Rate-induced tipping in natural and human systems
Reviewer 2 Responses

We are grateful for the further constructive reviewer comments received on our manuscript. These comments are repeated below in italic type. Our responses are coloured blue and given in normal type. New text copied from our revised manuscript is presented in quotations.

Response to Reviewer 2

Thanks to the discussion and the reformulation of some parts of the article, the aim and target group is now clearer to me, and makes sense. I think that my previous comments have been addressed to an extent that I find sufficient to agree with the publication.

I only have some minor additional suggestions:

- Sect 2 and 3: Although the applications and demonstrations come later, the authors might give an example here already (in 1-2 sentences). For example, could the case where lambda is in physical units of a rate, be the freshwater flux into the North Atlantic ("hosing") in kg/year (or Sv)?
  Yes exactly, we now provide the examples of the freshwater flux into the North Atlantic and population growth rates for the predator-prey system. In Section 3, we write "Furthermore, if the forcing λ itself is a physical rate of some sort (e.g. freshwater flux into the North Atlantic, measured in Sverdrups – millions of cubic metres per second, or population growth rate measured in individuals per unit area per year, from examples in Section 5)..."
- line 110 + footnote 2: I now understand why the authors show a stability diagram that deviates from the normal form in a specific way (to demonstrate "return tipping"), but I don't understand why this shape should be "more realistic". It would indeed be unrealistic if the normal form applied far from the saddle-node, but the particular shape the authors chose for demonstration might be even less realistic. I guess what is realistic always depends on the particular system. So I'd suggest to remove the word "realistic" in this context.
  We have removed "reflects more realistic bifurcation structures and" from the sentence and also removed the word "real" from the footnote.
- It took me a little while to understand in Fig. 3c why the tipping depends on the forcing rate as shown; what might help further is to plot the trajectories of forcing and state in the same plane as shown in Fig. 2d+e. This is what I meant with showing stable and unstable equilibria in the previous round. The authors point out in their reply that "the equilibria (for the static system) when plotted against time will be different due to the varying rates of forcing". What I meant was not to put time on any axis, but show trajectories in the state vs forcing space as in Fig. 2.
  Firstly, we would like to remind the reviewer that Figures 2(a,b,d,e) and Figure 3 correspond to the same conceptual model. Ideally, we would include all representations in the same figure but this would be too much. However, we believe it is more important to include the time series representation of the forcing and system response as this is commonly more familiar to climate and Earth system scientists than the state vs forcing plane (that is already plotted in Fig 2).
- Fig. 5b: The shape of the boundary between the green and red regime looks strangely non-smooth. Is this a numerical bug / lack of simulations and simulation time, or is it (to some extent) related to some non-smooth model property like taking the absolut value of q in Eq. 14+15?
  The non-smooth appearance is a result of insufficient resolution, instead there would be many small but smooth 'wiggles'. We now write "Additionally the (black) boundary has small 'wiggles' that appear as non-smooth corners. A similar 'wiggling effect' near a Hopf bifurcation has been observed in O'Keeffe and Wieczorek (2020)."
- Title, line 186 and elsewhere: To me "human systems" still sounds misleading. I would expect some system describing physiological or social behaviour of humans, not the engineering / technology related model of power grids. The authors may still want to consider rephrasing, e.g., human-made systems or technological systems.
  Although our specific example of a power grid is technological, it is also a human-created system. In

this sense, technological systems are a subset of human systems. In other human systems (e.g. the economy) we also expect to see rate-induced tipping. We therefore prefer to keep 'human' in the title, as this more clearly conveys the ubiquitous nature of rate-induced tipping.

- Line 258-262: It is still unclear to me why (near) power blackouts like during the 1990 semi-final are an example of R-tipping. I understand that the power demand was higher than expected, and that controllers needed time to catch up with the demand, but where does the critical rate come in here? If I interpret power demand minus supply as the forcing, then there would be one critical threshold, which is rather a B-tipping than R-tipping. Where does the memory of the system state come into play that is needed for R-tipping?
  As the reviewer points out, the fact that the controllers nearly did not have enough time to catch up, suggests by definition that it is a rate-induced problem. In this case, there is no issue with providing enough power to meet the new demand level, it is only the "rate" at which the demand is rising that would cause a blackout. We have added the following text to make this clearer, "Therefore in this case, the limiting factor was not the peak in demand, but instead the rate at which the demand on the network rose."

- Line 266, 268 and elsewhere: Still unclear to me how a "2π difference in the phase angle" or "a 2π shift in the phase angle" can be distinguished / defined at all. If I think of just one wave, a 2π shift gives the identical wave, i.e., 0 phase shift. Why should this be counted as another equilibrium?
  While the equilibria themselves would look the same, the variables are coupled. So, making the transition from one equilibrium to the next equilibrium (separated by a 2π phase angle) will result in a temporary drop in voltage. We now make this clearer with the following modification to the text, "An *alternative transient state* is a temporary drop in the voltage magnitude accompanied by a 2π shift in the phase angle, caused by the coupling within the system"

- Line 296: "slowly enough"?
  We have changed this to "sufficiently slowly". Note that in the same sentence we give the opposite side and mention for "...changes faster than some critical rate".

Additional comments I had not raised in the first round (so I think it's appropriate if the authors decide themselves whether they want to make changes or not):

- line 167: "For small overshoots of the Fold, even greater complexity is possible with the potential of three critical rates and two (red) tipping sub-intervals for a fixed peak change of return forcing." Maybe add a figure, e.g., in a supplement?
  This is already highlighted in the tipping diagram (Figure 3(c)), and we do not believe it to be necessary to show these suggested time profiles as the qualitative behaviour has already been shown for all three scenarios (tracking, points of return, points of no return).

- 167-175: the extreme cases of practically infinitely slow or fast forcing discussed here are not shown in Fig. 3c? If the system behaviour changes beyond the range shown, it could be insightful to extend the Figure.
  There is no major change in the behaviour beyond the range shown. In fact, as we explain at the end of page 6, the black dashed line separating tracking from tipping regions asymptotes to the Fold for small r and to the boundary of basin-instability for large r. We also already mention on page 8 that the return profile asymptotes to the Fold for small r and for large r tipping is prevented because the "system processes are too slow to react...".

I hope that my comments were somehow helpful to improve the article and compliment the authors for their contribution.
We thank the reviewer again for their helpful and constructive comments.