Review of Manuscript

'The Wasserstein distance as a hydrological objective function'

By J. Magyar and M. Sambridge

Dear Editor,

I have reviewed the aforementioned work. My conclusions and comments are as follows:

1. Scope

The article is well within the scope of HESS.

2. Summary

In their manuscript, the authors propose the Wasserstein distance as a useful method to quantify the distance between two spatial or temporal patterns, and to construct a single spatial/temporal pattern to represent an ensemble thereof. To motivate and illustrate the method, the authors use examples from hydrology, in particular discharge hydrographs. The authors start by describing the shortcomings of standard distance measures such as RMSE relying on pointwise comparison, if the main source of disagreement is spatial/temporal offset, also how this can lead to unwanted local minima when using such distance measures as an objective function. The authors then give a detailed overview on the concept of optimal transport, which is the basis of the Wasserstein distance, and then explain how the Wasserstein distance can be used to identify an ensemble representative as the spatial/temporal pattern which minimizes the transport distance to all ensemble members, the so-called "Wasserstein barycenter". Next, the authors discuss computational methods to calculate Wasserstein distances for general and specific cases, in particular the 1-d case, and discuss extensions of the Wasserstein distances when applied to patterns which are not proper distributions (with the integral summing up to 1), such as hydrographs. After the introduction of the method, the authors provide use-cases based on synthetic data, using Wasserstein distance for hydrological model calibration (in comparison RMSE) and for constructing "average" hydrographs by Wasserstein barycenters (in comparison to calculating the pointwise ensemble mean). The authors conclude that for the synthetic calibration examples used in the paper, Wasserstein provided better model fit than relying on RMSE as an objective function, that Wasserstein distances are best suited for spatial/temporal rather than magnitude offsets, and that a remaining key challenge for applying Wasserstein distance to hydrological time series is the issue of adding/removing mass, and that this may require the identification of matching subsections before calculating Wasserstein distances.

3. Evaluation

Overall, this is a well-written, well-structured article, which gives a very good introduction to Wasserstein distance and how it is calculated, and a thorough account of its potential and its limitations, neither over- nor underselling the method. As the Wasserstein distance is not common in Hydrology, this is a useful addition to the hydrological literature. That said, a few points remain which should be improved:

- Title: The paper discusses how the Wasserstein distance can be used i) as an objective function and ii) to construct ensemble representatives, but only the first aspect is reflected in the title. I suggest changing the title such that is contains both aspects.
- Use of the word "metric": In the article, the authors use the term "metric" to refer to distance measures in general, not to the strict definition of a metric being a distance measure with the following properties:

- d(a,b)>=0
- d(a,b)=0 then a identical b
- d(a,b)+d(b,c)<=d(a,c) (triangle inequality).

  As the authors go quite deep into the derivation of the Wasserstein distance, and the discussion of its properties, I suggest they should i) restrict usage of the term "metric" to true metrics only, and ii) mention if Wasserstein distance is a true metric or not.

- Similarly, it will be helpful for the reader to discuss if the Wasserstein distance is a symmetrical or non-symmetrical distance function, i.e. if d(a,b) = d(b,a) or not. In other words, does Wasserstein compare the distance of some "model" to a "reference truth", or the distance between to objects on equal terms.
- A comment (no changes requested): Apart from the illustrative results from the synthetic test cases, for the hydrological community it would be very useful to see some applications to real-world, and long, hydrographs. These will be the cases that are relevant from a practical point of view, but where the limitations of the Wasserstein "mass-problem" will become most obvious. I do not request extra work here, as the authors mention in lines 78-83 that the main purpose of this paper is to introduce the concept and show some illustrative examples, and I think this is enough to justify the paper. Nevertheless it would be helpful to provide such applications at least in a follow-up paper. This will greatly increase the chances that the method will be picked up by the community.
- Sect. 3.1: The authors apply the Wasserstein- and RMSE-optimization to the system with the rainfall timing errors. It would be helpful to initially mention that when using the true rainfall input, both Wasserstein- and RMSE-optimization would perfectly identify the true model parameters (which I assume should be the case).

Minor points

- Eq. 1: at this point, it is unclear why two different sets of data **x** and **y** are required for functions *f* and *g*. Either explain or replace **y** by **x**.

Overall, I think the manuscript needs **minor revisions** before publication.

Yours sincerely,

Uwe Ehret