Peer-Review of "Data Driven Regional Weather Forecasting"

December 2, 2022

1 Comments

1) About the role of time-delayed embedding

Let me elaborate more on what I meant. Taken's embedding theorem says that, under certain conditions, there exists an integer D_E so that $\mathbf{TD}(n)$ **DETERMINES** some $\mathbf{TD}(n+1)$. There exists a smallest such integer, which we denote by $D_E^{\max}$. Then for $D_E < D_E^{\max}$, there might be several different values of $\mathbf{TD}(n+1)$ that follows the same $\mathbf{TD}(n)$. In this case, the forecast system has uncertainty. And intuitively the more lags you use, the less uncertainty you would have. This is what I mean by "the authors use time-delayed observations to reduce the uncertainty of the forecast system". For instance, in Lorenz 63 system, suppose you observe the time series $x(t)$ from the 3-dimensioanly state vector time series $(x(t), y(t), z(y))$. For $D_E = 2$, numerically you still can construct a forecast system by parameterization and whatever method you use to estimate the parameters. But in fact there does not exist a deterministic function f, so that $f(\mathbf{TD}(n)) = \mathbf{TD}(n+1)$. Therefore no matter how you parameterize your forecast system, you can only get a "mean value" instead of the exact value of the future.

Theoretically, Taken's embedding theorem requires the dynamical system to be finite dimensional. This means that it must be a finite dimensioanl ODE or a PDE that has a finite dimensional inertial manifold. It is likely that the SWE with dissipation is the second case. But there is no demonstration about this point in this manuscript (and similarly in many others). Without a rigorous demonstration on whether the dynamical system is essentially finite dimensional, "Taken's embedding theorem" is merely a "faith" but might not be the fact. In this case, the time-delayed embedding technique is expected to reduce the uncertainty of the forecast system, instead of providing a deterministic and completely accurate forecast system.

It is true that your observations are taken from the 10×10 grid simulation of SWE, which in fact is a finite dimensional ODE. But this also implies that your obs are not taken from the true SWE dynamical system. For your interest, you can try increasing the resolution of your grid and do the same experiment for the same observed locations (the 3×3 sub-grid in the 10×10 grid). Maybe you will find that larger D_E is needed in order to provide an accurate forecast. If this is not obvious in your experiment, you can try using a smaller dissipation rate (A and ϵ) and a higher resolution.

As written in the introduction of this manuscript, the core idea of this manuscript is to provide a data-driven method to forecast the future of the observed time-series from the true nature (i.e. the infinite dimensional SWE). Therefore I think it is more reasonable to suggest in this manuscript that the time-delayed embedding is used to reduce the uncertainty of the forecast model, instead of providing a diffeomorphism between S and $\mathbf{TD}(n)$.

2) About the role of RBFs

Thanks the author for pointing out that the paragraph after Eq.(7) is equivalent to the interpretation of the reviewer, which I indeed did not notice. What I am curious about is that which term, the RBFs or the polynomial term, contributes more in the forecast model. It would be more clear if the authors can provide some numerical results on this. And, again, what is the rationale behind the choice of degree 1 polynomial? Is it due to some insight of the dynamics of the SWE or merely result-driven?