Reply to RC2

Manuscript information:

- Title: Mean age from observations in the lowermost stratosphere: an improved method and interhemispheric differences
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We would like to thank the reviewer for the kind words and the constructive comments. In the following document, the reviewers' comments are marked in *italic font* and indented, our answers are in regular font. Changes in the manuscript are marked-up in red and listed as framed screenshots below the respective comment. The line numbers in our listed changes refer to the marked-up version of the revised manuscript, that is provided separately.

Point-by-Point reply

1. *Minor Comment 1, Line 86: It is not clear to me why G(x,t') in equation (2) does not depend on the source region x_i. That is, G should also be conditionally dependent on x_i, such that G(x,t') should be rewritten as G(x,t'|x_i). The authors here have assumed that the transport operators propagating tracer concentrations for all regions i are the same, but I can envision several cases where this would not be true. For example, air propagating into the stratosphere at high latitudes will have no clear path into the stratosphere, as opposed to air straddling the midlatitude tropopause, where isentropic surfaces provide a clear pathway for stratosphere-troposphere exchange. The authors need to provide their rationale here.*

Thank you for your constructive comment. In case of an ideal inert linear evolving tracer the differences across individual *G(x,t'|x_i)* have no influence on calculating the mean age. In contrast, in the quadratic tracer case the mean age cannot be calculated without knowledge of $\Gamma(\boldsymbol{x}|\boldsymbol{x_i})$. However, if the quadratic term of the tracer mixing ratio time series is sufficiently low, then the concept of *G(x,t'|x_i)* can be neglected by using the Ansatz expressed in Eq. (2). We concluded that within the scope of this study, which focuses on relatively young mean ages derived from SF_6, we can neglect the influence of differences between different *G(x,t'|x_i)*.

We revised the Appendix A in the updated version of the manuscript to clarify our approach:

Appendix A: Calculating mean age in the LMS considering multiple entry regions and an ideal tracer

In case of an ideal inert linear evolving tracer, the tropical ground time series as a function of transit time t' is given by

$$\chi(\boldsymbol{x_{TR\ ground}}, t') = a - bt'. \quad \text{(A1)}$$

The negative sign points out, that looking at increasing transit times means looking backwards in time.

Assuming a constant time shift t_{xi} for each entry region i, the tracer time series at $\boldsymbol{x_i}$ is

$$\chi(\boldsymbol{x_i}, t') = a - b * (t' - t_{xi}). \quad \text{(A2)}$$

Considering individual transit time distributions $G_i(\boldsymbol{x}, t')$ for each origin fraction $f_i(\boldsymbol{x})$, the stratospheric mixing ratio $\chi(\boldsymbol{x})$ of a suitable age tracer at an arbitrary location $\boldsymbol{x}$ in the stratosphere is

$$\chi(\boldsymbol{x}) = \sum_{i=0}^{N-1}\left[f_i(\boldsymbol{x}) * \int_0^\infty \chi(\boldsymbol{x_i}, t') * G_i(\boldsymbol{x}, t')dt'\right] \quad \text{(A3)}$$

Hence, by inserting Eq. (A2) into Eq. (A3~~2~~), the stratospheric mixing ratio can be expressed as

$$\chi(\boldsymbol{x}) = \sum_{i=0}^{N-1}\left[f_i(\boldsymbol{x}) * \int_0^\infty (a - bt' + bt_{xi}) * G_i(\boldsymbol{x}, t')dt'\right]$$

$$= \sum_{i=0}^{N-1}[f_i(\boldsymbol{x}) * (a + bt_{xi})] - \sum_{i=0}^{N-1}\left[f_i(\boldsymbol{x}) * b * \int_0^\infty t' * G_i(\boldsymbol{x}, t')dt'\right]. \quad \text{(A4)}$$

~~$\chi(\boldsymbol{x}) = \int_0^\infty \sum_{i=0}^{N-1}\left(f_i(\boldsymbol{x}) * \left(a - b * (t' - t_{xi})\right)\right) * G(\boldsymbol{x}, t')dt'. \quad \text{(A3)}$~~

