Effects of including the adjoint sea ice rheology on estimating Arctic ocean-sea state

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Referee comment

GENERAL COMMENT:

The manuscript explores the impact of improved approximation of the adjoint code for the visco-plastic (VP) sea ice model on the overall quality of the optimized solution. The authors obtain quantitative estimates of the improvement and demonstrate that better approximation of the adjoint velocity fields provides more realistic distribution of the optimal corrections among other components of the sea ice state. Although the manuscript is of certain interest to ice modeling community, it has a number of deficiencies that need to be addressed before publication. In particular, the authors need to better articulate the position of their Arctic ice modeling system among other Arctic ice modeling systems with data assimilation capabilities. Presentation of the material also needs substantial improvements: I would suggest editing the manuscript by a person with a better command of English.

SPECIFIC COMMENTS:

Introduction: the authors need to expand their overview of the data-driven ice modeling. In particular, they should discuss advantages of the adjoint/variational approach in forecasting seasonal changes of the sea ice state as compared to sequential methods (e.g., EnKF). Attention should also be paid to limited of differentiability of the VP rheology that imposes certain constraints on the utility of variational methods in ice forecasting.

Section 2.1:

a) The utilized approximation to the adjoint of the tangent linear (TL) ice model should be described in more detail. Instability of the TL/adjoint codes of the Hibler's model is caused by violation of the positive semi-definite (PSD) property of the linearized C-grid discretization of the momentum equation, so it is instructive to give more details on what modifications of the code were made "to facilitate generation of the adjoint model". The cited paper by Losch et al (2010) is insufficient.
b) A description of the "control run" (first mentioned in Table 2) is missing.
c) Description of the assimilation window (line 100): It is unlikely that ice state in, say, December is controllable by the initial conditions on the 1st of January (Table 3). Provide more detail on the formulation of the assimilation window. Was it a sequence of twelve monthly windows (e.g., line 163)?

Section 2.3:

a) Second term in eq, 3 is incorrect: check arguments of the viscosity coefficients (no commas between the indices), replace the deformation rate tensor by its trace

b) Lines 153-156: instability of the TL/adjoint code is caused by violation of the PSD property by the system solved by the LSOR method, and could be eliminated by using weak formulation of the ice momentum equation (Mehlmann & Richter, 2017). Besides, stabilization of the linearized rheology term could be achieved by augmenting of the exact adjoint code by Newtonian damping (Panteleev et al, 2021) rather than eliminating dependence of the viscosities on the deformation rates. In that respect, I would suggest to use the term "approximate adjoint" rather than "stabilized adjoint" throughout the text and add some discussion of the issue either here or in Section 6. Stabilization of an [exact] adjoint model implies adding extra terms for the purpose of moving the eigenvalues of the TL/adjoint matrices inside the unit circle.
c) Line 161: does "global mean" imply spatial *and annual* average in this context?
d) Line 166, 172 etc: "along the SIEs" SIE should be explained. If it stands for "sea ice extent", it is better to use "along the sea ice margins" (SIMs). Also, SIT and SIC should also be explained after their first appearance in the text.

Section 3.1:

a) Line 189: provide details on "could not be further reduced": does M1QN3 stall at a certain step magnitude during the line search? What gradient reduction was achieved in both the FD and VP cases at the convergence?
b) Lines 196-197: replacing "adjoint of the full sea ice dynamics" by "approximate adjoint of the sea ice rheology" would be more appropriate.

Section 3.2:

a) Line 222: why sea ice in the Atlantic sector exhibits stronger non-linearity? Explain.
b) Figure 4d: why there is a gap from May to October? No satellite data?

Section 6:

I would suggest to add a few computational quantities related to the improvement in the approximation of the adjoint model, such as the comparison of the cost function gradient decline curves with iterations, extra computational time (needed to execute 32 instead of 13 iterations), and extra CPU time/memory requirements related to the necessity of memorizing background viscosities and computing extra terms in the updated adjoint model. I would also add more discussion on the issues of instability and differentiability of the linearized VP rheology, and on the utility of the variational methods in optimizing ice conditions in general, especially in view of alternative rheologies (e.g. Mohr-Coulomb, elasto-brittle) based on more sophisticated models parameterizing sub-grid ice dynamics.

TECHNICAL CORRECTIONS:

There are numerous grammar and stylistic errors that should be corrected: To name a few:

L 47 and after: change hereafter to hereinafter.
L 61: "may overestimate", also change "towards" to "with respect to"
L174: change "contradicts the impacts" to "resist [or oppose] the impact"
L214: add "in Fig. 3d" after RMSEs
L222: change "exist" to "persist"
L226: unclear which "three simulations", change "may related" to "may be related"
L227-228: change to "threshold thickness… which triggers sea ice formation in open water"
L236 remove "larger"; change "constant" to "spatially uniform"
L293 change "which" to "but"

Figure 8: Color bar units? Both m/s and degrees Celsius? Comment in the caption.
L315: “By replacing [long list of variables] with their values and estimate [] contributions to cost reductions.” Incomplete sentence. Split into several, clarify what you meant.
L319: variables
L323 “relies”, “the wind vectors”, “the temperature and salinity”.
L326-330: Appears repetitive of the previous paragraph. Reformulate more clearly.
L362: move “thin**n**er” after “sea ice”
L363” “while sea ice in the Eurasian Basin and central Arctic is thicker by 0.4m”
L367: remove “thickened”
L420: change “seems” to “is evident”
L421: “is less efficient to reduce” to “are less efficient in reducing”
L429-430: change “which is melted” to “which destroys sea ice by”
L431: This statement looks trivial (it would be surprising if the effect were opposite).