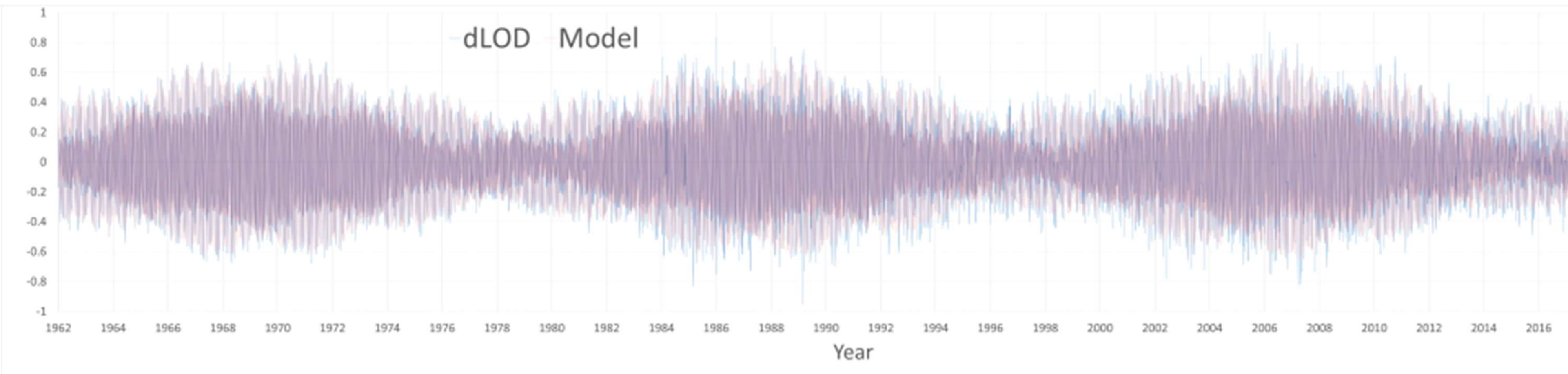
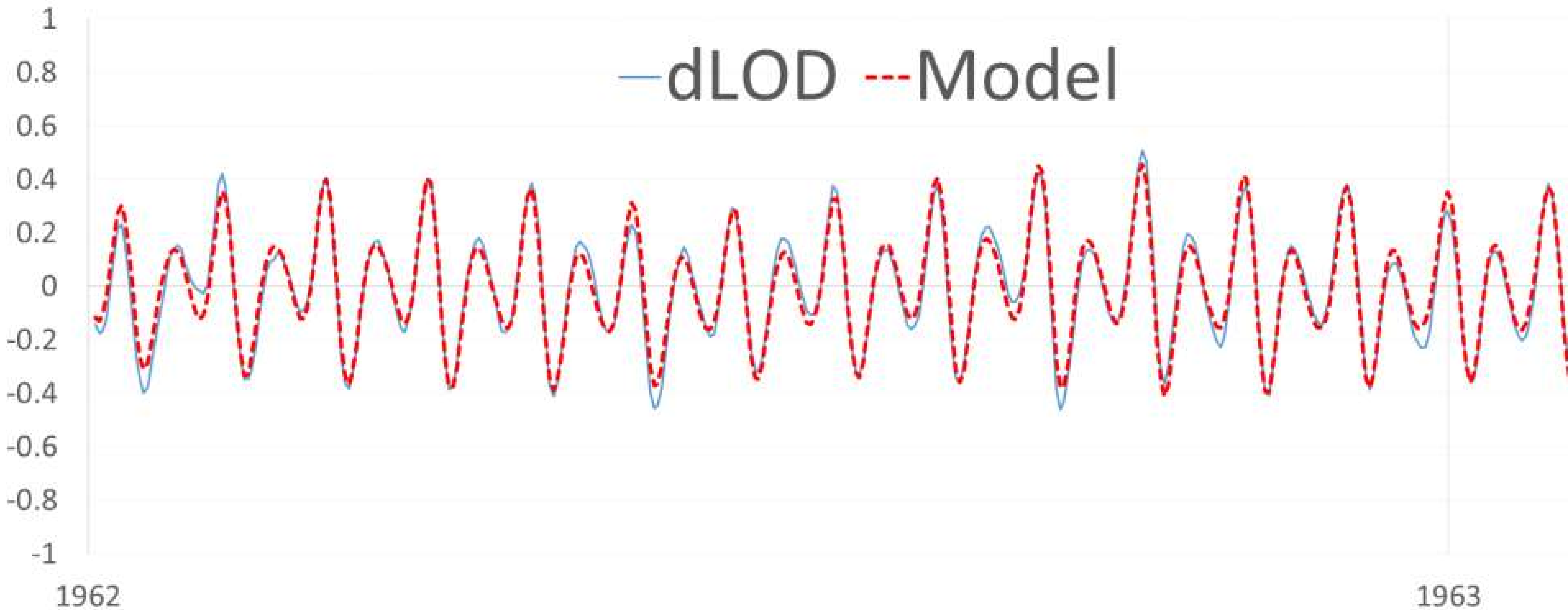
Paper under review "The modelled climatic response to the 18.6-year lunar nodal cycle and its role in decadal temperature trends"

