



1                    **Producing SWOT measurements with a multiscale data assimilation system**  
2                    **during the prelaunch field campaign**

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15

## 16 **Abstract**

17

18 A data assimilation system for a high-resolution model has been developed to address the  
19 opportunities and challenges posed by the upcoming Surface Water and Ocean Topography  
20 (SWOT) satellite mission. This developed system is based on a three-dimensional variational  
21 data assimilation scheme (3DVAR), which is computationally highly efficient and thus can be  
22 applied to a very high-resolution model. A crucial consideration of the system is to use a  
23 multiscale data assimilation approach (MSDA) to first assimilate routinely available  
24 observations, including conventional satellite altimetry, sea surface temperature (SST) and  
25 salinity (SSS), and temperature/salinity vertical profiles, to constrain large scales and large  
26 mesoscales. High-resolution (dense) observations and future SWOT measurements can then be  
27 effectively and seamlessly assimilated to constrain the smaller scales. The 3DVAR is extended to  
28 assimilate observations over a time interval, which specifically enhances the efficacy of the  
29 assimilation of satellite along-track altimetry observations, which are limited by large repeat time  
30 intervals. Using this system, a reanalysis dataset was produced for the SWOT pre-launch field  
31 campaign that took place in the California Current System from September through December,  
32 2019. An evaluation of this system with assimilated and withheld data demonstrates its ability to  
33 effectively utilize both routine and campaign observations to produce sea surface heights with  
34 the accuracy close to that required by SWOT. These results suggest a promising avenue for data  
35 assimilation development in the SWOT altimetry era, which will need the capability of jointly  
36 assimilating existing routine observations with SWOT measurements to resolve small-scale  
37 ocean processes.

38

## 39 **Key words**

40

41 SWOT satellite mission, field campaign, high-resolution modeling, multiscale data assimilation,  
42 extended three-dimensional data assimilation, reanalysis, California Current



## Introduction

The Surface Water and Ocean Topography (SWOT) satellite mission will launch in late 2022. SWOT will carry a new-generation altimeter – a Ka-band Radar Interferometer (KaRIn) – measuring sea surface height (SSH) in two-dimensions (2D) at unprecedented spatial resolution (Durand et al., 2010; Fu & Ubelmann, 2014). KaRIn has lower instrument noise ( $\sim 2$   $\text{cm}^2/\text{cycle}/\text{km}$ ) than conventional nadir-looking altimeters ( $\sim 100$   $\text{cm}^2/\text{cycle}/\text{km}$ ). The low noise allows for an effective spatial resolution down to a scale of 15 km. The effective spatial resolution is the minimum spatial wavelengths that can be resolved. The SWOT resolution is a significant improvement over the  $\sim 150$ -200 km 2D resolution of altimetry measurements from a constellation of conventional nadir-looking altimeters (Fu & Ubelmann, 2015; Dufau et al., 2016; Wang et al., 2019). The KaRIn instrument measures SSH over a nominal 120-km wide swath with a 20-km gap around the satellite's nadir track.

The SWOT satellite has a repeat orbit of about 21 days with global coverage. While the SWOT mission will provide unprecedentedly high spatial resolution measurements, it is not matched by high temporal resolution. The SWOT satellite will fly at 7 km/s, the same as conventional altimetric satellites. In 1-min, the measurements can cover a region of 420-km along-track, and thus can effectively provide a synoptic map of 2D SSH structures. The temporal resolution is limited by a 21-day repeat time. Fast evolving dynamical processes may develop, decay, or move unobserved between two passes. The current mapping methods (e.g., Le Traon, et al., 1998; Archer et al., 2020) for traditional altimetry observations will not be appropriate for SWOT measurements (Morrow et al., 2019).

When the SWOT satellite launches, a 90-day fast-sampling phase has been designated for Calibration and Validation (CalVal) to support understanding of the novel measurements. During this phase, the SWOT satellite will fly on a one-day repeat orbit to gather high-temporal resolution data at specific locations; two overpasses will be made every day at the crossover points. An 'Adopt-a-Crossover' consortium has been organized (Morrow et al., 2019), in which researchers around the globe will deploy in-situ instruments to augment the SWOT daily observations, to study small-scale ocean features and understand the nuances of SWOT's novel measurements. These observing systems can be coupled to data assimilation systems that



73 combine the measurements with high-resolution models to produce best-estimates of the ocean-  
 74 state. However, there are challenges in applying existing DA systems to the novel SWOT data.

75 We have developed a multi-scale data assimilation (MSDA-SWOT) system with a high  
 76 resolution-model to specifically address some of the challenges posed by the SWOT satellite  
 77 mission, which include:

- 78 i. A 21-day repeat cycle that limits our ability to directly observe the time-evolution of  
 79 the fast-moving small-scale ocean variability, requiring a dynamical model to fill-in  
 80 the time gaps
- 81 ii. SWOT measurements should be effectively combined with measurements from  
 82 conventional nadir-looking altimeters measurements, which have a fundamentally  
 83 lower spatial resolution but relatively higher temporal resolution.
- 84 iii. Measurements of small-scale ocean features embedded in larger-scale structures,  
 85 which requires an ability to effectively constrain different scales of variability

86 For MSDA-SWOT to successfully address the specific challenges of SWOT, it is  
 87 fundamentally important that the system has the capability of assimilating all routinely available  
 88 observations from operational observing networks. During the last two decades, global routine  
 89 observing networks have been established, enhanced and sustained for operational purposes.  
 90 There are more than five satellite altimeters presently in operation, from which SSH  
 91 measurements are routinely available in near real time and may be merged to resolve eddies in  
 92 two-dimensions down to almost 100 km in size at the mid-latitudes (Archer et al., 2020). Global  
 93 temperature/salinity vertical profiles are provided by the operational Argo float network and  
 94 augmented by mooring arrays, glider lines and a variety of other observing platform networks.  
 95 Although temperature/salinity vertical profiles are spatially sparse and temporally infrequent,  
 96 they have allowed the production of monthly-average data sets, which have an effective  
 97 resolution of a few hundred kilometers (e.g, Good et al., 2013). Furthermore, a fleet of satellites  
 98 carry infrared and microwave sea surface temperature (SST) sensors that provide maps on a daily  
 99 basis. The MSDA-SWOT algorithm has been formulated to effectively constrain mesoscale  
 100 variability using observations from these routine observing networks. We argue that the  
 101 effectively constrained mesoscale variability down to about 100 km is a prerequisite for the  
 102 dense observations from the field campaigns and future high resolution SWOT measurements to



103 be effectively and seamlessly assimilated, and the MSDA methodology is formulated to achieve  
104 such a goal (Li et al., 2015a; Li et al., 2019).

105 MSDA-SWOT is a three-dimensional variational data assimilation system (MS-3DVAR) that  
106 has been documented in Li et al. (2015b, 2019). The model used is the Regional Ocean Modeling  
107 System (ROMS, Shchepetkin & McWilliams 2005, 2011). The MS-3DVAR has been extended  
108 to address the particular challenges of the SWOT mission. In MSDA-SWOT, a 3DVAR  
109 formulation is extended to assimilate observations over a time interval and thus has the objective  
110 similar to the First Guess at Appropriate Time (FGAT) (e.g. Martin et al., 2015). In extended  
111 3DVAR, a formulation has been given to account for the error due to the difference between data  
112 assimilation and observing time, which is termed as sampling time error (see section 3.1). By  
113 taking into account the sampling time error, it allows the selection of a relatively large time  
114 interval so that many temperature/salinity vertical profiles and along-track altimetry  
115 measurements can be assimilated jointly to constrain large-scale and mesoscale circulation.

116 The MSDA-SWOT system is based on the MSDA system described in Li et al. (2019b) with  
117 the implementation of the extended 3DVAR. There are other major differences in the treatment  
118 of observations, and accordingly, in background error covariances, discussed later. It has been  
119 implemented at the primary SWOT CalVal site in a region encompassing the California Current  
120 System (Fig. 1). This eastern boundary current site has been selected for the moderate tides, and  
121 moderate mesoscale and sub-mesoscale dynamics (e.g., Hickey, 1998; Capet et al., 2008).

122 In Li et al. (2019), we presented a set of twin experiments, also known as OSSE (observing  
123 system simulation experiment). The results showed that the MSDA system could produce  
124 appropriately accurate SSH for CalVal with a dedicated glider array on top of the routine  
125 observations available from operational observing networks. A pre-launch field campaign that  
126 was dedicated to SWOT CalVal took place from September through December, 2019 (for more  
127 details, see Wang et al., 2021). MSDA-SWOT has been used to produce analyses and forecasts  
128 by assimilating measurements from the field campaign and routine observing networks. We here  
129 evaluate and illustrate the performance of MSDA-SWOT to produce appropriately accurate SSH  
130 as the Li et al. (2019) twin-experiment OSSE demonstrated. We focus on the fine-scale  
131 performance of MSDA-SWOT in comparison to withheld glider and mooring data in a  
132 companion paper (Archer et al. 2021).



133 The paper is organized as follows. Section 2 summarizes observations that are assimilated  
 134 and used for evaluating the system. The MSDA-SWOT algorithm is presented in section 3.  
 135 Implementation strategies and practical considerations are described in section 4. Section 5  
 136 presents an evaluation of the performance of MSDA-SWOT with an emphasis on the illustration  
 137 of the algorithm and implementation strategies. Finally, a brief summary of the key results is  
 138 given in Section 6.

