In this document, we respond to the comments of reviewer 1 one by one. Whenever some entirely new text has been added to the manuscript, it has been added in italics and in red.

Reviewer 1

This manuscript presents a detailed exploration of the thermal regime of the Grigoriev ice cap and the Sary-Tor glacier, which are located in the Inner Tien Shan in Kyrgyzstan. Using a wide range of observations, such as observed temperature profiles and GPR profiles, model parameters are tuned and subsequently evaluated against observations. The study finds that the Sary-Tor can always be considered a polythermal glacier unlike the Grigoriev ice cap which can always be considered as a cold structure. It is also suggested that the found parameters can be generalized to similar type of glaciers in the region. The author(s) did a great job reviewing and referencing previous work. Overall this was found to be a very extensive and thorough study.

We would like to thank the reviewer for the useful review which helped us to improve the quality of the manuscript.

Specific comments

[RC1.1] Geothermal heat. The Geothermal heat flow parameterization used seems too casual. The study is using the same constant value for the geothermal heatflux for both the valley glacier and the ice cap, however, it has been showed that geothermal heatflux is focused in valleys like Sary-Tor and diminished on ridges, like Grigoriev Ice Cap. It is possible that Grigoriev Ice Cap has double the heat flow as Sary-Tor. See the papers by Colgan et al. 2020 (doi:10.1029/2020JF005598), section on topographic correction and Van der Veen et al. 2007 (doi:10.1029/2007GL030046). The chosen value for the geothermal heat boundary condition of 50mW/m2 seems quite high given the altitude of the glaciers. Another study by Zhong et al. 2013 from Journal of glaciology doi:10.3189/2013JoG12J202, studying the East Rongbuk Glacier, have derived a lower geothermal heatflux value of approximately 19mW/m^2.

We would like to thank the reviewer for these relevant suggestions. We kept the average heat flux of 50 mW m^{-2} as different previous research showed this value to be the average geothermal heat flux in our study region. We added two more references to previous studies on this subject (Duchkov et al., 2001; Vermeesch et al., 2004). The East Rongbuk Glacier is located in the Himalaya, and it is also situated about 3000 meters higher which may explain the difference between the geothermal heat flux used. Furthermore, the Tien Shan is assumed to be associated with crustal thickening and thus is expected to display an increased surface heat flow (Vermeesh et al., 2004).

Two references were added:

Duchkov, A.D., Yu.G. Shvartsman, and L.S. Sokolova, Deep heat flow in Tien Shan: advances and drawbacks, Geologiya i Geofizika (Russian Geology and Geophysics), 42, 10, 1516–1531(1436–1452), 2001.

Vermeesch, P., Poort, J., Duchkov, A., Klerkx, J. and De Batist, M.: Lake Issyk-Kul (Tien Shan): Unusually low heat flow in an active intermontane basin. GEOLOGIYA I GEOFIZIKA, 45(5), 616–625, 2004

Following your suggestion, we implemented a topographical correction following Colgan et al. (2021). As such, we account for the influence of topographic relief on the geothermal heat flux within the study region.

Colgan W., MacGregor J.A., Mankoff K.D., Haagenson R., Rajaram H., Martos Y.M., Morlighem M., Fahnestock M.A. and Kjeldsen K.K.: Topographic correction of geothermal heat flux in Greenland and Antarctica, Journal of Geophysical Research. Earth Surface, 126, https://www.doi.org/10.1029/2020JF005598, 2021

In lines 446-461, we added:

To account for the influence of topography on the geothermal heat flux, we apply the empirically determined topographic correction procedure described in Colgan et al. (2021). This concerns a high-pass filter with a dimensionless correction factor applied to the average geothermal heat flux, which makes the geothermal heat flux spatially variable depending on the local elevation. Using the correction factor, the geothermal heat flux is magnified in incised valleys such as for the Sary-Tor glacier and attenuated on ridges such as for the Grigoriev ice cap. More specific, in the correction procedure, the average geothermal heat flux is perturbed by an anomaly $\left(\frac{\Delta g_{hf}}{\overline{g_{hf}}}_{ij}\right)$ *to obtain the local geothermal heatflux* $(g_{hf,ij})$.

$$g_{hf,ij} = \overline{g_{hf}}\left(1 + \frac{\Delta g_{hf}}{\overline{g_{hf}}}_{ij}\right) \quad (14)$$

The anomaly is estimated as a function of local relief using

$$\frac{\Delta g_{hf}}{\overline{g_{hf}}}_{ij} = \frac{1}{\alpha}(\bar{h}_{ij} - h_{ij}) \quad (15)$$

with α *an empirically determined characteristic height (950 m) and* $\bar{h}_{ij}$ *the mean elevation averaged within a moving window of 10 x10 km centred over location ij.*

Applying this topographic correction, the obtained geothermal heat flux on the summit of the Grigoriev ice cap is substantially lower (29 mWm^{-2}), which is more in line with observation of Arkhipov et al. (2004) that the ice temperature at the summit did practically not depend on the vertical coordinate at depths from 10-45 m (~lower geothermal heat flux).