The premise of the paper is that the ocean will show modulation of mixing with a cycle of ~18 years corresponding to the 18.6-year lunar declination cycle. That may indeed be the case, but it likely pales in comparison to the other so-called *long-period* tidal cycles. In particular, every ~2 weeks the moon makes a complete north-south-north declination cycle that likely has a huge impact on the climate as it sloshes the subsurface thermocline (cite the paper by Lin & Qian[1]). Unfortunately, this much shorter cycle is not directly observed in the observational data, making it a challenge to determine how the pattern manifests itself. In the following, I will describe how this is accomplished, referring to the complete derivation found in Chapter 12 of Mathematical Geoenergy[2].

Consider that the 2-week lunar declination cycle is observed very clearly in the Earth's rotational speed, measured in terms of small transient changes in the length of day (LOD). From the IERS site, we can plot the differential LOD (dLOD) and fit to the known tidal factors, leaving a clean closed-form signal that one can use as a forcing function to evaluate the ocean response, in this case comparing it to the well-defined ENSO climate index.

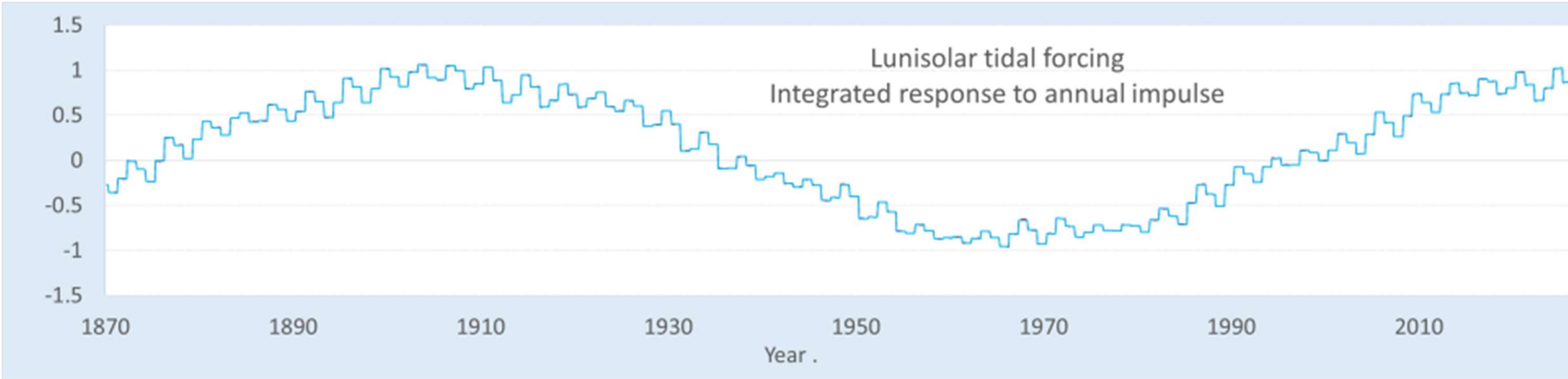


The 18.6-year nodal cycle can be seen in the modulation of the cyclic dLOD data. At a higher resolution, the comparison is as follows:



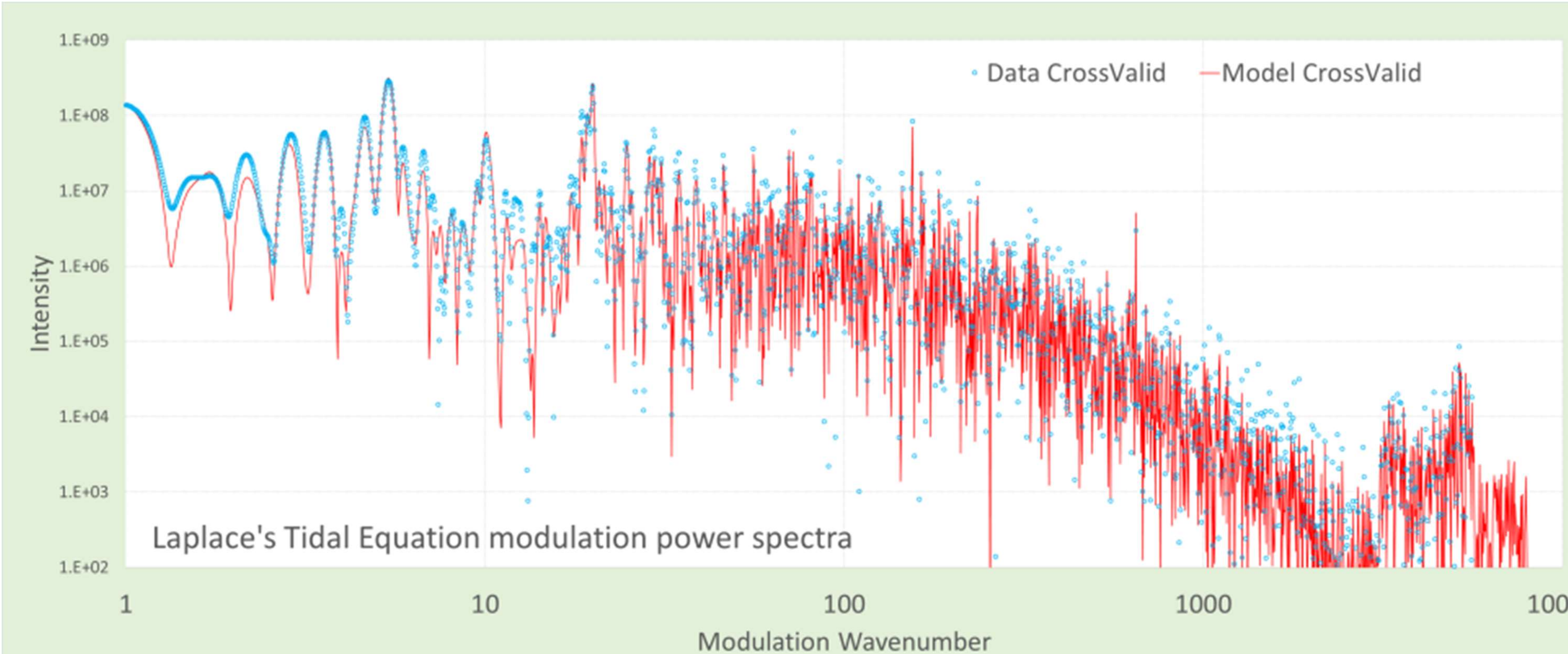
As the high-resolution dLOD measurement only goes back to 1962, but the ENSO NINO34 time-series goes back to 1870 [3], the closed form model fit can be used across the entire duration of a climate index (assuming that tidal factors are stationary).

To do that, we first make the assumption that the tidal cycle is modulated on an annual cycle, corresponding to the well-known "spring predictability barrier". So, by integrating a sequence of May impulses against the value of the tidal forcing at that point, the following time series is generated.



