Replies to Reviewers' comments on

[previous title] "A two-step method to derive combined Fourier-wavelet spectra from space-time data for studying planetary-scale waves, and its Matlab and Python software (cfw v1.0)"

I am thankful to the reviewers for taking the time to review the manuscript and provide useful comments. I have taken all the comments into account in revising the manuscript. My response to individual comments can be found below, where the comments from the reviewers are highlighted in blue, and the text from the revised manuscript is highlighted in red. The line numbers in the response refer to those in the revised manuscript.

Before going into my replies to individual comments, I would like to highlight important changes that both reviewers may want to be informed of.

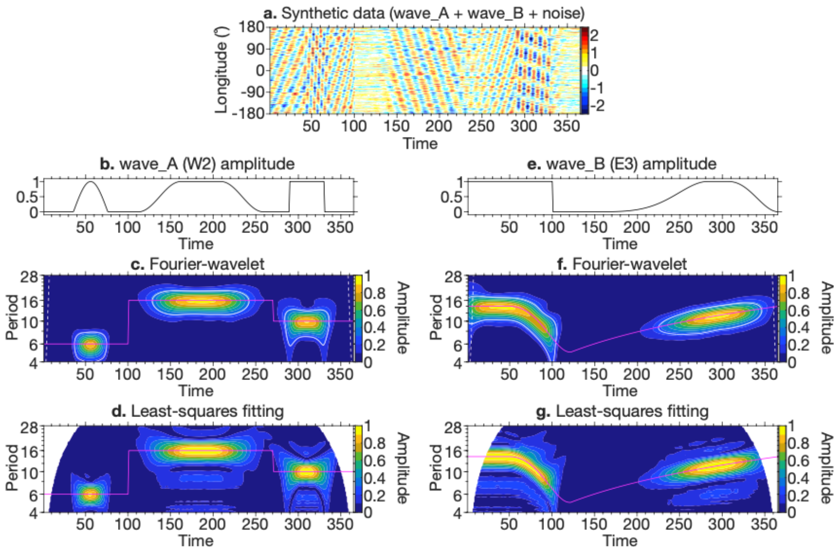
[1] The title of the manuscript has been changed. This is mainly because RC1 pointed out that the term "combined Fourier-wavelet" is a misnomer. The new title reads:

"A method to derive Fourier-wavelet spectra for the characterization of global-scale waves in the mesosphere and lower thermosphere, and its Matlab and Python software (fourierwavelet v1.1)"

In the revised manuscript, the technique is referred to as Fourier-wavelet analysis. Accordingly, the name of the software has been changed from "cfw" to "fourierwavelet".

The new title also emphasizes that the main target is global-scale waves in the mesosphere and lower thermosphere.

[2] Figure 1 has been updated by including the results obtained by the least-squares fitting method for a comparison. This is as suggested by RC2. Also, Figure 5 and the relevant text have been removed, as RC1 pointed out that the figure does not take any advantage of the Fourier-wavelet technique.



New Figure 1: "(a) Synthetic data containing wave_A (westward-propagating with zonal wavenumber 2, W2) and wave_B (eastward-propagating with zonal wavenumber 3, E3), along with noise. (b) Amplitude of wave_A. (c) Phase of wave_A (magenta line), and Fourier-wavelet amplitude spectrum for W2 (contour plot). The white curves indicate the 95% confidence level, while the white dashed lines show the cone of influence. (d) Same as (c) except that the amplitude spectrum is derived with the least-squares fitting method. (e–g) Same as (b–d) but for wave_B. The amplitude spectra are for E3."

RC1 (Dr. Jun-Ichi Yano)

[1-1] This is an interesting piece of work, potentially worthwhile for a publication: there are already extensive literature performing the continuous wavelet transform to time series in atmospheric science. However, this work is new: the continuous wavelet transform is applied to time series, that itself is nothing new, but in the wavenumber space. The author shows that this methodology can characterized certain wave activities rather well, as shown in Figs. 2-4.

[**Response**] Thank you for accurately articulating the novelty of this work. Below are my replies to individual comments.

[1-2] The main problem with the present manuscript, as it stands for now, is to present it with a rather sensational fanfare, calling it specifically a "combined Fourier-wavelet spectrum". As far as I can follow, it is nothing other than just performing a continuous wavelet transform in time and a Fourier transform in space (longitude): these are two independent linear operations, that can be performed in any order. There is nothing to "combine", but just to perform two independent things in sequence. In other words, the adjective "combined" is nothing other than just a misnomer.