139

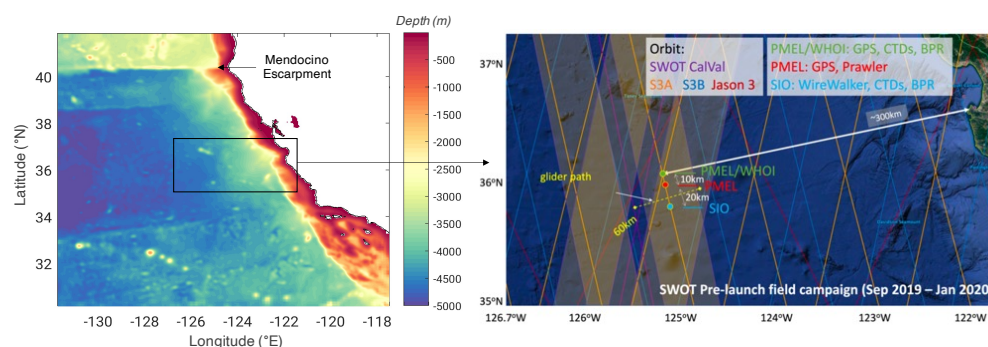
## 140 2 Data

141

142 In the context of this paper, distinct spatial scales are considered. For clarity, they are defined as  
 143 follows: the large-scale is greater than 400 km, the large mesoscale is from 400 km down to 150  
 144 km, the small mesoscale is from 150 km down to 50 km, and the submesoscale is smaller than 50  
 145 km. These scale definitions are not used for dynamical analysis, but they are associated with the  
 146 ability of currently existing observing networks to resolve spatial scales. SWOT measurements  
 147 aim to accurately resolve scales from 150 km down to 15 km in two-dimensions.

148 In the following, we briefly describe the observations that are collected during the pre-launch  
 149 field campaign as well as those from routine observing networks. We also provide information  
 150 on a global reanalysis dataset we use to benchmark our system.

151



**Figure 1.** Model domain and bathymetry (left). The model domain encompasses the Mendocino escarpment, which is the primary source of semi-diurnal internal tides at the CalVal site. In the zoom-in area (right), the observing assets are indicated during the SWOT pre-launch campaign. The ground tracks of Jason-3 (red), Sentinel-3B (S3B, blue) and Sentinel-3A (S3A, orange) are shown. Three moorings – south (SIO), middle (PMEL) and north (PMEL/WHOI) – were deployed within a SWOT swath cross over (light brown diamond) along a S3A track (three dots). A glider flew along a track perpendicular to the mooring line, crossing between the south and middle moorings.



152

## 153 **2.1 Pre-launch Field Campaign**

154 The pre-launch field campaign observations provide information for the design of the post-  
155 launch in-situ observing system. Three different moorings were deployed. The first (south) is a  
156 hybrid mooring with a profiling Conductivity, Temperature, and Depth (CTD) instrument called  
157 a WireWalker installed on top of fixed CTD sensors. The WireWalker measures vertical profiles  
158 of temperature/salinity down to a depth of about 500 m with high temporal resolution. The  
159 second (north) mooring comprises a GPS altimeter installed on top of a line of CTDs. And the  
160 third (middle) is a GPS/Prawler mooring, on which was installed a GPS altimeter above a  
161 Prawler. Like the WireWalker, a Prawler is a profiling CTD that measures T/S vertical profiles at  
162 very high resolution and frequency down to a depth of about 500 m. Another observing platform  
163 deployed was a Slocum glider that measures T/S vertical profiles. Three moorings were deployed  
164 along a Sentinel-3A altimetry track, and the glider flew along a track perpendicular to the  
165 mooring line. (Fig. 1).

166

## 167 **2.2 Temperature/salinity vertical profiles**

168 Temperature/salinity vertical profiles are measured by the global Argo float network, local glider  
169 networks, moorings, and during various surveys. The profiles are spatially and temporally sparse  
170 and heterogeneous, but they have allowed the production of monthly gridded products at a low  
171 spatial resolution (e.g. Good et al., 2013). We assimilate individual T/S profiles processed and  
172 quality controlled by the EN4 program at the Met Office Hadley Centre (Good et al., 2013).  
173 Figure 2 shows the locations of all the vertical profiles from September 1 through Nov 30, 2019.  
174 An appropriate assimilation of those profiles should help constrain the large-scale circulation in  
175 the model.

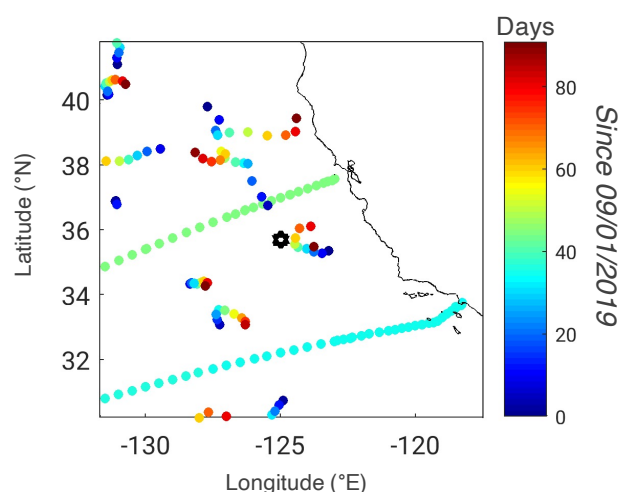
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## 177 **2.3 Along-track and gridded altimetry data**

178 In this study, both along-track and gridded altimetry data are assimilated. The altimetry data of  
179 absolute dynamic topography is assimilated although it is called SSH for convenience. We use  
180 the level 3 (L3) along-track data and level 4 gridded data that are publicly available through the  
181 Copernicus website (<http://marine.copernicus.eu/services-portfolio/access-to-products/>). We use  
182 5 altimeters that were in orbit: Jason-3 (J3), Sentinel-3A (S3A) and Sentinel-3B (S3B), SARAL-



183 DP/AltiKa (ALD), and Cryosat-2 (C2). Table 1 presents the main characteristics of each satellite  
 184 altimeter, and details of the L3 dataset (Pujol et al. 2016) for more information on altimeter  
 185 standards and the DUACS processing chain). DUACS-DT2018 provides the 1 Hz along-track  
 186 data in two resolutions; unfiltered (~7 km spacing between along-track grid points), and filtered  
 187 (~14 km spacing, low passed with a 65 km cut-off). The 65 km threshold was chosen based on  
 188 signal-to-noise ratio of one in the wavenumber spectra (Dufau et al.,2016).  
 189



**Figure 2.** Locations of vertical temperature/salinity profiles from Argo floats, gliders and other routine observing platforms that are available from September 1 through Nov 30, 2019. The colors show the observing time (days) from September 1, 2019.

190  
 191

**Table 1** DUACS L3 along-track unfiltered (filtered) data

Acronym	<i>J3</i>	<i>S3A</i>	<i>S3B</i>	<i>ALD</i>	<i>C2</i>
Along-track grid spacing (km)	7 (14)	7 (14)	7 (14)	7 (14)	7 (14)
White noise level (cm)	2.9 (1.1)	2.4 (0.9)	2.4 (0.9)	2.1 (0.8)	2.5 (1.0)
Repeat interval (day)	10	27	27	35	29

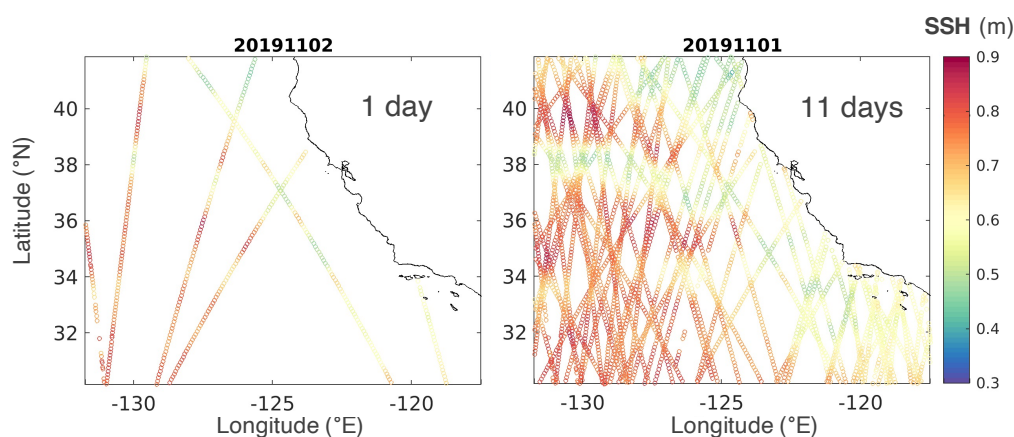
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 193

194 Even with five altimeters, the long repeat intervals limit coverage for one day to only a few  
 195 tracks over the model domain (Fig. 3a). It is challenging to assimilate such highly heterogeneous





196 observations into a model with a spatial resolution on the order of 1 km. Our experience has  
 197 shown that the assimilation of such few tracks of altimetry observations often causes problems –  
 198 distorting and/or mis-locating eddies, and even generating spurious eddies. As such, we focused  
 199 in previous studies on assimilating only the gridded data products that are highly smoothed. The  
 200 high-resolution information from along-track altimetry was thus not used in our previous DA  
 201 systems (e.g., Li et al., 2019a, b). To address this limitation, we have formulated the MSDA-  
 202 SWOT data assimilation scheme to assimilate multiple days of along-track altimetry  
 203 observations, by extending the framework of 3DVAR. As we will show, MSDA-SWOT can  
 204 effectively and reliably assimilate along-track altimetry observations into a high-resolution  
 205 regional model (Section 3).