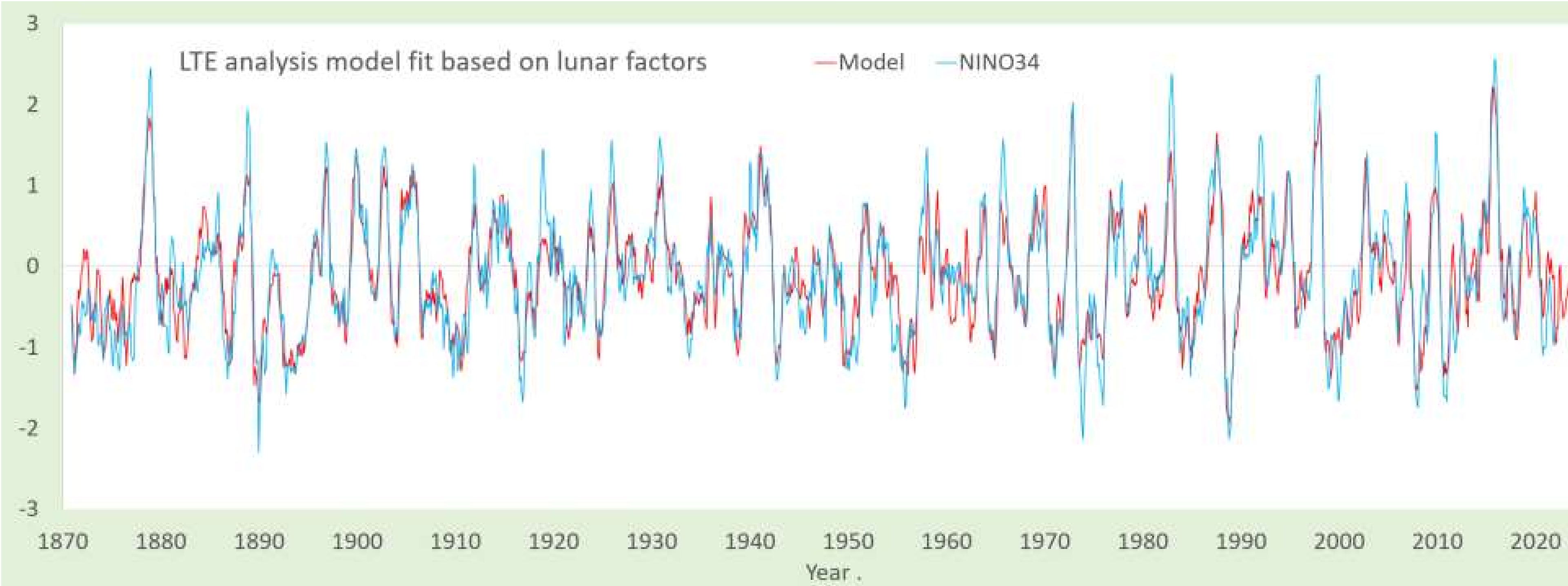
Obviously, this does not match the ENSO NINO34 signal, but assuming that the subsurface response is non-linear (derivation in cite #2 below) and creates standing wave-modes based on the geometry of the ocean basin, then one can use a suitable transformation to potentially extract the pattern. The best approach based on the solution to the shallow-water wave model (i.e. Laplace's Tidal Equations) is to map the input forcing (graph above) to the output corresponding to the NINO34 index, using a Fourier series expansion.

The result is the Laplace's Tidal Equation (LTE) modulation spectra, shown below in a particular cross-validation configuration. Here, the NINO34 data is split into 2 halves, one time-series taken from 1870-1945 and the second from 1945-2020. The spectra were calculated individually and then multiplied point-by-point to identify long-lived ***stationary*** standing-wave nodes in the modulation. Thus, it isolates modulations that are common to each interval.



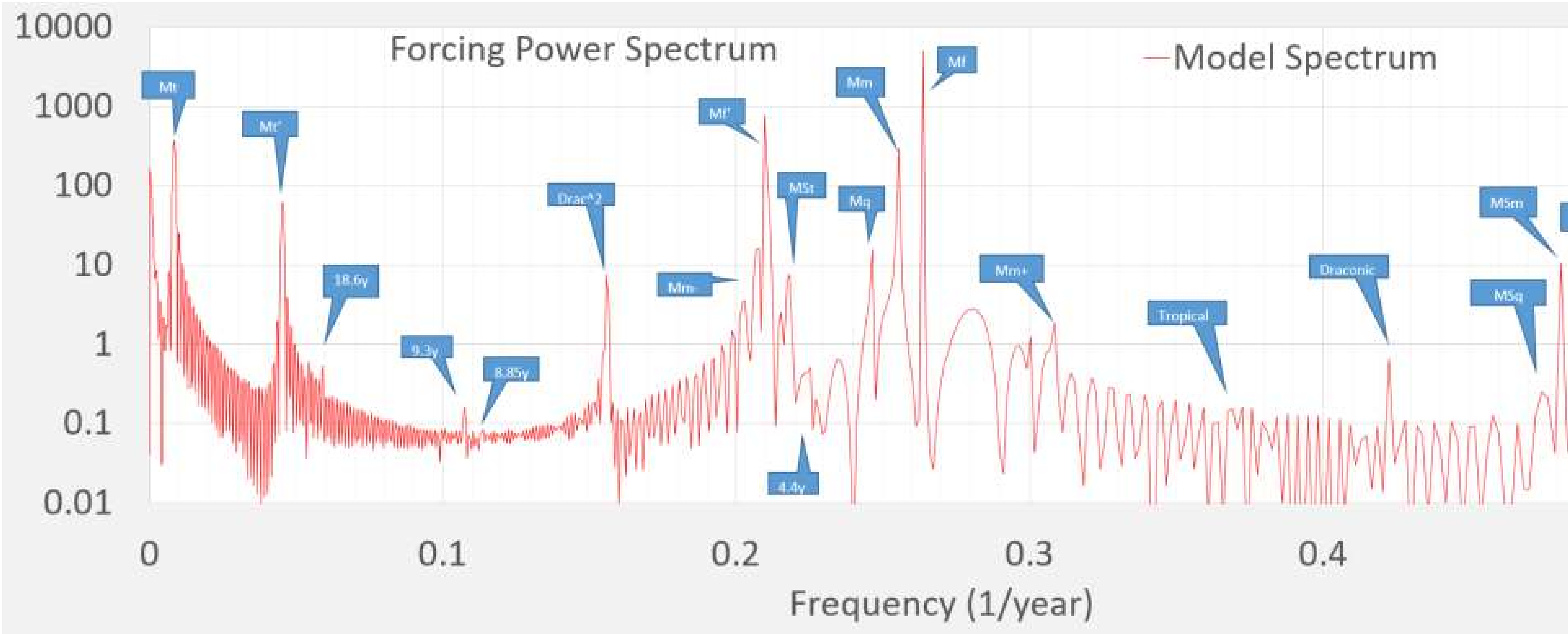
This is a log-plot, so the peak excursions shown are statistically significant and so can be modeled by a handful of quantifiable standing-wave modulations. The lowest wavenumber modulations are associated with the ENSO dipole modes and the higher wavenumber modulations are potentially associated with tropical instability waves (TIW)[2].

