are less dependent on changes in the sensor's excitation behavior over temperature, which contributes to sensitivity (Marusenkov 2006) and offset (Ripka 2014) instability over changes in temperature. Felch and Potter (1953) demonstrated that an inhomogeneous feedback field causes a residual out-of-phase cosine signal to appear at the output, that in turn causes baseline to drift. This signal can also leak into the 2f, which in addition to causing offset and temperature instability, could raise the instrumental noise floor.

We have cited previous work that highlights the importance of feedback homogeneity by adding the following text on line 190: "A homogeneous magnetic null around the cores improves stability over *temperature by reducing in dependance of the sensitivity on the temperature dependance of the excitation current (Korepanov and Marusenkov 2012). A study by Petrucha et al (2015) determined that in a three-axis null sensor, changing the inhomogeneity of the magnetic feedback by 1.3%, can alter the measured alignment of the sensor's axes by as much as +/- 0.12 degrees. An inhomogeneous feedback field can also degrade instrumental linearity. Brauer et al (1997) demonstrated that inhomogeneities as little as 1% causes 25 nT deviations from linearity when an uncompensated earth field is applied in a transverse axis."*

7. The authors present a nice sensor with an excellent performance. The only drawback is the mass, which may be a little bit too heavy for constellation missions or small satellites. However, with respect to the mass of boom and harness, a few 100g should be acceptable for the most important part of a magnetic field experiment, the sensor.

The Authors agree with Referees assessment that the Tesseract Sensor is on the heavier side to be accommodated on a small satellite. Tesseract could be a good fit for small-medium scale constellation satellites (missions like the proposed Geospace Dynamics Constellion (GDC) and MagneToRE).

8. The discussion of thermal expansion of the feedback system is not as simple. A high scale value stability of <10ppm/K, achieved by a combination of materials with different expansion coefficients must not be better than a scale value stability of 20ppm/K, if this one is linear over the whole temperature range and has a lower hysteresis. Thus linearity, the reaction on fast temperature changes (e.g. during eclipse) and the hysteresis are criteria which have to be discussed and compared, not the number itself.

We agree with the reviewer that the linearity of the dependance of gain on temperature matters more than the magnitude of that dependance (in ppm/K). We understand that the time varying effective temperature, sometimes described as thermal gradient, is of great interest to the community and has been observed in-situ (Bromund et al., 2021) and in the laboratory (Brauer et al., 1999). We are interested in addressing this issue are currently running an entirely separate set of tests, that are out of the scope of this paper.

In this paper, we focus on thermally homogenous effects. We test this by changing the sensor temperature very gradually (roughly one degree Celsius per hour as shown in Figure 10). However, in an uncontrolled warming there is always the possibly for thermal gradients or transients. We are currently building a new purpose-build testing facility similar to the one used in Magnes et al (1998) that will allow thermal controlled testing environment and limit the possibly for thermal 'gradients', 'shocks' or 'transients'.

We have added text to address the concern of a hysterisis effect due to thermal gradients in section 4.1 from lines 556 – 561: "The accuracy of these measurements is limited by the magnetic noise of the laboratory setup and in an uncontrolled warming there is always the possibly for thermal gradients or transients. A more sophisticated experimental set up will be required to characterize the stability of Tesseract's calibration parameters to an accuracy acceptable for most space science applications (greater than ± 0.1 nT). We are currently developing a new calibration facility at the University of Iowa that will be purpose built for controlled temperature characterization fluxgate sensor's calibration parameters which will incorporate better shielding and limit the possibly for thermal gradients or shocks." Response to Peer Review Referee Comments

---

Response to RC3 "Tesseract – A High Stability, Low-Noise Fluxgate Sensor Designed for Constellation Applications" by Kenton Greene et al. by an Anonymous Referee #3 on May 23, 2022:

We thank the referee for the constructive comments which we have incorporated into the manuscript. The referee raised an important issue about the description of the thermal testing procedure, which we address below. Referee comments are in plain text our responses in italics and any content added to or changed in the manuscript are in "quoted italics".