**Figure 3.** Typical distribution of observations from along-track altimetry for five satellites: Jason-3, Sentinel-3A and Sentinel-3B, SARAL-DP/AltiKa, and Cryosat-2. SSH measurements for 1 day (left) on November 2, 2019, and 11 days (right) from November 1 through 11, 2019. Color indicates SSH in meters.

206

207

208 Using the measurements from multiple conventional satellite altimeters, gridded SSH  
 209 products can now resolve SSH length-scales larger than approximately 150-km wavelength in  
 210 the region near the CalVal site (e.g., Chelton et al., 2007; Pujol et al., 2016; Dufau et al., 2016;  
 211 Archer et al., 2020). By merging SSH measurements from all available altimetry satellites, daily  
 212 SSH maps can be generated even though they involve smoothing with a time window much  
 213 longer than one day. Daily gridded maps by AVISO (Archiving, Validation and Interpretation of

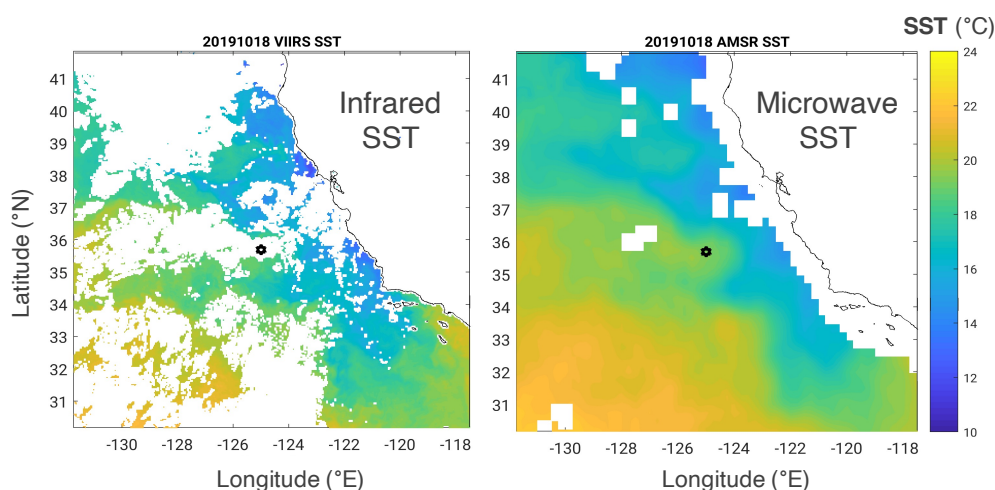


214 Satellite Oceanographic) have been extensively used. The root-mean-square difference between  
 215 the AVISO gridded and filtered along-track data is around 2.5 cm in this region (Archer et al.,  
 216 2020).

217

#### 218 **2.4 Satellite sea surface temperature**

219 Satellite remote sensing has provided accurate global sea surface temperatures (SST) for  
 220 decades. SST measured by satellite microwave (MW) sensors has been gridded at a spatial  
 221 resolution of 25 km with an error of 0.6°C. SST measured by infrared (IR) sensors that are  
 222 carried on polar orbit satellites has a spatial resolution of about 1 km with an error of 0.7°C, and  
 223 they are well into the submesoscale. There are several satellites measuring MW and IR SST,  
 224 jointly producing SST observations a few times daily. In Li et al. (2019b), we assimilate  
 225 nighttime and morning (Local 12 am-9 am) satellite SSTs, and daytime SSTs with wind speeds  
 226 higher than 3 m/s to minimize the impact of skin temperature differences that arise in low wind  
 227 daytime conditions. In this study, we will not assimilate IR SST but use them as independent  
 228 data for evaluation.



**Figure 4.** Infrared VIIRS (left) and microwave AMSR-2 (right) SST on October 5, 2019. Infrared VIIRS SST has a resolution of 0.7 km. No infrared SST is available over cloudy areas. This VIIRS SST map shows a day with a relatively good coverage. Microwave AMSR SST has a resolution of 25 km. No microwave SST is available near shore because of land contamination. VIIRS SST is not assimilated but used as independent data for evaluation.



## 229 **2.5 HYCOM**

230 We benchmark MSDA-SWOT against the Global HYbrid Coordinate Ocean Model (HYCOM),  
 231 which is coupled with the Navy Coupled Ocean Data Assimilation (NCODA) to produce a  
 232 global reanalysis product (Cummings, 2005; Cummings and Smedstad, 2013). This is one of the  
 233 leading global reanalysis products available. NCODA uses the 24-hour model forecast as a  
 234 background field in a 3DVAR scheme. It assimilates satellite SSH, in situ and satellite SST,  
 235 vertical profiles of temperature and salinity from XBTs, Argo floats and moored buoys. It can be  
 236 considered an approximate equivalent to the routine DA run of MSDA. More information on the  
 237 system is available at <https://www.hycom.org/>. We here use a high-resolution run provided by  
 238 the Navy, at 0.04 ° longitude by 0.02° latitude, and hourly time sampling, for the pre-launch field  
 239 campaign period. To remove the tides from the SSH field, we take a daily average and detrend  
 240 the SSH in two-dimensions around the CalVal site.

241

## 242 **3 An extended three-dimensional multiscale scheme**

243

244 The MSDA-SWOT system has been developed to address new challenges relating to the fine-  
 245 scale resolution and data density of satellite observations in the SWOT altimetry era. As such,  
 246 the numerical ocean model used should be high resolution with a grid spacing on the order of 1  
 247 km or finer. This imposes a major computational challenge to formulate MSDA-SWOT. Both  
 248 the field campaign and SWOT measurements are localized to a limited area. Another major  
 249 challenge is to assimilate these localized measurements seamlessly along with the broad routine  
 250 observations. Otherwise, a spurious circulation surrounding the observing area may develop, and  
 251 data assimilation could even fail. To address these two major challenges, we have formulated a  
 252 particular MSDA scheme, which is based on a 3DVAR algorithm that is extended to assimilate  
 253 observations over a time window.

254

### 255 **3.1 An extended 3DVAR scheme**

256

257 For a very high-resolution model, 3DVAR is a scheme extensively used in both  
 258 meteorological and oceanic applications (e.g., Gustafsson, et al., 2018), because of its  
 259 computational efficiency. However, a 3DVAR scheme can formally assimilate observations  
 260 taken only at an instantaneous time. In practice, observations over a time window are assimilated



with the assumption that they are taken at the DA time. As such, there is a difference between the observation time and DA time. This time difference inputs error to the system, which we term as ‘sampling time error’. In practice, a short time window is selectively chosen as a compromise between incorporating more observational information, and keeping the sampling time error at an acceptable level.

To alleviate this short time window limitation in 3DVAR, we here formulate a scheme to extend the ability of 3DVAR to assimilate observations in a longer time window. Suppose that at time  $t_k$  ( $k = 1, 2, \dots, K$ ), a number  $m$  of observations (note that  $m$  could vary in time) are available and placed into an  $m$ -vector  $\mathbf{y}_k^o$ .  $\mathbf{R}_k^o$  is observational error covariance, and  $\mathbf{R}_k^o = \langle \mathbf{e}_k^o (\mathbf{e}_k^o)^T \rangle$ , where  $\mathbf{e}_k^o$  is the observational error associated with  $\mathbf{y}_k^o$ . The state variable at  $t_0$  can be denoted as an  $n$ -vector,  $\mathbf{x}_0$ .

We extend the standard 3DVAR cost function as

$$J(\mathbf{x}_0) = \frac{1}{2} (\mathbf{x}_0 - \mathbf{x}_0^b)^T \mathbf{B}_0^{-1} (\mathbf{x}_0 - \mathbf{x}_0^b) + \frac{1}{2} \sum_{k=0}^K [H_k(\mathbf{x}_0) - \mathbf{y}_k^o]^T (\mathbf{R}_k^o)^{-1} [H_k(\mathbf{x}_0) - \mathbf{y}_k^o] . \quad (1)$$

Here  $\mathbf{B}_0$  is the background error covariance, and  $\mathbf{B}_0 = \langle \mathbf{\epsilon}_0^b (\mathbf{\epsilon}_0^b)^T \rangle$ , where  $\mathbf{\epsilon}_0^b$  is the background error associated with the background state  $\mathbf{x}_0^b$ .  $H_k$  is often known as an observation operator that maps the state variable to  $\mathbf{y}_k^o$ .