The mean age Γ is the first moment of the age spectrum, given by

$$\Gamma(\boldsymbol{x}) = \int_0^{\infty} t' * G(\boldsymbol{x},t')dt'. \quad \text{(A5)}$$

In case of $G_i(\boldsymbol{x},t')$ Eq. (A5) translates into the mean age of air originating from source region i ($\Gamma_i(\boldsymbol{x})$)

$$\Gamma_i(\boldsymbol{x}) = \int_0^{\infty} t' * G_i(\boldsymbol{x},t')dt'. \quad \text{(A6)}$$

Inserting Eq. (A6) into Eq. (A4) yields:

$$\chi(\boldsymbol{x}) = \sum_{i=0}^{N-1}[f_i(\boldsymbol{x}) * (a + bt_{xi})] - \sum_{i=0}^{N-1}[f_i(\boldsymbol{x}) * b * \Gamma_i(\boldsymbol{x})]. \quad \text{(A7)}$$

Since the sum of all origin fractions equals 1, Eq. (A73) can also be written as

$$\chi(\boldsymbol{x}) = a + \sum_{i=0}^{N-1}[f_i(\boldsymbol{x}) * bt_{xi}] - b * \sum_{i=0}^{N-1}[f_i(\boldsymbol{x}) * \Gamma_i(\boldsymbol{x})]. \quad \text{(A8)}$$

The mean age $\Gamma(\boldsymbol{x})$ equals the sum of individual $\Gamma_i(\boldsymbol{x})$, weighted by their respective origin fraction $f_i(\boldsymbol{x})$:

$$\Gamma(\boldsymbol{x}) = \sum_{i=0}^{N-1}[f_i(\boldsymbol{x}) * \Gamma_i(\boldsymbol{x})]. \quad \text{(A9)}$$

By inserting Eq. (A9) into Eq. (A8), we can thus reduce the number of unknown parameters:

$$\chi(\boldsymbol{x}) = a + \sum_{i=0}^{N-1}[f_i(\boldsymbol{x}) * bt_{xi}] - b * \Gamma(\boldsymbol{x}) \quad \text{(A10)}$$

~~$\chi(\boldsymbol{x}) = \int_0^{\infty}[a - bt' + b * \sum_{i=0}^{N-1}(f_i(\boldsymbol{x}) * t_{xi})] * G(\boldsymbol{x},t')dt',$ (A4)~~

~~which is equivalent to~~

~~$\chi(\boldsymbol{x}) = a + b * \sum_{i=0}^{N-1}(f_i(\boldsymbol{x}) * t_{xi}) - b * \int_0^{\infty} t' * G(\boldsymbol{x},t')dt'.$ (A5)~~

~~The mean age Γ is the first moment of the age spectrum, given by~~

~~$\Gamma(\boldsymbol{x}) = \int_0^{\infty} t' * G(\boldsymbol{x},t')dt'.$ (A6)~~

~~Inserting Eq. (A6) into Eq. (A5) yields:~~

~~$\chi(\boldsymbol{x}) = a + b * \sum_{i=0}^{N-1}(f_i(\boldsymbol{x}) * t_{xi}) - b * \Gamma(\boldsymbol{x}).$ (A7)~~

Equation (A107) can be solved for Γ, which yields

$$\Gamma(\boldsymbol{x}) = \frac{a-\chi(x)}{b} + \sum_{i=0}^{N-1}(f_i(\boldsymbol{x}) * t_{xi}), \quad \text{(A9)}$$

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which is equivalent to Eq. (5). The same result can be obtained mathematically when we use the origin fractions as weights only for the mixing ratio time series and neglect the concept of $G_i(\boldsymbol{x},t')$ (starting with Eq. (2) instead of Eq. (A3)). Differences across individual $G_i(\boldsymbol{x},t')$ thus have no influence on calculating the mean age from an ideal inert linear evolving tracer. In contrast, in case of an ideal inert quadratic evolving tracer the Ansatz expressed in Eq. (A3) cannot be solved for $\Gamma(\boldsymbol{x})$ without knowledge of individual $\Gamma_i(\boldsymbol{x})$. However, if the quadratic term of the tracer mixing ratio time series is sufficiently low, then the concept of $G_i(\boldsymbol{x},t')$ can be neglected by using the Ansatz expressed in Eq. (2).