Specific Comments:

1. The references (Brauer et al., 1999; Miles et al., 2017) exploit some other approaches in comparison with that described in the manuscript. Brauer et al. (1999) used so-called "thin shell" calibration - the calibrating signals "were randomly distributed over shells of fixed field magnitudes". The data processing was also different – the overdetermined system of linear equations was solved for a parameters matrix by singular value decomposition. Miles et al. (2017) did not estimate orthogonality. In both references, a fluxgate magnetometer as a whole unit was calibrated, whereas the manuscript estimates the temperature characteristics of the feedback coils only, without magnetic cores inside.

It was not our intention to imply that the temperature test procedure described in section 3.2 uses the same methods as these previous experiments (Brauer et al., 1999; Miles et al., 2017). We agree with the reviewer that there are important differences between the calibration method presented in this paper and methods in previous temperature tests referred to: Miles et al. (2017). Brauer et al. (1999). While the temperature calibration test presented in this work draws form the designs of their experimental apparatus, we use different method to estimate sensitivity and orthogonality which address below in our response to comment #4.

We have made the following changes on lines 339-341 in order to clarify this: "This test usually requires sophisticated equipment. simpler, low-cost experimental setups have been created (i.e., Brauer et al., 1999; Miles et al., 2017) to calibrate a magnetometer using an insulated cooler filled with dry ice placed within some form of calibration coil. Here, we extend the method of Miles et al. 2017 to characterize both sensitivity and orthogonality"

The reviewer highlights another important distinction from the referenced experiments: the experiment described in this manuscript measures the temperature characteristics of the feedback coils only, without magnetic cores or driving fluxgate electronics.

We have changed the title of the section and added clarifying context in the opening paragraph of the section 3.2 and to highlight this important distinction in line 335: "Here, we describe a test to measure the sensitivity and orthogonality of the Tesseract sensor's feedback windings over temperature by temporarily configuring it as an air-core search coil magnetometer. This allows us to assess the temperature stability of sensor base and feedback windings without any dependence on cores or electronics."

2. Were the feedback coils used in the air-core search coil magnetometer to form feedback signals or to serve as sense windings? What was a sense winding in the first case? In the second case (the feedback winding is used as a sense one) the temperature dependence of the sensitivity or gain of such air-core search coil magnetometer was actually tested. The temperature stability of the fluxgate magnetometer's scale factor depends on the stability of the coil constant of the feedback winding. Is it assumed that the gain of the air-core search coil magnetometer based on the feedback winding depends on the temperature in a similar way as the coil constant of the feedback winding does?

We agree with the reviewer's remarks about the importance of distinguishing between Tesseracts sense coils and feedback coils. Unlike some other fluxgate instruments, such as the Ringcore design described in this paper, which use the sense winding to provide magnetic feedback, The Tesseract Sensor has purpose-built (Merritt Coil) windings to provide magnetic feedback and separate solenoidal windings (shown in figure 3) wound directly on the racetrack core bobbin to act as sense windings.

For temperature testing (Section 3.2), The feedback coils alone were used as an air-core search coil. It is assumed that the gain of the air-core search coil magnetometer based on the feedback winding depends on the temperature in a similar way as the coil constant of the feedback winding does once the cores are inserted since the dominant effect should be the coefficient of linear thermal expansion of the single-piece sensor base.

We have added text to address this in lines 138-140: "Production cores are interleaved with a polymer between the foil layers to prevent them from moving during the magnetizing drive pulses. A plastic lid closes the core and serves as a base upon which to wind a quasi-toroidal drive of AWG 32 magnet wire (Figure 2b). Finally, a solenoidal sense winding of AWG 34 magnet wire is wrapped around the bobbin"

We have made the following changes at the end of line 335 to make clear that this test is not a calibration of a full fluxgate (with cores and driving electronics) over temperature: "In this section, we describe a test to measure the stability of the Tesseract sensor's feedback windings over temperature by temporarily configuring it as an air-core search coil magnetometer to directly access the attributes of the sensor base and feedback windings without any dependence on cores or electronics."

3. The mutual orientation of the axes of the calibrating system and the device under test is not clear in Figure 9a. How accurately were aligned the magnetometer feedback coil axes with that of the Merritt coil system and what method was used to achieve this?