The cost function (1) has a form similar to the four-dimensional variational data assimilation (4DVAR) algorithm (e.g., Li & Navon, 2001), which has been implemented for oceanic applications (e.g., Weaver et al., 2003; Moore, et al., 2004; Zhang et al., 2010; Ndgodock & Carrier, 2014). In 4DVAR,  $\mathbf{x}_0$  in the observation related terms in (1) is replaced by  $\mathbf{x}_k$ , so that it can naturally assimilate observations over a time window. The cost function (1) can be considered as a reduced 4DVAR cost function, in which a forecast approximation of  $\mathbf{x}_k = \mathbf{x}_0$  is applied, or in which a persistence forecast model is used in 4DVAR.

Rather than the standard 3DVAR cost function, the cost function (1) is used to account explicitly for the observation error due to the difference between the observation and DA time. By definition, the observational error in (1) has the form (e.g., Li et al. 2015),

$$\mathbf{e}_k^o = \mathbf{y}_k^o - H_k(\mathbf{x}_0^t), \quad (2)$$

where  $\mathbf{x}_k^t$  is the unknown true state. We can write (2) in the expansion form

$$\begin{aligned} \mathbf{e}_k^o &= (\mathbf{y}_k^o - \mathbf{y}_k^t) + (\mathbf{y}_k^t - H_k(\mathbf{x}_k^t)) + (H_k(\mathbf{x}_k^t) - H_k(\mathbf{x}_0^t)) \\ &= \mathbf{e}_k^m + \mathbf{e}_k^r + \mathbf{e}_k^{mp}, \end{aligned} \quad (3)$$



291 where  $y_k^t$  is the unknown true value of  $y_k^o$ .

292 In (3), the first term is the measurement error, and the second term the representation error  
 293 due to the inaccurate observation operator. These two terms are well known. The last term is a  
 294 new type of observational error. We can understand it in the following way. If all the  
 295 observations are taken at  $t_0$ , this type of error does not occur. We follow the formulation (3) to  
 296 estimate the error arising from the difference between the observation time and DA time.

297 The MSDA-SWOT system is based on (1) and (3). Because  $t$  is allowed to be negative in (1),  
 298 we assimilate observations prior to and after the DA time. We note that this extended 3DVAR  
 299 has the same objective as First Guess at Appropriate Time (FGAT), another scheme used in  
 300 conjunction with 3DVAR (e.g. Martin et al., 2015), which is discussed in Archer et al. (2021).

301

### 302 **3.2 Multiscale Framework**

303 We have stated that data assimilation is a method to use imperfect observations to correct the  
 304 error in the forecast. The essence of a multi-scale data assimilation framework is to correct  
 305 forecast errors sequentially from large to small spatial scales in multiple steps, where the number  
 306 of steps depends on the characteristics of the observing networks. An MSDA formulation hinges  
 307 on the fact that the scales of forecast error that are corrected can be defined and determined by  
 308 the background error covariance  $B_0$  (Li et al., 2015, 2016).

309 The background error covariance  $B_0$  can be decomposed as  $B_0 = \Sigma C \Sigma$ , where  $\Sigma$  is a  
 310 diagonal matrix whose elements are the background root-mean-square error (RMSE) associated  
 311 with  $x_0^b$ , and  $C$  is the correlation matrix whose elements consist of the spatial correlations.

312 This error covariance decomposition allows us to separately examine RMSE and correlation.  
 313 The background error amplitude is given by  $\Sigma$ , which plays a role of weight as we can see in the  
 314 cost function.

315 It is well known that the correlation plays a key role in spreading the observation innovations  
 316 to the surrounding areas. However, a more important role in MSDA is its filtering effect. The  
 317 larger the correlation scale is, the stronger the filtering effect that data assimilation imposes on  
 318 observation innovations (Li et al., 2016; Jacobs et al., 2020). A rule of thumb is that all the scales  
 319 smaller than twice the correlation length scale are filtered out. This filtering effect dictates that  
 320 the data assimilation can correct only those scales larger than twice the correlation length scales.  
 321 By defining a proper background state and associated correlation length scales, MSDA allows



for specifying scales that are corrected. In MSDA-SWOT, we implement a three-step MSDA for  
 constraining different spatial scales (section 4.2).

324  
 325

## 326 **4 Implementation**

327

328 The implementation of the scheme formulated above leverages the MSDA system described in  
 329 Li et al. (2019b). The difference here is in the use of the extended 3DVAR scheme, as well as in  
 330 the treatment of observations, and accordingly, in the adjustment of the multiscale background  
 331 error covariances, which will be described in this section.

332

### 333 **4.1 Modeling System**

334 The MSDA-SWOT configuration of the modeling system is similar to the one described in Li et  
 335 al. (2019b). Here we give a description. The modeling system is based on ROMS (Shchepetkin  
 336 & McWilliams, 2005, 2011). A one-way nesting procedure is used as described in Mason et al.  
 337 (2010) with successive, nearly isotropic grid resolutions, varying from 9 km covering a large  
 338 region of the Northeast Pacific, 3- km for an extended region of the California coast. The  
 339 bathymetry for three domains is constructed from the ETOPO1 1 arc-minute dataset of Amante  
 340 & Eakins (2009). The model domain and its bathymetry are shown in Fig.1.

341 The lateral boundary conditions for the 9-km domain are derived from version GOF3.1 of  
 342 the global analysis of HYCOM (<https://www.hycom.org/dataserver/gofs-3pt1/analysis>). The  
 343 temperature, salinity, velocity components and SSH fields are all used. To smooth out possible  
 344 unrealistic variability, a 15-day average is applied. The three-hourly atmospheric forcing uses a  
 345 bulk flux formula (Fairall et al., 2003). The required atmospheric fields (10m-wind speed and  
 346 direction, net shortwave radiation, downward longwave radiation, 2m-air temperature and  
 347 relative humidity) are obtained by interpolation from the 25-km resolution of the NCEP GFS  
 348 (National Centers for Environmental Prediction Global Forecast System) operational  
 349 atmospheric model 3-hourly outputs. In the calculation of wind stresses, ocean surface currents  
 350 are subtracted.

351 The model is forced by barotropic tides at the open boundaries of the 3-km resolution  
 352 domain, and the Flather boundary condition (Flather, 1976; Wang et al., 2009) is used. The tidal  
 353 sea level and barotropic velocity amplitudes and phases for the 10 dominant tidal constituents  
 354 (M2, S2, N2, K2, O1, K1, P1, Q1, Mf, and Mm) were extracted from the 1/12th degree



355 resolution tidal model solution for the Pacific basin, which was constrained by the assimilation  
 356 of satellite altimetry in the Oregon State University Inverse Tidal Software (OTIS, Egbert and  
 357 Erofeeva, 2002). An evaluation against mooring observations shows that the model realistically  
 358 produces baroclinic tides in the region near the CalVal site (Li et al., 2019b).

359

## 360 **4.2 Three-Step MSDA**

361 Following the definition given in section 2, a three-step MSDA is configured respectively to  
 362 constrain three scales: 1) large and large mesoscale DA for scales larger than 150 km, 2) small  
 363 mesoscale DA for scales larger than 50 km, and 3) submesoscale DA for scales larger than 15  
 364 km.

365 As discussed in section 3.2, the scales that are constrained are dictated by background error  
 366 correlation length scales. We define the background correlation using a Gaussian function,  
 367  $e^{-r^2/2L_D^2}$ , where  $r$  is a spatial distance between grid points and  $L_D$  is a decorrelation length scale,  
 368 known as the Daley correlation scale (Daley, 1991). Following the rule of thumb that only the  
 369 scales larger than two times of the correlation length scale are constrained, we use a  
 370 decorrelation length scale  $L_D$  of 75 km, 25 km and 9 km.

371 We note that these scales are defined empirically and will benefit from further tuning. Those  
 372 three scales are specified mainly by considering the observations assimilated, which are listed in  
 373 Table 2. An important consideration is that the observation error should be specified consistently  
 374 with the decorrelation length scale and the assimilation window for the use of observations.

375 For the large mesoscale DA, a decorrelation scale of 75 km is given mainly because the  
 376 AVISO gridded data and daily mean microwave SST can have an effective constraint on about  
 377 150 km (Archer et al., 2020), roughly two times the decorrelation length scale. Their observation  
 378 errors given in section 2 are used. Since the boundary conditions are derived from the 15-day  
 379 average of the HYCOM analysis, we also assimilate the same 15-day average T/S profiles twice  
 380 a month to ensure that the state inside the model domain is not drifted away from the lateral  
 381 boundary conditions.

382 For the small mesoscale DA, a decorrelation scale of 25 km is used for maximizing the  
 383 impact of along-track altimetry observations, because the along track observations are filtered  
 384 with a cut-off of 65 km (Pujot et al., 2016). Leveraging the extended 3DVAR formulation, we  
 385 assimilate along-track altimetry and routine T/S profile observations over a time window of 11





386 days. We choose 11 days because the Jason-3 has the smallest repeat interval among five  
 387 altimetry satellites, and it is 10 days. It is not uncommon that a few days of observations are  
 388 assimilated in 3DVAR. For example, an assimilation windows of 5 days for SSH and 12 days for  
 389 T/S profiles is used in Jacobs et al. (2014). The cost function (1) allows the use of a larger  
 390 assimilation window, but the sampling time error given in (3) should be added to the observation  
 391 error.