[RC1.2] Line 650-655: There is something unclear in this section. First it is described that average ice temperatures are most sensitive to changes in geothermal heat, but the paragraph is ended with 'geothermal heatflux do not have any major influence'. I assume the latter statement is the conclusion the author draws from the sensitivity analysis, and the former just means that relative to the other tested parameters geothermal heat influence the mean temperature the most. Maybe the section can be written slightly different?

We agree with the reviewer and changed this paragraph in lines 1261-1264:

"For both ice bodies, the average ice temperature changes only slightly when the parameters and the average geothermal heat flux are altered within the predefined ranges (Table 4). Hence, based on the sensitivity analysis, it can be concluded that for the selected ice masses, parameter uncertainty and the magnitude of the geothermal heat flux do not have a major influence on the obtained thermal regime, making the results of this study robust."

[RC1.3] Temperature calculations. As far as I can tell the ice flow model is a 'cold ice' model like most ice flow models. Modelling the flow of temperate and polythermal ice is different to modelling cold ice since it is essential to know the spatial distribution of the water content in the ice (See Dynamics of Ice sheets and Glaciers by Ralf Greve and Heinz Blatter). According to a study by Andy Aschwanden (doi:10.3189/2012JoG11J088) 'cold-ice' models are not energy-conserving when used with temperate ice since they cannot account for the part of the internal energy which comes from latent heat of liquid water. This could be included as a discussion point. Technically both glaciers could be considered polythermal, following the IACS glacier terminology Cogley 2011.

We agree with the reviewer, and we added a section in the methodology section describing that the applied model is a cold ice model in lines 337-341:

"The applied HO model is a cold ice model which implies that it is not energy conserving when temperate ice is present, as it does not account for the part of internal energy coming from the latent heat of liquid water (Aschwanden et al., 2012, 2017). Hence, it does not take into account the presence of water in the ice. Nevertheless, the tuning of the enhancement factor (m) is an implicit way to include softening effects due to factors such as water content and impurity content (Huybrechts et al., 1991)."

We are aware that technically, if the temperate ice of the Grigoriev ice cap is not only present at the contact surface between the ice and the bedrock, we can technically classify this ice cap as being polythermal. However, since the surface area and volume for which temperate ice was found is very limited (< 0.01% under standard conditions, and only the bottom layer in contact with the bedrock), the ice cap is predominantly cold, and we decided to preserve the terminology.

[RC1.4] Line 309-311: The explained effect is also described in Hooke 1976 J. glaciology, study of Barnes ice cap. It might be good to cite that paper.

Thank you for the suggestion. We added a reference to Hooke (1976) in line 511.

General questions and comments on unit, equations and notation:

[RC1.5] There are many parameters and m appears several times it might be nice to include an annotation table for clarity? This could also help to clearly define units of every variable. Sometimes variables in text does not have units e.g. the melting point depression factor, gamma in line 198.

We agree with the reviewer and added a table with all variables, constants, and their symbols, values and units (Table 1).

[RC1.6] How is the internal heat production P from equation (9) calculated?

P is calculated from strain heating. We added this in the text on line 395: *"The internal heat production (P) is calculated from strain heating (Huybrechts, 1996)."*

[RC1.7] Where does equation (15) come from?

We added a sentence to make this clearer in lines 461-462:

"Then, as boundary condition at the bed, the temperature gradient is defined as the sum of the local geothermal heat flux and the heat due to basal sliding (Huybrechts and Oerlemans, 1988)."

[RC1.8] Equation (16) The last term of the surface temperature (Ts) parameterization for H<ELA does not seem to have units of degree if ms has units of m w.e. pr yr? units don't seem to match with Ts having units of degree Celsius.

Thank you for this remark. The '10' value in the denominator of the fraction has unit metres of ice equivalent. We added the units to m_s and 10 in this equation to make it clear for the reader.

[RC1.9] The introduction mentioned that Grigoriev Ice Cap is losing mass by calving, but this is not mentioned in the mass balance model section. Does the 'm_s at the front' in table 2 refer to mass loss from calving? What do the tuning parameters c0 and c1 represent, I don't think they are explained fully in the text? It seems like one would have to read Van Tricht et al. 2021b to understand it the section does not stand completely on its own. It might be helpful to explain a bit further?

The mass loss by calving for the Grigoriev ice cap (at the north side) is situated at the highest elevations (>4500 m a.s.l.). It is not included in the mass balance model. To make this clearer, we added in lines 771-772:

"Concerning the northern boundary of the Grigoriev ice cap, when ice flows beyond the predefined calving front, it is automatically removed."

Concerning more explanation about the mass balance model, we added in lines 619-621:

"This simple model is based on incoming solar radiation, temperature and precipitation and contains two additional tuning parameters (c_0 and c_1) representing the sum of the longwave radiation balance and the turbulent sensible heat exchange."

And we repeated in line 627:

"(c_0 and c_1)"

We also added the parameters in the annotation table (Table 1).

[RC1.10] Line 396: could we get more information on how the mass balance correction works? I assume it is not spatially constant correction in order reach the 1990 geometry but the text says 'constant correction'?