The Tesseract sensor was roughly aligned with the coil system by hand (i.e: X axis of the feedback coil was lined up with the X axis of the calibration coil system) and then rotated slightly until the measured signal was maximized in each axis. The maximum of the 10,000 nT applied field was found to within +/- 5 nT which corresponds to an alignment accuracy within +/- 1.8 degrees. This alignment is not critical to the calibration, as long as the sensor does not rotate over the course of the test, since we express changes in sensitivity and orthogonality with respect to a reference (i.e. The sensitivity measured at room temperature).

We have made changes on line 388: "The Tesseract sensor's axes are manually aligned with the coil system's axes (Figure 9b) and then slowly rotated until the measured 23 Hz signal is maximized in each axis. The sensor base is then firmly fastened to a mount, so that it does not rotate over the course of the test."

4. The equations (2), (3), and (4) for estimating sensor orthogonality have to be explained in detail or appropriate reference should be added. The way Equations (2), (3), and (4) for estimating orthogonality angles were derived is not clear. Why are these equations different for the XY pair and the XZ, and YZ pairs? How was the total magnitude (A) of the applied field calculated or measured?

We define the orthogonality angles based on figure 1 and equation 2 in Olsen et al. 2003. The equations (2) (3) and (4) are different for the XY pair because the Olsen et al. 2003 convention define the x axis to be projection invariant, the y axis has a single degree of freedom in the XY plane, and the z axis has two degrees of freedom. We have also changed the notation to be in agreement with that in Olsen et al. 2003

Line 419 has been changed to include this reference: "The Tesseract sensor's three orthogonality angles; the angles between the X and Y axes u_1, X and Z axes, u_2 and Y and Z axes u_3 as defined in (Olsen et al., 2003)"

We have added a new figure as Figure 11 (taken from Olsen et al. 2003 Figure 1 Left) that illustrates the definition of the three angles.

We have added a definition of the total magnitude (A) and a description of how it was measured. Lines 376-380 now reads: "A is the total magnitude of the applied field which is defined as $A = \sqrt{Xx^2 + Yy^2 + Zz^2}$, Where Xx is the field measured in the X axis when the coil system applies a field in the X axis and Zz is the measured field in the Z axis when the coil system applies a field in the Z axis. Figure 11b plots the change in these angles, u_1, u_2 and u_3, over temperature."

We have added text to the caption to Figure 11 to explain the differences between equations (2) (3) and (4) in this convention: "The equations (2) (3) and (4) are different for the u_1 because the Olsen et al. 2003 convention define the x axis to be projection invariant, the y axis has a single degree of freedom in the XY plane, and the z axis has two degrees of freedom."

Technical Changes:

1. Which component of the magnetic field generated by the feedback coil is presented in the color map in Figure 4a? Bx? It would be useful to clarify.

The magnetic field rendered in Figure 4 is generated by the x-axis feedback coil (Line 207). We have clarified this in the Figure caption as well in Line 203: "Here we show a render of the x axis feedback coil"

2. The length of the racetrack sensor is equal to 31.45 mm (Subsection 2.1, p. 5, line 132), but the Racetrack boundaries are equal to +/-14.5 mm in Figures 6, 7, and +/-15 mm in Figure 8.

The racetrack boundary line on Figures 6, 7, and 8 have been changed to the actual core length which is 31.45 mm long (or +/- 15.725 mm)

3. The last part of the caption of Figure 6: "...Configuration (b) was optimized for best homogeneity while sensor while (c) was chosen for good homogeneity with very low power consumption." Should it be "...Configuration (b) was optimized for best homogeneity within the sensor while (c) was chosen for good homogeneity with very low power consumption." or "...Configuration (b) was optimized for best homogeneity while sensor (c) was chosen for good homogeneity with very low power consumption."?

This has been corrected in the caption of Figure 6: "Configuration (b) was optimized for best homogeneity while sensor (c) was chosen for good homogeneity with very low power consumption.".

We have also added a sentence in the text to avoid misinterpretation of our prototype selection process: "Prototype #1 was lowest power consumption configuration; Prototype #2 was the low inhomogeneity and Prototype #3 was chosen for a balance of low inhomogeneity and low power consumption."