[**Response**] Output spectra, which are previously called "combined Fourier-wavelet" spectra, are now referred to as "Fourier-wavelet" spectra without "combined". Accordingly, the title of the paper and the name of the software have been changed as stated above.

[1-3] With an attempt of the author convincing the readers that this is "revolutionary", the author provides rather lengthy technical details of the so-called Hayashi's method (that itself is nothing other than just performing the Fourier transform in time and longitude) and the continuous wavelet in the last half of the introduction section. However, I do not see those technical details to be any importance for this method. As I just said above, what is done in practice is very simple: perform FFT in longitude, and the continuous wavelet analysis in time. We just need few more extra words on specific.
On the other hand, the presentation of the proposed methodology in Sec. 2 is rather "muddled", and difficult to get this very simple point straight.

[**Response**] I do not completely agree with the reviewer. The main idea of this work is to combine the Hayashi's method with the wavelet technique (as mentioned at lines 82-84). Thus, detailed explanation of Hayashi's method in Sec. 1.2 is necessary. As stated at lines 90-91, Sec. 1.2 describes how the amplitude and phase of individual wave components are associated with the 2-D

Fourier transform. Once Sec. 1.2 is understood, Sec. 2. is easy to follow because there are a lot of similarities between the formulae for the Fourier-wavelet method (Sec. 2) and Hayashi's method (Sec. 1.2). No changes have been made to the manuscript.

[1-4] A simple methodology is always a beauty. However, if the author intends to present the present manuscript as a proposal paper of a new methodology, a more careful review of the existing methodologies is required.

First of all, planetary waves can be extracted in a straightforward manner, at least in principle, by the normal-mode decomposition. A full description of this methodology is provided in Zager et al. (2015, GMD, https://gmd.copernicus.org/articles/8/1169/2015/) with a software publicly available to apply this methodology. Also please refer to a workshop report for further backgrounds: Zagar et al. (2016, BAMS, https://journals.ametsoc.org/view/journals/bams/97/6/bams-d-15- 00325.1.xml).

In this respect, the introduction is slightly confused as it stands for now: its

[**Response**] Thank you for the interesting reference. Although normal modes are addressed in Sec. 1, it is not the aim of this paper to identify the normal modes of linear wave theory as in Zager et al. The present study focuses on global-scale waves in the mesosphere and lower thermosphere (MLT) region, where the behavior of traveling planetary waves deviate considerably from classical normal modes. In the revised manuscript, I have emphasized these two facts: (1) the Fourier-wavelet method has been developed for global-scale waves in the MLT region and (2) the behavior of traveling planetary waves in the MLT region deviate considerably from classical normal modes.

For (1), the new title explicitly state "in the mesosphere and lower thermosphere" and the first line of abstract now reads, (lines 1-2) "This paper describes a simple method for characterizing global-scale waves in the mesosphere and lower thermosphere (MLT), such as tides and traveling planetary waves, using two-dimensional longitude-time data."

For (2), it is stated in the revised manuscript (lines 34) "The behavior of traveling planetary waves in the real atmosphere deviates from what is anticipated from the linear wave theory due to dissipation and mean winds."

[1-5] Second is lack of a proper review of the wavelet method. The most important question, in this context, for me is a choice between the continuous redundant wavelet and the discrete orthogonal wavelet. Here, the author chooses the former, but without explanation. The choice is just puzzling for me considering a very fact that the latter is much more robust, with much more potential applicabilities, as my series of work suggest: see a list of reference below.

Performing a continuous wavelet analysis is like a decomposition of a finite domain data into continuous wavenumbers, when only the discrete integer wavenumbers have a meaning.

[**Response**] The advantage of the continuous wavelet transform has been addressed in the revised manuscript

(lines 128-131): "One advantage of the 'continuous' (in contrast to 'discrete') wavelet transform is that the user can arbitrarily select the frequency resolution of the output spectrum. This is helpful especially in investigating traveling planetary waves, as the user has no prior knowledge of the dominant frequency of the wave. On the other hand, the discrete wavelet transform has its own advantage such as non-redundancy and straightforward invertibility (e.g., Yano et al., 2001, 2004), which, however, will not be explored in this study."

Thank you for the references. The Yano et al. (2001, 2004) papers have been cited.

[1-6] Finally, if detection of a planetary-wave packet is the main issue, a discrete set of wavelets can be constructed based on normal modes, in an equivalent manner as the Meyer wavelet is constructed based on the Fourier modes. Though I do not think that the author has to try this possibility in the present work, all those potential possibilities must be clearly mentioned in the manuscript.