We use a constant mass balance correction which means that we add/subtract over the entire glacier/ice cap a constant mass balance value. To clarify this, we added *"over the entire ice mass"* in line 782.

[RC1.11] Line 415-418: No error bars are given on the area and volume estimates?

Thank you for this suggestion. We added error bars following the same approach as in Van Tricht et al. (2021a) to estimate the uncertainty of the reconstructions.

[RC1.12] Figure 5. Very nice sensitivity plot – as I understand from the figure these values are constant in space and time? It might be nice to make that clear in the text, it also means there is potential for further work for examining the spatial variability of the parameters i_s and w_f.

Thank you. We added a sentence in lines 980:

"In the applied model, the calibrated values of w_f and i_s are kept constant in space and time."

[RC1.13] Line 512: why is the enhancement factor chosen to be different (3 and 4 respectively) for 2 glaciers which are stated to be similar?

The enhancement factor is calibrated to match the modelled thickness optimally with the ice thickness reconstructions. The enhancement factor indirectly takes into account softening effects actors such as water content. This is why the Sary-Tor glacier and the Grigoriev ice cap, although located close to each other, can have a slightly different value for m.

[RC1.14] Would it be possible to include an evaluation of modelled velocities against observed velocities e.g. using the ITS_LIVE product? If it was showed the velocities were reasonably well captured this would also strengthen the confidence in the calculated temperatures. approximately line 540

We compared the modelled velocities with observations from mass balance stake displacements and we found a close correspondence. We added this in the text in lines 1090-1091:

"Comparison between modelled velocities and velocities derived from mass balance stake displacements at the Sary-Tor glacier shows a close correspondence (RMSE = 1.5 m yr^{-1})."

[RC1.15] Line 569: It is written that: 'the difference between the thermal regime of the two ice masses can be attributed to several factors' – this also includes geothermal heat focused in valleys like Sary-Tor and diminished on ridges like Grigoriev ice cap (see Colgan 2021, JGR), topographic corrections.

This is correct. We added in line 1107: *"the geothermal heat flux corrected for topographic relief"*

[RC1.16] It would be helpful if you include a description of how it possible to distinguish between temperate and cold ice from the radargram.

Thank you for the suggestion. We added a short description in the caption in lines 1163-1165:

"The figure clearly shows two sections of cold (upper part) and temperate (lower part) ice. The latter is characterised by the presence of small hyperbolic diffraction features due to the presence of water in the ice, typical for temperate (shown here as warm) ice."

[RC1.17] Financial support section: Kumtor mining company has been funding glaciology in this valley. Maybe include a clear statement that no industry funding was used or who did pay for the logistics of the field work?
We added a sentence in lines 1475-1476: "Local logistics were organised and funded by the Tien Shan High Mountain Research Center and the Kumtor mining company."

(Minor) Technical corrections and suggestions

[RC1.18] Line 38: using GPR abbreviation but it is not explained until line 64-65.
Thank you for this remark. We explained the abbreviation in line 51.

[RC1.19] Line 117: 'down to a depth' instead of 'up to a depth of 102 m'?
Done.

[RC1.20] Line 135: 'down to bedrock' instead of 'up to bedrock'
We followed your suggestion and replaced 'up to the bedrock' into 'down to the bedrock'.

[RC1.21] Line 136: 'depth' missing p
We added a 'p'

[RC1.22] Line 164 +166: include Eq. when referring to equation (2) and (4) to be consistent with the other times you refer to an equation.
Done.

[RC1.23] Line 289: 'assumed not to be' instead of 'to be not'
Done.

[RC1.24] Line 440: 'On' instead of 'At' in sentence 'At the other hand…'
Modified.

[RC1.25] Line 511: the number of decimal places vary
We added a second decimal for the RMSE of the Grigoriev ice cap.

[RC1.26] Line: 622: write ‘1960’ instead of ‘the sixties’

Done.

[RC1.27] Line 669: missing space between table and 1.

Modified.

[RC1.28] If possible it would be nice to see the location of the observed temperature profiles on a map.

We added the location of the borehole of 2007.

[RC1.29] Equation (7) should be A(T_pmp) instead of A(T)

Replaced.

[RC1.30] Figure 1: The light blue colour indicating the glaciers are not very clear, it is very similar to the blue showing lower elevations of the background map.

We changed the light blue colour to a white colour to create a greater contrast between the lower elevations and the glaciers/ice caps.

[RC1.31] Figure 5 right figure: suggest removing ‘at 10-15m depth’ from the x-axis label

Done.

[RC1.32] Figure 8:
* subfigure c has wrong label of the flowline, should be C and D instead of A and D to fit subfigure d.
* figure text missing f in flowlines in line starting with (b,d) Horizontal ice velocities along the flowlines
* plot elevation on y-axis of subplots (b,d) to be consistent with the other similar figures

- Done.
- Adjusted
- We added the elevation on the y-axis

[RC1.33] Table 3
* Initial values for Sary-Tor glacier have %value on top and temperature below to be consistent
* Suggestion: include the actual parameter values in the table not just the relative change

- We replaced the % value and the temperature to be consistent with the layout for the Grigoriev ice cap.
- We agree and now include the actual parameter values in the table (Table 4).