392 In implementation, a 11-day time window comprises 5 days prior to and 5 days  
 393 succeeding the DA Day. To add the sampling time error, we simply assume that the sampling  
 394 time error has the form as

$$395 \quad \mathbf{e}_k^{mp} = \gamma_k (\mathbf{e}_k^m + \mathbf{e}_k^r), \quad (4)$$

396 where  $\gamma_k \geq 0$  and  $k = 1, 2, \dots, 5$  in day. With (4), the observational error given in (3) becomes  
 397  $\mathbf{e}_k^0 = (1 + \gamma_k)(\mathbf{e}_k^m + \mathbf{e}_k^r)$ . Thus, the observational error is inflated. The value of  $\gamma_k$  is estimated  
 398 using observations or model simulation outputs. We note that the assumption (4) is not  
 399 necessary, since  $\mathbf{e}_k^{mp}$  can be directly estimated. We use (4), so that  $\gamma_k$  can be used as an  
 400 adjustable parameter to account for the possible inaccurate representation error. The formulation  
 401 (4) is applied to altimetry and T/S vertical profile observations. For altimetry observations,  $\gamma_k =$   
 402  $0.09k$ . For five days, for example, the estimated sampling time error is 1.45 times of the sum of  
 403 measurement and representation error. For temperature and salinity profile observations,  $\gamma_k =$   
 404  $0.1k$  and  $\gamma_k = 0.08k$ .

405 For the submesoscale DA, a decorrelation length scale of 9 km is tentatively given. This is  
 406 because of the SWOT CalVal baseline requirement resolution of 15 km. This scale will smooth  
 407 out spatial structures smaller than 18 km. We emphasize that DA smooths out the observation  
 408 innovation, but not the background state. The small scales generated by the model during the  
 409 forecasting stage will remain, although no correction will be made to them by DA. We do not  
 410 use a smaller decorrelation length scale because the model grid space is 3 km. The decorrelation  
 411 length scale has been as small as the three-grid point size. When a smaller model grid spacing is  
 412 used, this decorrelation length scale can be reduced accordingly.

413 This three-step implementation is computationally efficient. In the configuration described  
 414 above, the wall-clock time for the execution of MSDA is close to that required to carry out the  
 415 model forecast using the same CPUs. This implies that MSDA can be implemented on a very  
 416 high-resolution model if the configuration is computationally feasible for carrying out





417 simulations without data assimilation; that is – the MSDA implementation does not require any  
 418 compromise in the configuration of the forecast model.

419

420

421 **Table 2** Observations assimilated

Large mesoscale (> 150 km)	Small mesoscale (150 – 50 km)	Small scale (< 50 km)
11-day routine T/S profile	11-day along-track SSH	Campaign T/S profile
AVISO gridded SSH	Daily mean campaign T/S profile	Satellite IR SST*
Daily mean MW SST	Daily mean HF radar velocities*	Other observations*
Daily mean MW SSS*		

\* These observations may be assimilated during the SWOT post-launch field campaign but not in this study.

422

423

424

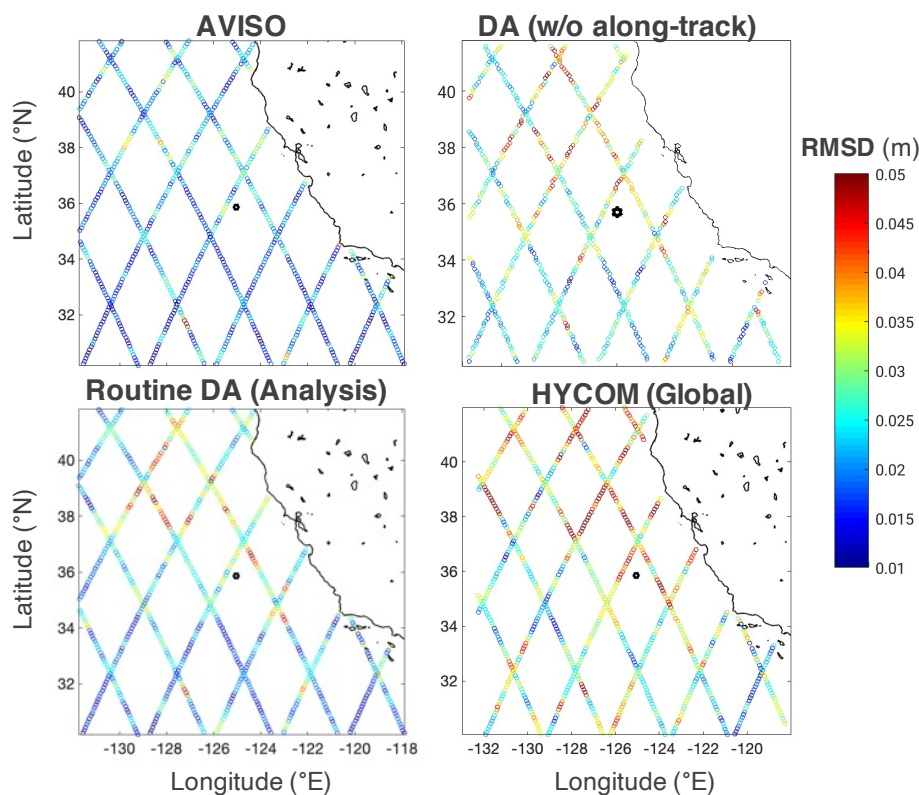
## 425 **5 Evaluation and Illustration of Performance**

426

427 The MSDA-SWOT outlined here is largely based on the DA system presented in Li et al.  
 428 (2019b). As described in previous sections, we have implemented three major new capabilities:  
 429 (1) assimilation of observations over a longer time window via extended-3DVAR; (2)  
 430 assimilation of multi-satellite along-track altimetry observations; and, (3) optimization of the  
 431 MSDA implementation. The evaluation presented in this section primarily serves to illustrate the  
 432 effectiveness of these new capabilities. A systematic evaluation related to the SWOT pre-launch  
 433 field campaign is presented Archer et al. (2021).

434 The evaluation here focuses on data assimilation analyses and forecasts for four months from  
 435 August 10 to December 10, 2019. The model is initialized on July 30 using a 15-day average of  
 436 the HYCOM analyses that are also used to derive the lateral boundary condition as described in  
 437 section 4. 1. The data assimilation analyses and forecasts are generated by four experiments: 1)  
 438 Routine DA, which assimilates the routine observations; 2) NODA, which has no data  
 439 assimilation; 3) Routine DA without along-track altimetry observations; and, 4) DA Cal, which  
 440 assimilates routine observations and the SWOT pre-launch field campaign observations.

441



**Figure 5.** RMSD against filtered along track J3 data from August 10 through December 10, 2019. RMSD of the AVISO gridded data product (top left), the MSDA without assimilation of along-track altimetry observations (top right), the DA analysis (bottom left), and HYCOM (bottom right). The space-time RMSD values are 2.1 cm, 2.8 cm, 2.6 cm, and 3.4 cm, respectively.

442

443 We emphasize that interpretation of the evaluation results must take into account observation  
 444 errors, since most observation errors are close to the analysis and forecast error. Therefore, when  
 445 comparing analyses and forecasts with the observations, we use the terminology root-mean-  
 446 square difference (RMSD) rather than use the term root-mean-square error (RMSE).

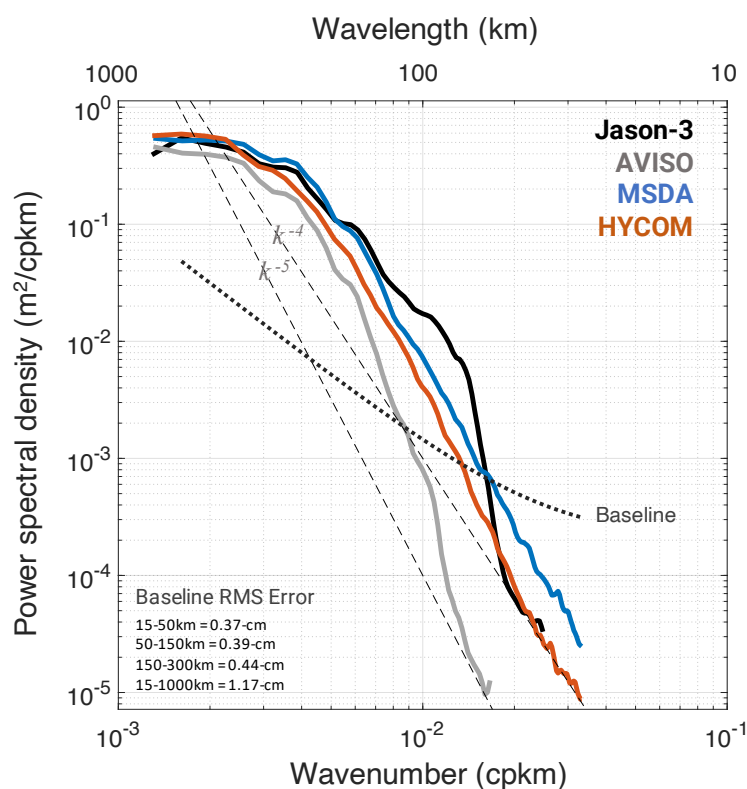
447

## 448 5.1 DA Analysis Evaluation

449 Since the assimilation of along-track altimetry data is a new implementation, we examine  
 450 whether they are assimilated effectively. Figure 5 shows the RMSD between filtered Jason-3  
 451 along-track data and the DA analysis. We use Jason-3 data because of its spatially fixed ground  
 452 tracks and a short repeat time of 10 days. The mean RMSD over the entire model domain is 2.6