It is obvious that the author is only taking a small first step forwards for exploring all those wider possibilities.

[**Response**] Thank you for your understanding. As the reviewer accurately pointed out, this work is just a small first step towards a more comprehensive diagnostic tool for MLT wave dynamics.

The detection of planetary-wave packets is important for this study. In the revised manuscript, I have made it clear that the technique is intended for characterizing global-scale waves in the MLT region (lines 1-3) and that the behavior of traveling planetary waves in the MLT region deviates from that of linear wave theory (lines 34-35). I believe that these changes partly address the reviewer's comment. Also, general future directions are mentioned in Sec. 4, which addresses the rest of the reviewer's comment:

(lines 337-339) "Although this study has focused on tides and traveling planetary waves in the MLT region, the Fourier-wavelet method can be easily applied to data from other regions of the atmosphere. Also, the applicability of the technique in research areas outside atmospheric science is yet to be explored."

Specifics

[1-7] L79-80, the standard wavelet technique is not directly applicable to longitude-time data: false. 2D wavelet transform can easily performed in analogy with the 2D Fourier transform: refer to my publications below.

[**Response**] This was about 1D wavelet transform. The manuscript has been revised to make it clear:

(lines 80-81): "However, the standard 1-D wavelet technique is not directly applicable to two-dimensional (2-D) longitude-time data ..."

[1-8] L90, parameter -> variable

[**Response**] The change has been made as suggested.

[1-9] Eqs. (5)-: the frequency, omega, must be discrete, as the case for the wavenumbers. please comment on this

[**Response**] Yes, omega is discrete. This has been made clear:

(lines 94-95) "ω (>0; = ω_0, ω_1, ω_2, ...)"

[1-10] L122: state explicitly that continuous wavelet is applied in time

[**Response**] The text now clearly states so:

(line 125) "A continuous wavelet analysis is applied in time."

[1-11] Eq. (16): the actual data set only has a finite length in time. comment on this

Eq. (21): if not, this expression is puzzling: since continuous wavelet is applied here, obtained coefficients must also be continuous: why we suddenly get a discrete expression?

[**Response**] Yes, the data set has a finite length in time and sampled at a finite rate. That is the reason for the discrete expression. The text has been revised to make this clear:

(line 140) "If x(t) is sampled with the sampling interval Δt for a finite length in time from t_0 to t_{N-1},"

[1-12] Sec. 2: as far as I can follow, the longitudinal dependence does not play any role in the presentation, though Eqs. (29) and (30) retain it. at least a word would be required for a clarification: otherwise, in my own reading, Sec. 2 is essentially just repeating Sec. 1.3. If not, what is a difference except for a longitudinal dependence added in Eqs. (29) and (30)?

[**Response**] As already described in the text, Eqs (29) and (30) [Eqs (31) and (32) in the revised manuscript] are modifications of Eqs (3) and (4), respectively, taking into account the modulation of the wave amplitude with time. The longitudinal dependence of Eqs (29) and (30) plays an important role in constraining the zonal wavenumber and wave phase. There is no repetition of Sec. 1.3 in Sec. 2. So, no changes have been made to the manuscript.

[1-13] Eqs. (29), (30): the exponent, -t^2/2 must be replaced by -t^2/2s? if not, I do not know how to connect this expansion with (17), as invoked after Eq. (34).

[**Response**] Thank you for spotting the error. Eqs (29) and (30) [Eqs (31) and (32) in the revised manuscript] have been revised by including *s*.

[1-14] Eqs. (29), (30): the given decomposition modes are only localized in time, thus it appears to me that the author essentially fails to address a question of the propagation of a wave packet, that should happen both in time and space.

[**Response**] As this study is concerned only with temporally-localized global-scale waves, the Fourier-wavelet technique does not take into account the propagation of a longitudinally-localized wave packet in zonal direction. A 'wavelet-wavelet' technique would be required for resolving longitudinally-localized waves. I believe that it is already well emphasized in the manuscript that this paper focuses on global-scale waves. So, no changes have been made.

[1-15] Eqs. (35) and (36): Psi* here must depend on both s and omega: how do you specify them?

