Following is my review of the revised manuscript entitled "Effects of including the adjoint sea ice rheology on estimating Arctic ocean–sea ice state" by Guokun Lyu, Armin Koehl, Xinrong Wu, Meng Zhou, and Detlef Stammer (egusphere-2022-1099).

General Comment

In this study, motivated by Toyoda *et al.* (2019), the adjoint sea-ice model with viscous–plastic rheology (adjoint-VP) is applied to a coupled ocean and sea-ice state estimation system for the Arctic Ocean, and compared with the previous version in which the simplified adjoint sea-ice model of free drift (adjoint-FD) is used to avoid numerical instability. One year of optimization experiment for 2012 shows that the adjoint-VP can produce better state of the ocean and seaice through more appropriate dynamic and thermodynamic processes than the adjoint-FD.

The revised manuscript became much understandable than the original one in many aspects, but still needs to explain or respond adequately to the following points to be accepted for the publication in *Ocean Science*.

Response:

We thank the reviewer for carefully reading the manuscript again and apologize for missing the "Technical Corrections" in the last review report.

Based on the reviewer's comments below and in the last review report, we have revised the manuscript and response the reviewer's comment below.

Specific Comments

1. Introduction: Indeed, the adjoint method has a characteristic that optimized fields strictly obey the model governing equations, but the control variables are subject to bad influences in some cases as shown in this study (unrealistic adjustment of 2-m air temperature when using the adjoint-FD). In addition, statistical methods can estimate atmospheric forcing and model parameters as well as the initial conditions by augmenting the state vector. Therefore, the reviewer recommends not to exaggerate the advantages of the adjoint method over statistical methods.

Response:

We thank the reviewer's suggestion. The main propose of such a comparison is to explain advantage/disadvantage of different methods.

To achieve a more complete and fair comparisons between statistical and adjoint methods, we revised the manuscript in the two aspects:1) we remove the improper statement "In addition, the adjoint method adjusts all uncertain inputs, including initial conditions, atmospheric forcing, and model parameters, rather than only the initial conditions as in statistical-based methods."; 2) we point out the disadvantage of adjoint method "However, the qualities the reanalysis dataset depends on the accuracy of the tangent linear approximation."

In this way, we hope the readers could have better and fair understandings on different data assimilation methods.

2. Line 182: Clarify whether the diffusivity of 500 m^2 s^{-1} is for the vertical or the horizontal.

Response: 500 m^2 s^{-1} is horizontal diffusivity. We have clarified it in the manuscript.

3. Line 182: Is a harmonic viscosity used in the adjoint model in spite that a biharmonic viscosity is used in the forward model (line 101)?

Response:

We thank the reviewer's comment. Here we explain why we "**increase (or add)**" harmonic viscosity in the adjoint model, while in the forward model we use biharmonic viscosity.

1) In the adjoint model, the biharmonic viscosity is included with the same coefficient as in the forward model.
2) We further add harmonic viscosity and tracer diffusion to stabilize the adjoint model over a large assimilation window. We explain the reason as follows.

Assuming the forward model is expressed as:

$$y = Mx$$

Where x is model input, y is model state, and M represents model integration matrix. Its tangent linear approximation is

$$\Delta y = M'(x)\Delta x.$$

M'(x) is tangent linear model, Δx is perturbations of model inputs and Δy is the resulting model state perturbations. For chaotic nonlinear system, such as the coupled ocean-sea ice system in this study, the nonlinear system (M) and its tangent linear system (M'(x)) have positive Lyapunov exponents and eigenvalues, respectively, limiting the assimilation window. We need to add extra terms in the tangent linear/adjoint systems to reduce/remove the positive eigenvalues in the adjoint model to extend the assimilation window. In the model implementation, both harmonic and biharmonic viscosity and diffusion can be used to damp the positive eigenvalues in the adjoint model. We choose to use harmonic viscosity because it is a more efficient to damping out fast growing modes than the biharmonic ones.

4. Section 3.2.1: As the authors mention that "the normalized RMSEs in Figure 3d should be close to 1.0 if the optimization found a model simulation consistent with the observations and the prior uncertainties" (line 243), the normalized SIC errors of about 0.5 indicates that simulated SICs are overfitted to observations or the prior uncertainties are overestimated. The same can be said of the normalized RMSEs of SIT (Figure 4). Again, discuss this point.