As a final step, by applying this set of modulations to the lunisolar forcing (the blue chart above), a fit to the NINO34 time-series results. The chart shown below is a very good fit and can be cross-validated via several approaches[10].



The mix of incommensurate tidal factors, the annual impulse, and a nonlinear response function is what causes the highly erratic nature of the ENSO waveform. It is neither chaotic nor random, as some researchers claim but instead is deterministically tied to the tidal and annual cycles, much like conventional tidal cycles have proven over the course of time.

To further quantify the decomposition of the tidal factors that force both the dLOD and the sloshing ENSO response, the paper by Ray and Erofeeva is vital[8]. When trying to understand the assignment of frequencies, note that after the annual impulse is applied, the known tidal factors corresponding to such tidal factors labelled **Mf**, **Mm**, *etc* get shifted from normal positions due to signal aliasing (see chart below in gray). This is a confusing factor to those who have not encountered aliasing before. As an example, the long-term modulation (>100 years) displayed in the blue chart above is due to the aliased 9.133 day **Mt** tidal factor, which almost synchronizes with the annual cycle, but the amount it is off leads to a gradual modulation in the forcing -- so overall confusing in that a 9 day cycle could cause multidecadal changes.



Ding & Chao[9] provide an independent analysis of LOD that provides a good cross-check to the non-aliased cross-factors. It may be possible to use lunar ephemeris data to calibrate the forcing but that adds degrees-of-freedom that could lead to over-fitting [10].

The reason that Lin & Qian were not able to further substantiate their claim of tidal forcing lies in that they could not associate the seasonal aliasing and a nonlinear mapping against their observations, only able to demonstrate the cause and effect of tidal forcing on the thermocline and thereby ruling out wind forcing. Other sources to cite are "Topological origin of equatorial waves" [4] and "Solar System Dynamics and Multiyear Droughts of the Western USA" [5], the latter discussing the impact of axial torques on the climate. Researchers at NASA JPL including J.H. Shirley, C. Perigaud[6], and S.L. Marcus[7] have touched on the LOD, lunar, ENSO connection over the years.

Bottom-line take aways:

1. Tidal factors are numerous so a measure such as dLOD is critical for calibrating the forcing.
2. Use the knowledge of a seasonal impulse, a la the spring predictability barrier, to advantage, while considering the temporal aliasing that it will cause.
3. The solution to the geophysical fluid dynamics produces a non-linear response, so clever transform techniques such as Fourier series are useful to isolate the pattern.

A recent citation to use: Pukite, Paul. "Nonlinear long-period tidal forcing with application to ENSO, QBO, and Chandler wobble." *EGU General Assembly Conference Abstracts*. 2021. https://ui.adsabs.harvard.edu/abs/2021EGUGA..2310515P

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