In order to derive mean age from an ideal inert quadratic evolving tracer with multiple entry regions, we extended the equations given by (Volk et al., 1997). In this case the TR ground mixing ratio time series is given as a function of transit time by

$$\chi(\boldsymbol{x}_{TR\ ground}, t') = a - bt' + ct'^2. \quad \text{(A9)}$$

2. *Minor Comment 2, Section 2.2.2: I am curious about the calculation of t_xi. The procedure outlined in steps (i)-(iii) essentially sounds like a description of how to calculate the SF6-age, which previous studies have used to calculate the tropospheric mean age (albeit using an SF6 surface boundary condition that only averages stations over northern midlatitudes). The details of the regions considered may be slightly different, but the procedure is basically the same. So why not reference this literature? In particular, the authors should review these studies:*
    - *Waugh, Darryn W., A. M. Crotwell, E. J. Dlugokencky, G. S. Dutton, J. W. Elkins, B. D. Hall, E. J. Hintsa et al. "Tropospheric SF6: Age of air from the Northern Hemisphere midlatitude surface." Journal of Geophysical Research: Atmospheres 118, no. 19 (2013): 11-429.*
    - *Orbe, Clara, Darryn W. Waugh, Stephen Montzka, Edward J. Dlugokencky, Susan Strahan, Stephen D. Steenrod, Sarah Strode et al. "Tropospheric Age of Air: Influence*

of SF6 Emissions on Recent Surface Trends and Model Biases." Journal of Geophysical Research: Atmospheres 126, no. 19 (2021): e2021JD035451.

Thank you for your constructive comment. We referenced the proposed literature in the updated version of the manuscript:

> the subsequent decade from 2008 on. This decade is not covered by the model from Rigby et al. (2010) that we used to derive t_{xi}, however it is covered by $\chi(\boldsymbol{x}_{TR\ ground}, t')$ (Laube et al., 2022, updated from Simmonds et al., 2020). Previous studies used a similar procedure as outlined above (steps (i) to (iii)) to estimate transport time scales while referencing the NH midlatitude ground (Orbe et al., 2021; Waugh et al., 2013). We found that t_{xi} varies less over the time period 1973 to 2008 when referencing the tropical ground in the MOZART data set. In order to derive more robust entry mixing ratio time series for our exTR-TR method, we thus decided to use the tropical ground as a reference. We emphasize that each t_{xi} as defined here is an integrated empirical measure. t_{xi} does neither contain useful information on transport paths nor on transit times from the TR ground to the entry regions. We only use t_{xi} to derive entry mixing ratio time series at locations, where suitable long term time series are not available from measurements.

3. *Technical Comments: Line 83: The concept of "origin fraction" referred to here certainly precedes the Hauck et al. (2020) study and the authors should properly reference the literature. For example, see these studies:*
    - *Orbe, Clara, Mark Holzer, Lorenzo M. Polvani, and Darryn Waugh. "Airmass origin as a diagnostic of tropospheric transport." Journal of Geophysical Research: Atmospheres 118, no. 3 (2013): 1459-1470.*
    - *Orbe, Clara, Darryn W. Waugh, and Paul A. Newman. "Airmass origin in the tropical lower stratosphere: The influence of Asian boundary layer air." Geophysical Research Letters 42, no. 10 (2015): 4240-4248.*

Thank you for pointing that out. We referenced the proposed studies in the updated version of our manuscript:

> $\chi(\boldsymbol{x}_i, t')$ by calculating a weighted mixing ratio time series. The relative importance of individual source regions can be described by so-called origin fractions (e.g. Orbe et al., 2013, 2015). We use the origin fractions $f_i(\boldsymbol{x})$ as ~~introduced~~ derived by Hauck et al. (2020) as weights for each $\chi(\boldsymbol{x}_i)$. $f_i(\boldsymbol{x})$ is the fraction of air at $\boldsymbol{x}$, that entered the stratosphere through $\boldsymbol{x}_i$. By