453 cm. As a reference, the RMSD between the filtered Jason-3 along-track data AVISO gridded  
 454 data is also shown and has a domain mean RMSD of 2.1 cm. The RMSD with the DA analysis is  
 455 larger than that with the AVISO gridded data. This result is expected, because the spatial  
 456 resolution of the model is higher than the AVISO grid and thus retains smaller scales. This result  
 457 is consistent with the same analysis using Sentinel-3A and 3B (not shown). The RMSD with  
 458 HYCOM is 3.4 cm. This indicates that MSDA-SWOT performs well for this region in which it is  
 459 optimized.  
 460



**Figure 6.** Power spectrum of Jason-3 filtered along track observations (black), de-tided daily DA analysis (blue), daily AVISO gridded data (grey), and de-tided daily HYCOM (red) averaged from August 10 to December 10, 2019, and the SWOT baseline spectrum (dotted black line). Data are interpolated to the along track grid for computing power spectra. The two dashed lines represent spectral slopes of  $k^{-4}$  and  $k^{-5}$ . The SWOT baseline root-mean-square errors, integrated over a variety of wavelengths, are shown in the bottom left-hand corner.

461



462 To explore the variability across different length scales, we compute wavenumber power spectral  
 463 densities, where the DA and AVISO data is interpolated to the Jason-3 ground tracks (Fig. 6).  
 464 For wavelengths larger than 120 km, the spectral density of the DA analysis is very close to that  
 465 of the Jason-3 along-track observations. For wavelengths from 120 km down to round 65 km, the  
 466 spectral density of the DA analysis is smaller than that of the Jason-3 along-track data. We  
 467 hypothesize this may be due to internal tidal residuals in the along-track altimetry observations  
 468 (e.g., Zhao et al., 2016). For wavelengths smaller than 65 km, the spectral density of the DA  
 469 analysis is much larger, because the scales smaller than 65 km are significantly filtered in the  
 470 along-track altimetry observations (Pujol et al, 2016). The results indicate that the DA has the  
 471 capability of excluding along-track altimetry observation errors due to internal tidal  
 472 contamination and smoothness.

473 It is desirable that the spectral density of the DA analysis is larger at wavelengths smaller  
 474 than 65 km. It is actually an objective of MSDA-SWOT. The MSDA-SWOT system is  
 475 formulated to allow the assimilation of smoothed data without smoothing the DA analysis. This  
 476 is demonstrated in Fig. 6. For example, the AVISO gridded data is assimilated, and Figure 6  
 477 shows that the spectral density of the gridded AVISO data is much smaller than that of the along-  
 478 track altimetry data, but the spectral density of the DA analysis closely follows the along-track  
 479 data. This shows that assimilating the highly smoothed AVISO gridded data does not smooth the  
 480 DA analysis. In contrast, the HYCOM power spectrum, while similar in character to MSDA-  
 481 SWOT, does not exhibit small-scale variability, but rather follows the filtered along-track power  
 482 level.

483 To quantify the impact of along-track altimetry observations in the analysis, they are  
 484 withheld in a DA experiment. The RMSD increases by a substantial amount (Fig. 5 top right),  
 485 with a space-time mean RMSD increase from 2.6 cm to 2.8 cm.

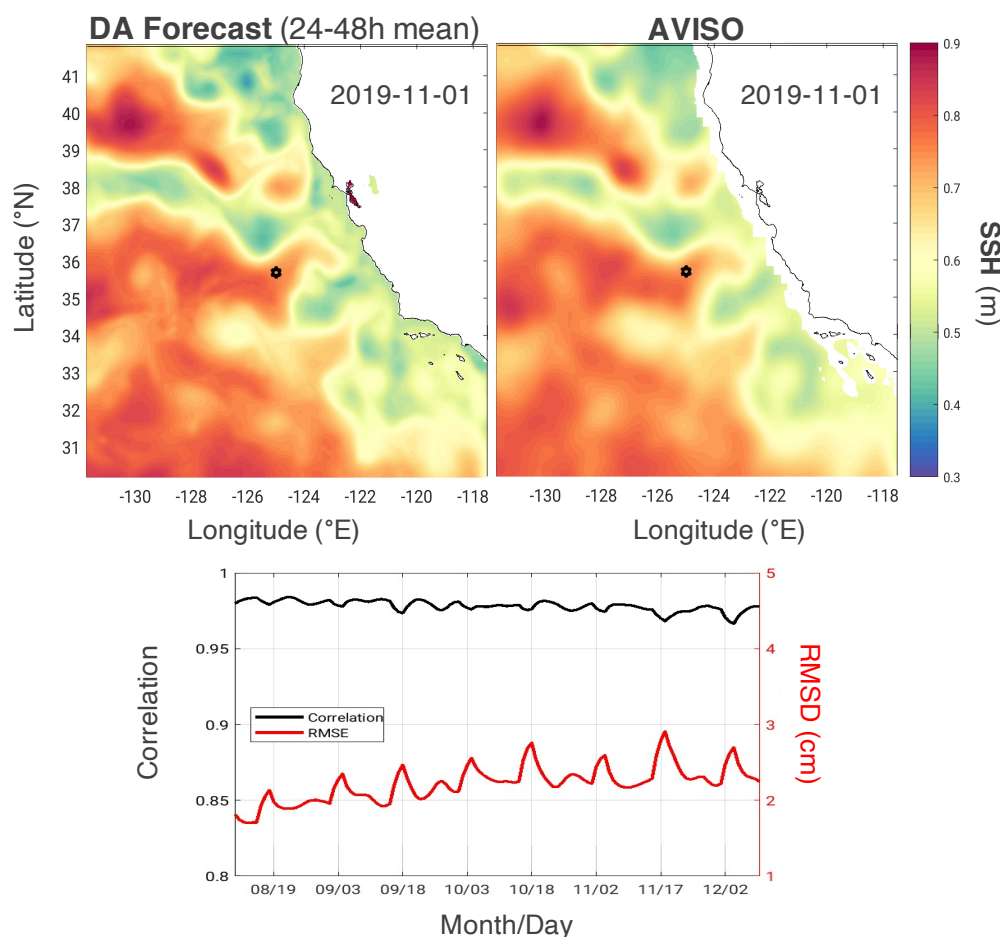
486

## 487 **5.2 DA Forecast Evaluation**

488 As stated in the introduction, the MSDA-SWOT system is developed not for prediction but for  
 489 state estimation. One question for DA analysis is whether observations are assimilated with  
 490 dynamical consistency. In particular, the SSH increment from the assimilation of altimetry  
 491 observations must be consistent with the increment in T/S vertical profiles, otherwise the SSH  
 492 increment could create external gravity waves that may propagate away in a few hours. Also, all



493 observations can be considered as independent data when we evaluate forecasts. We thus here  
 494 focus on the evaluation of short-time forecasts.  
 495



**Figure 7.** (top) SSH field on November 1<sup>st</sup>, 2019. The time mean of forecasts between 24 h to 48 h (left), the AVISO SSH field (right). The star indicates the pre-launch campaign site. (bottom) the timeseries of spatial RMSE and correlation between DA forecast 24-hr to 48-hr mean and AVISO, with an average of 2.3 cm over 4 months from August 10 to December 10, 2019. In the calculation, the daily domain average error, which ranges from 0.0 cm to 1.5 cm, has been removed.

496

### 497 5.2.1 Comparison with altimetry data

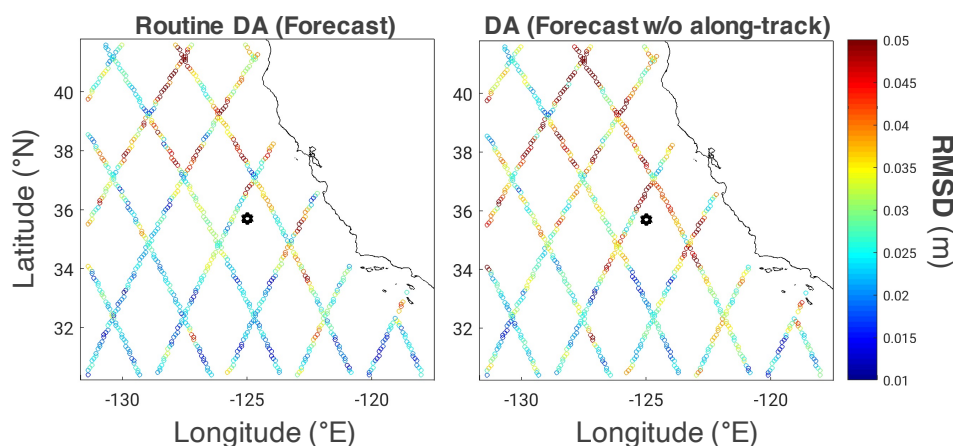
498 The gridded AVISO data can resolve wavelengths down to 150-200 km in this region (Pujol,  
 499 2016; Taburet et al., 2019; Archer et al., 2020). Therefore, the AVISO gridded data can be



reliably used to evaluate large mesoscale eddies from the forecast. Here we compare the mean of the forecast from 24 h to 48 h to AVISO daily gridded data

As an example, Fig. 7 shows the 24–48 hr mean of the forecast and the AVISO gridded data on November 1. The model forecast reproduces the large mesoscale eddies and filaments. Because of the high resolution of the model, there are finer scale eddies and other circulation features in the forecast. To quantify the similarity between the forecast and the AVISO data, we compute RMSD and spatial correlation over the entire model domain (Fig. 7 bottom). The spatial correlation ranges from 0.96 to 0.97 day-by-day, indicating a high resemblance in spatial structure. The RMSD ranges from 1.8 cm to 2.9 cm, while the mean RMSD over this time period is 2.3 cm. We conclude that the eddies that the AVISO gridded data resolves are realistically reproduced.