[**Response**] Yes, Psi* depend on both s and ω. The ω values are given in Eqs (27) and (28), but there was no description on how the scale s was determined. In the revised manuscript, I have explained how the scale s is handled:

(lines 144-149):
"The scaling factor s can be arbitrarily selected. Torrence and Compo (1998) used a set of scales that is fractional powers of two, and it is also adopted here. That is,

$$s_j = s_0 2^{j\Delta j} \qquad (21)$$

where $s_0 = 2\Delta t$ and $j = \{0, 1, 2, ..., J\}$. Δj controls the scale resolution, which the user can arbitrarily select. J determines the largest scale and is given by

$$J = 1/\Delta j \log_2 (N/2) \qquad (22)$$"

[1-16] Fig. 5b, c: they should be better presented by standard Fourier transforms: the plots do not take any advantage of wavelet, either.

[**Response**] I agree that Fig. 5 does not take any advantage of the Fourier-wavelet analysis. As such, the figure has been removed along with the relevant text.

RC2

[2-1] This paper introduces a simple method to perform combined Fourier-wavelet (CFW) transform to extract planetary-scale waves in the gridded 2-D longitude-time atmospheric data set. Although the concept of this method is not new, or the idea is not difficult to understand, the implementation of this method has always been a difficulty for the space physics or mesosphere and lower thermosphere community. After testing the programs provided by the authors, I think this method has the following advantages:

1. The calculation speed is fast since this method only reads the data once. This advantage is meaningful, especially to the extraction of tides, which usually

requires data with a high temporal resolution and is a challenge for the memory and computing resources of a personal computer;

2. The output is directly the real amplitudes of the waves, which is similar to the S-transform, but this method obtains the amplitudes in the 2-D data set;

3. In fact, there has been no good way to accurately extract the planetary wave activity during sudden stratospheric warmings in the longitude-time satellite observations or simulations. However, the results in this manuscript show that this method does a relatively good job on this issue, which is important for related research.

[**Response**] Thank you for the accurate summary of the advantage of the Fourier-wavelet analysis.

[2-2] Overall, the authors implemented the CFW transform in a relatively simple way with MATLAB and Python, which is very worthy of recognition and will bring a lot of convenience to the middle and upper atmosphere community. I only have two minor comments on the current manuscript:

1. The authors may consider comparing the results obtained by different methods (e.g., CFW transform, 2-D fast Fourier transform, and least squares fitting) to better demonstrate the superiority of the CFW transform in studying the temporal variations of planetary-scale waves.

[**Response**] Figure 1 has been revised by including a comparison with the least-squares fitting method, which is most simple and arguably most popular in studies of global-scale waves in the MLT region. The comparison serves the two purposes: (1) the Fourier-wavelet spectra are similar to those derived from the least-squares fitting method, and (2) the computation time for the Fourier-wavelet is much shorter than that for the least-squares fitting method. Accordingly, the text has been revised:

(lines 245-253): "Figure 1d is the same as Figure 1c but derived with the least-squares fitting method, which is often used for studying global-scale waves in the MLT region (e.g., Fan et al., 2022; Qin et al., 2022). The analysis was performed using time windows that are 3 times the wave period, which is a common choice in investigations of traveling planetary waves (e.g., Forbes and Zhang, 2015; Yamazaki and Matthias, 2019). The amplitude is not computed at the beginning and end of the data, where the length of the data is less than 3 times the wave period. There is good agreement between the results derived with the Fourier-wavelet (Figure 1c) and least-squares fitting (Figure 1d) methods. However, the computation time for the Fourier-wavelet method is approximately 1/100 that for the least-squares fitting method, highlighting the advantage of the Fourier-wavelet method in computation speed. Figures 1e–1g correspond to Figures 1b–1d but for wave_B."

Also, in Sec. 4:

(lines 333-336): "The Fourier-wavelet method has an advantage over other existing methods in that the computation is fast. For the example presented in Section 3.1, the computation time for the Fourier-wavelet method is approximately 1/100 that for the least-squares fitting method."

[2-3] 2. The authors can appropriately show the results on the planetary waves or tides in the ionospheric parameters (e.g., total electron content) extracted by the CFW transform.

[**Response**] For this paper, I would prefer to focus on global-scale waves in the MLT region. I am currently working on another manuscript describing the application of the Fourier-wavelet analysis to ionospheric data. In the revised manuscript, the applicability of technique to non-MLT data is addressed as future work.

(lines 337-339) "Although this study has focused on tides and traveling planetary waves in the MLT region, the Fourier-wavelet method can be easily applied to data from other regions of the atmosphere. Also, the applicability of the technique in research areas outside atmospheric science is yet to be explored."