Response:

We thank the reviewer's comment. We have added discussion on the too large SIT uncertainties and pose the requirements on more accurate SIT observations in the future.

5. Section 4: Briefly describe the differences between the ERA5 and NCEP-RA1 reanalyses, especially from the viewpoint of the treatment of sea-ice boundary conditions. The reviewer remembers that the NCEP-RA1 does not use a fractional sea-ice concentration but 0 or 1.

Response:

We thank the reviewer's comment. We have added more comments on differences between the NCEP-RA1 and ERA5 lower boundary conditions "The ERA5 uses fractional SIC as surface boundary conditions, but NCEP-RA1 uses 0 and 1 for ice-free and ice-covered ocean, respectively." (L359-L361).

6. Section 4: Similar to the specific comment 4, the normalized RMS of adjustments of the atmospheric variables of around 0.1 indicates that their estimated prior uncertainties are too large, or equivalently, the relative contribution of the last term in Equation (1) is too small. Discuss this point.

Response:

We thank the reviewer's comment. By now, the uncertainties of atmosphere states remain uncertain. We use the standard deviations of the non-seasonal signals from NCEP-RA1. Nguyen et al. (2021) base on differences/deviations of different atmosphere reanalysis. But, both work show that the adjustments are much smaller than the estimated prior uncertainties. We have added comments on the small values of normalized adjustments "The normalized adjustments of 0.1-0.6 indicate that the estimated prior uncertainties of atmospheric state remain too large." L357.

7. Figure 9: It is confusing to use the blue line for adjoint-FD and the black line for adjoint-VP, because they are opposite in other figures.

Response: we thank the reviewer's comment. We have revised Figure 9 to make the line colors/experiments the same as in the other Figures.

8. Figure 11, caption: Explicitly describe the contour intervals.

Response: we have added "The contour intervals are 2 °C" in Figure 11 caption.

Technical Corrections

1. The reviewer pointed out the followings in the previous report, but they are not corrected in the revised manuscript.

Response:

We thank the reviewer for carefully reading throughout the manuscript and apologize for missing the "technical corrections" in the previous review report.

Based on the previous report (technical corrections) and manuscript, corrections below and revised manuscript last time, we have revised the manuscript.

1.1 Lines 67: Dynamic should read dynamics.

Response: we have corrected the mistakes throughout the paper.

1.2 Line 142: 0.25% should read > 25%.

Response: we thank the reviewer and we have revised the mistake.

1.3 Line 170: *C** should be 20.0 rather than –20.0.

Response: we have corrected this mistake.

1.4 Line 187 and 192: Dynamic should read dynamics.

Response: we thank the reviewer. We have corrected the mistakes throughout the manuscript.

1.5 Line 231: Visual should read visible.
Response: we have change "visual" to visible in the context.

1.6 Line 470 and 473: Dynamic should read dynamics.
Response: we thank the reviewer. We have corrected the mistakes throughout the manuscript.

1.7 Figures 1, 3, 4, 7, and 11: Paint the Great Britain Island gray.
Response:
We thank the reviewer for pointing out the mistakes. We have replotted Figures 1,3,4,7,11 and painted Great Britain Island gray.
2 Line 101: m–2 s–1 should read m2 s–1.
Response: we have revised the units m^{-2} s^{-1} to m^2 s^{-1}
3 Line 114: Right hand should read right hand side.
Response: We have changed "right hand" to "right hand side".
4 Line 447: –6ºC should read 6ºC.
Response: we have removed "–"
5 Line 449: Figure 10b should read Figure 11b
Response: we thank the reviewer for pointing out the mistakes. We have changed "Figure 10b" to "Figure 11b" here.

Nguyen, A. T., Pillar, H., Ocaña, V., Bigdeli, A., Smith, T. A., & Heimbach, P. (2021). The Arctic Subpolar Gyre sTate Estimate: Description and Assessment of a Data-Constrained, Dynamically Consistent Ocean-Sea Ice Estimate for 2002–2017. *Journal of Advances in Modeling Earth Systems*, *13*(5), e2020MS002398. https://doi.org/https://doi.org/10.1029/2020MS002398