As discussed, the MSDA-SWOT system is based on the previous MSDA system described in Li et al. (2019b), but with a set of new implemented formulations. In Li et al. (2019b), a similar comparison with the AVISO data was made. The average correlation was 0.91 and RMSD 3.3 cm. Although the experiment was for a different period of time and thus the results are not directly comparable, the significant difference in both correlation and RMSD indicate that the MSDA-SWOT shows a significant improvement in SSH prediction.



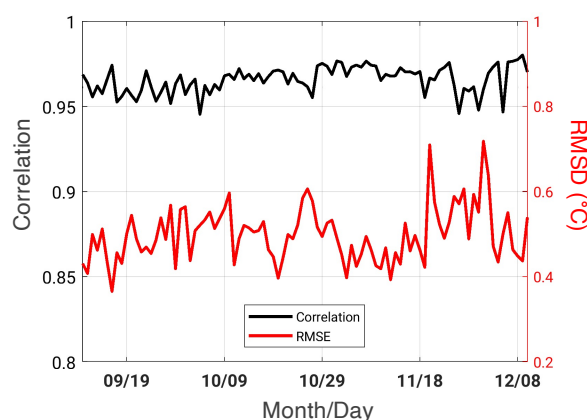
**Figure 8.** RMSD of model forecast to Jason-3 SSH observations for DA with along-track assimilation (left) and without along track assimilation (right). The space-time RMSD for each experiment is 2.9 cm, and 3.3 cm.



519 In subsection 5.1, the DA analysis was evaluated against the Jason-3 along-track altimetry  
 520 data. The RMSD with the 2-day forecast is given in Fig. 8a. The RMSD shows an increase over  
 521 the majority of the model domain. The domain average RMSD is 2.9 cm. As we can see in Fig.  
 522 7, mesoscale eddies have an amplitude ranging from about 10 cm to 40 cm in the region. The  
 523 relatively small RMSD implies that mesoscale eddies can be reproduced in the forecast.

524 To further illustrate the accuracy of the forecast, we compare the RMSD to the experiment  
 525 without DA. Without DA, the RMSD is much larger (not shown), and the domain average  
 526 RMSD is as large as 10.0 cm, more than 3 times larger than the forecast with DA. Thus, the DA  
 527 effectively reduces the forecast error.

528



**Figure 9.** Daily forecast RMSD and spatial correlation against MW SST that is not assimilated.

529

530 We also evaluate the extent to which assimilating along-track altimetry data reduces the  
 531 error (Figure 8). Without assimilating along-track altimetry data the domain-averaged forecast  
 532 error is 3.3 cm. After assimilating along-track data, it is 2.9 cm, so the RMSD is reduced by as  
 533 much as 14 %. Therefore, improvement in the DA analysis from along-track altimetry  
 534 assimilation remains in the 24-48-hr forecast.

535

### 536 **5.3 Comparison with independent satellite SST**

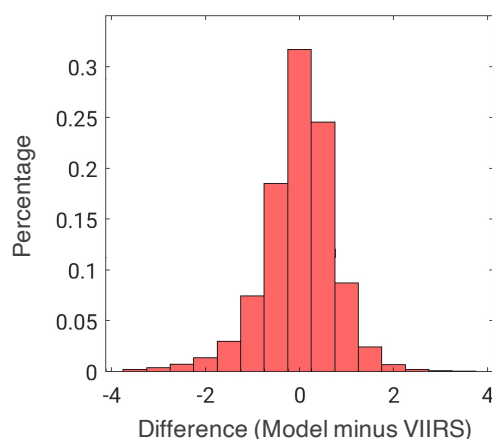
537 Since microwave (MW) SST measurements are not affected by clouds, they provide more  
 538 consistent spatial coverage than infrared (IR) SST (compare Fig. 4a to 4b). We thus compute  
 539 RMSDs and spatial correlations over the entire model domain day-by-day. MW SST is





assimilated at UTC 03 using observations within a time interval of 12 h from UTC 21 to UTC 09 of the subsequent day. Therefore, the observations from UTC 9 to UTC 21 are not assimilated and so are independent observations that can be used for evaluating the DA system. The RMSD is around 0.5° C, and the spatial correlation over the entire model domain higher than 0.95 day-by-day (Figure 9). Given expected differences between the model upper-bin temperature and MW satellite-observed skin temperature, this RMSD is very acceptable.

VIIRS SST is a high-resolution product. Its resolution of 0.7 km allows for resolving submesoscale features down to a few kilometers. Unfortunately, the CCS region is prone to low-elevation cloud coverage, and the VIIRS IR sensor cannot measure SST over cloudy areas. Over the majority of the model domain VIIRS has no observations. We thus interpolate the model data to the time and location of VIIRS SST observations. Figure 10 shows the distribution of the temperature difference. More than 75% of the difference is smaller than 0.75 °C. The overall RMSD is 0.78 °C, which is close to the measurement error.



**Figure 10.** Distribution of difference between forecast and VIIRS SST observations. The RMSD is 0.77°C, and the correlation 0.96.

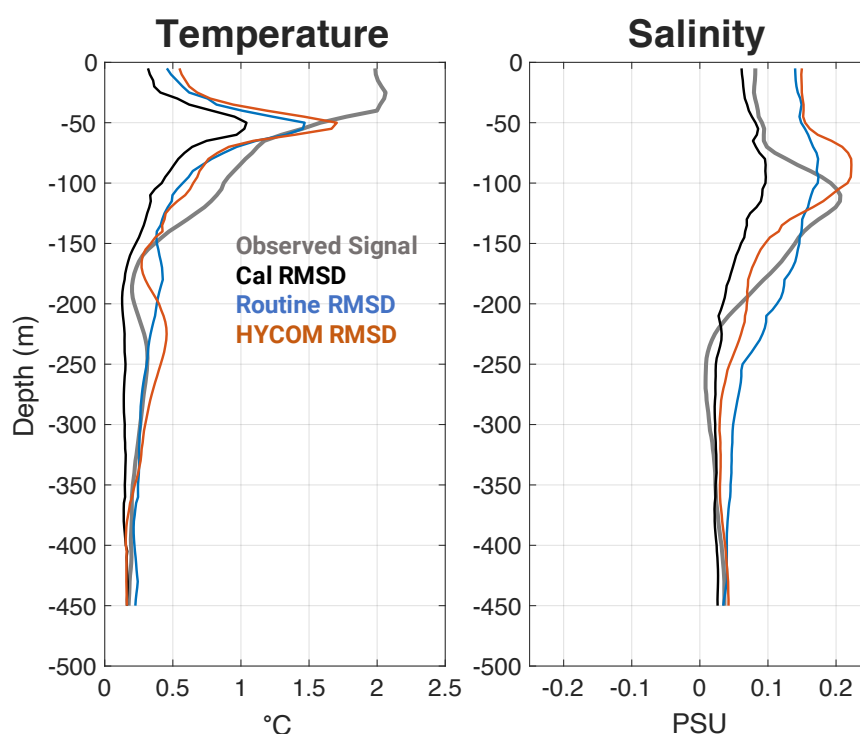
#### 5.4 Evaluation of subsurface temperature and salinity profiles

The MSDA-SWOT goal is to be able to assimilate observations concentrated in a very limited area, and avoid the so-called spurious “campaign area circulation” that can occur when small-scale observations are assimilated into the background field of a numerical model that may have a bias.





560 The strategy used in MSDA-SWOT is to assimilate all routine observations to minimize  
 561 the model bias and constrain mesoscale variability down to about 100 km or further, then the  
 562 innovations of the dense but localized field campaign observations become small enough to be  
 563 effectively assimilated without being smoothed, via the multiscale data assimilation  
 564 methodology (Li et al., 2015a). The evaluation of the SSH and SST forecasts has shown that the  
 565 forecast errors are reduced to less than 3 cm and 1°C, respectively. These errors are a few times  
 566 smaller than the amplitudes of their signal standard deviation (not shown).  
 567



**Figure 11.** The vertical profile RMSD against the middle mooring observations for temperature (left) and salinity (right), for: DA Cal, DA Routine, and HYCOM. In DA Cal, both routine and campaign observations are assimilated (including the middle mooring), while only routine observations assimilated in DA Routine and HYCOM.

568  
 569 Compared against the T/S profiles from the middle mooring CTD (Fig. 1 and 11), which  
 570 are independent observations for the routine DA, there is a significant reduction in the forecast  
 571 errors at all depths for both temperature and salinity when compared to the NODA run (not  
 572 shown). For depths below 150 m, the temperature error is reduced to 0.25°C from about 1°C in

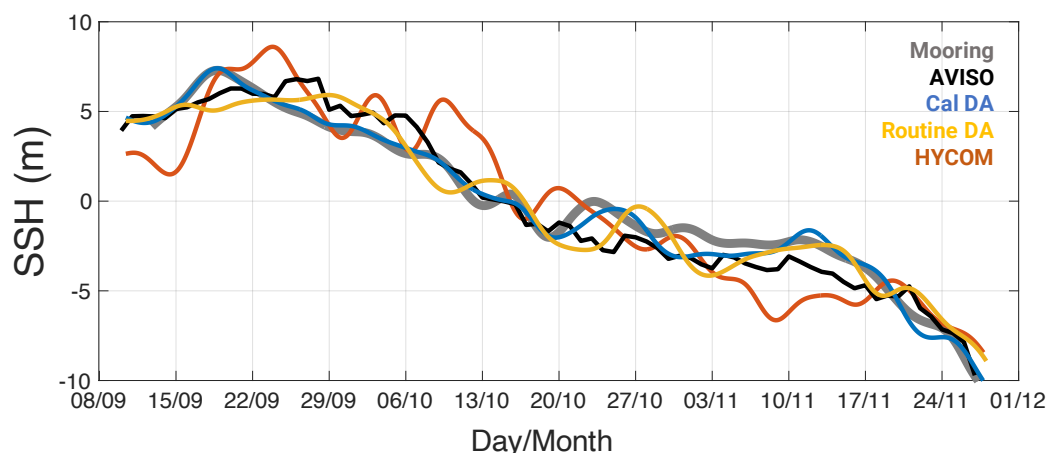


the experiment without DA, while the maximum error located near the mixed layer base reduces from 4.0°C to 1.2°C. Overall, the assimilation of routine observations reduces the error to less than one third of the error without DA. Except the depth between 200 m and 300 m, in which the NODA experiment shows an error as small as 0.07 psu and is close to the Routine DA, the impact of the routine observations is also significant in salinity. The largest error reduction occurs at a depth of 100 m, from 0.55 PSU in the NODA experiment to 0.14 PSU.

The comparison of HYCOM to the middle mooring shows similar performance to the routine DA run. There are some differences however, most notably in salinity. HYCOM exhibits largest error at the top of the halocline, but then the error quickly falls away. In contrast, the routine MSDA has less error above the halocline, but the error more slowly reduces with depth than HYCOM. Both the routine MSDA and HYCOM have errors greater than the signal variance for depths above ~100-m.

In the DA Cal experiment, the T/S profiles from the south and middle mooring CTD observations (Fig. 1) are assimilated in the small mesoscale and small-scale DA (Table 2). Compared to the middle mooring CTD observations, the assimilation of the campaign T/S profiles further reduces the forecast error (Fig. 11). The subsurface error reduces to below 0.2°C, except near the bottom of the mixing layer depth. The largest error is 0.5°C and located at a depth of around 40 m. The salinity error reduces to be smaller than 0.02 psu below 200 m, and the largest error is 0.1 psu and occurs at a depth of around 100 m. For both temperature and salinity (except for 250-300-m), the DA Cal experiment shows errors smaller than the signal for all depths.

Fig. 12 shows the SSH at the middle mooring location (Fig. 1). All DA experiments and the AVISO product can effectively represent the longer period variability exhibited by the mooring. However, DA Cal shows a much improved ability to resolve the shorter-scale fluctuations. The analysis by Archer et al. (2021) shows that the forecast steric height can reach the basic SWOT CalVal requirement of an accuracy of the order of 1 cm.



**Figure 12.** Time evolution of 1000-m steric height at the middle mooring location for: middle mooring (grey), AVISO SSH (black), Cal DA (blue), Routine DA (orange), and HYCOM (red).

602

603 We have cautioned that the assimilation of observations from a localized area may create a  
 604 spurious “campaign area circulation”. Following Fig. 12, we can articulate it further. The RMSD  
 605 with the AVISO gridded data is as large as 16.0 cm for the NODA experiment (not shown). This  
 606 large RMSD arises because the mooring is located in the high SSH ridge extended from the open  
 607 ocean toward the coast, but in the lower SSH zone extended offshore (Fig. 7) in the NODA  
 608 experiment. Consider the width of the SWOT swath of 50 km and an error of 16.0 cm. The  
 609 assimilation of the mooring T/S vertical profiles may correct the SSH locally, create large  
 610 spurious SSH gradients or geostrophic velocities that may be as large as 0.32 m/s, and thus a  
 611 sporous “observing system circulation”. In contrast, after the routine observations are  
 612 assimilated, the SSH error has been reduced to around 1.5 cm. The localized observations can be  
 613 aggressively assimilated, and no intense spurious “observing system circulation” would be  
 614 generated.

615

## 616 **6 Conclusions and Discussion**

617

618 A data assimilation system for a high-resolution model has been developed to address the  
 619 opportunities and challenges posed by the upcoming SWOT satellite mission. This system,  
 620 dubbed MSDA-SWOT, is based on a multi-scale data assimilation scheme documented in Li et  
 621 al. (2015b, 2019b). Three major changes have been implemented. First, the conventional



622 3DVAR formulation has been extended to effectively assimilate observations over a longer time  
623 window, which is 11 days for the large and small mesoscale DA in the present configuration  
624 (Table 2). In this extended 3DVAR, the error due to the difference between the observation and  
625 DA time is explicitly accounted for. Second, the assimilation of multi-satellite along-track  
626 altimetry observations has been implemented, and the effectiveness has been demonstrated. And  
627 third, the multi-scale data assimilation configuration has been optimized for inputs, such as  
628 background error covariances.

629 The system has been used to produce a reanalysis for the SWOT pre-launch field campaign  
630 that took place at the planned SWOT CalVal site in the California Current System, from  
631 September–December 2019. The reanalysis dataset has been preliminarily evaluated against  
632 assimilated and independent observations. MSDA-SWOT showed a significantly improved  
633 performance on top of the MS-3DVAR (Li et al., 2019b). Archer et al. (2021) presents a  
634 comprehensive evaluation of the system based on the pre-launch campaign moorings and glider  
635 observations.

636 As we reiterated, the most important consideration in the formulation and configuration of  
637 MSDA-SWOT is the assimilation of all routine observations to minimize the model bias and  
638 constrain mesoscale variability down to about 100 km or further. The background error is  
639 relatively dominated by small scale errors. As a consequence, the background error correlation  
640 length scale becomes small. A small correlation length scale ensures that the dense but localized  
641 field campaign observations are effectively assimilated without being smoothed (Li et al., 2015a,  
642 b). Further, the evaluation of the SSH, SST and subsurface vertical profile forecasts has shown  
643 that the forecast errors can be a few times smaller than the amplitudes of their signal standard  
644 deviation (STD). Mathematically, only in a pure linear system can observations be assimilated  
645 fully, reduce the error in the analysis that is the initial condition for the subsequent forecast, and  
646 ensure a reduction in forecast errors (e.g., Li & Navon, 2001). With nonlinearity in the model, a  
647 reduction in the initial condition does not ensure a reduction in forecast errors. In MSDA-SWOT,  
648 the forecast error has been significantly reduced by assimilation of routine observations. The  
649 innovation associated with the campaign observations and thus the analysis increment has been  
650 small. With a small analysis increment, the model performs more linearly in the time evolution  
651 related to the increment.



652 The impact of routine observations in MSDA-SWOT has an important implication for  
653 observing system design for the SOWT postlaunch field campaign. It strongly supports the  
654 suggestion proposed in Li et al. (2019a) that a field campaign should be designed to leverage  
655 routine observing networks. A field campaign should make observations that augment the  
656 routine observing networks for resolving smaller scales and higher frequency variability, and/or  
657 measure variables that the routine observing networks do not provide.

658

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665 The authors have arranged for the datasets, which are used to produce the results in this paper, to  
666 be available at <https://doi.org/10.5281/zenodo.4602095>.

667

668 **Code and data availability:** The source codes of the multiscale data assimilation system and  
669 some of the diagnostics tools developed in this study are shared in an open repository  
670 (<https://doi.org/10.5281/zenodo.5787332>). The basic model outputs and used observation data  
671 are available at <http://doi.org/10.5281/zenodo.4602095>.

672

673 **Author contributions:** Zhijin Li: Conceptualization, modeling and data assimilation  
674 methodology and implementation, numerical experiments, formal analysis, software,  
675 visualization, writing - original draft, writing - review & editing; Matthew R. Archer:  
676 Conceptualization, modeling and data assimilation implementation, formal analysis, observation  
677 processing, visualization, writing - original draft, Writing – review & editing; Jinbo Wang:  
678 Conceptualization, modeling and data assimilation implementation, formal analysis, resources,  
679 Software, observation processing, visualization, writing - original draft, writing – review &  
680 editing; Lee-Lueng Fu, Conceptualization, funding acquisition, project administration, modeling  
681 and data assimilation methodology and implementation, writing - original draft, writing - review  
682 & editing.



683

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686



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