Response to Referee

Following is my review of the manuscript entitled "Effects of including the adjoint sea ice rheology on estimating Arctic ocean–ice state" by Guokun Lyu, Armin Koehl, Xinrong Wu, Meng Zhou, and Detlef Stammer (egusphere-2022-1099).

General Comment

In this study, motivated by Toyoda *et al.* (2019), the adjoint sea-ice model with viscous–plastic rheology (adjoint-VP) is applied to a coupled ocean and sea-ice state estimation system for the Arctic Ocean, and compared with the previous version in which the simplified adjoint sea-ice model of free drift (adjoint-FD) is used to avoid numerical instability. One year of optimization experiment for 2012 shows that the adjoint-VP can produce better state of the ocean and sea ice through more appropriate dynamic and thermodynamic processes than the adjoint-FD.

Such findings are important for a further development of global-scale ocean state estimation and data assimilation studies, and could be worth to be published in *Ocean Science*. However, the manuscript has many deficiencies listed below and needs to be substantially revised before acceptance.

Response:

We thank the reviewer's time for reviewing the manuscript and helping us to improve the manuscript. We have revised the manuscript following the reviewer's suggestions. Our responses to the reviewer's comments are listed below.

Specific Comments

1. Line 44: ECCO should be defined here.

Response:

We thank the reviewer's comment and have defined ECCO here (L51).

2. Figure 1: It might be better to indicate important seas and straits.

Response: We thank the reviewer's suggestion. We have labeled the major basins and straits in Figure 1 and described them in the caption.

3. Line 86: Describe the bulk formulae and related parameters used in this study, or cite appropriate references.

Response:

We thank the reviewer's advice. Surface fluxes computations in MITgcm are based on the bulk parameterization of Large and Yeager (2004). We use their default parameter values. In the manuscript, we added a reference here (L96).

4. Explain why the open boundary conditions and the river runoff are not included in the control variables.

Response:

We thank the reviewer's comment. The choice of the control variables is for reasons of "simplicity and the robust performance of this coupled data assimilation system." Since we concentrate on developing the adjoint of sea ice rheology, we think using a configuration that works well now is safe. Therefore, we use our previous setup. In the future development of Arctic reanalysis, we will include river runoff, the open boundary conditions, and parameters in the control variables. In the manuscript, we explain the reasons in L122-128.

5. Line 101: Explain what "effective" thickness means.
Response:
We thank the reviewer's comment. In the model formulation, **"effective ice thickness" means "sea ice thickness multiply sea ice concentration" or "volume per unit area."**

In L87, we have defined it to "mean sea ice thickness (in volume per unit area, mean SIT hereinafter)" and used the terminology "mean SIT" throughout this manuscript.

6. Line 118: BGEP should be defined here.
Response:
We have defined BGEP in L135.

7. There are no explanations for SIC, SIE, SIT, SLA, and SST.

Response:

We thank the reviewer for pointing out these mistakes. We have added the full names of SIC (L86), SIE (L428), SIT (L87), SLA (L143), and SST (L145) at their first appearance.

8. Section 2.3: Briefly describe the treatment of snow on sea ice, which may affect surface albedo and thermodynamic processes, or cite appropriate references.
Response:
We thank the reviewer's comment. A diagnostic snow model is applied on sea ice, which modifies the heat flux and surface albedo. In the revised manuscript, we describe the snow model in Section 2.1 and cite the related reference (L84-87).

9. Line 191: It is better to write explicitly that satellite-observes SST (J_{SST}) and SIC (J_{SIC}).
Response:
We thank the reviewer's comment. We have written the "satellite-observed SST (J_{SST}) and SIC (J_{SIC})" explicitly (L221).

10. Line 196: It is misleading to call "the adjoint of full sea ice dynamics", because adjoint-VP still uses an approximated form of viscous–plastic rheology.
Response:

We thank the reviewer's comment on "approximated form of viscous–plastic rheology. ". In this manuscript, we change "adjoint of VP rheology" to "approximated adjoint of VP rheology" (L212).

11. Section 3.1 and Table 2: The reviewer supposes that relative costs of individual constituents depend on their number of observations. If this is true, it might be better to indicate the total number of each measurement in Table 1.
Response:
We thank the reviewer's comment. We have added the number of individual observation types in Table 1.

12. Figure 3, caption: Explicitly mention that (a)–(c) are average of 2012
Response:
We thank the reviewer's suggestion. We have included "averaged over 2012" in the Figure 3 caption.

13. Line 217: Explain what "sea ice extent regions" means.

Response:

We thank the reviewer's comment. Here, it means "sea ice margins (SIMs)". We have revised it to "sea ice margins (SIMs)" in L199.

14. Section 3.2.1: The normalized SIC errors of about 0.5 indicates that simulated SICs are overfitted to observations. Discuss this point.

Response:

We thank the reviewer's comment. In winter, the normalized SIC errors in the control run (averaged over the sea ice-covered regions) are also small (~0.5), indicating that the model simulated SIC matches the satellite measurements well. Significant SIC errors are mainly along the sea ice margins, while the errors in the central Arctic Ocean are usually smaller than the observational errors. We added "indicating that the control run and the satellite SIC measurements match well." In L247-248, we explain why normalized SIC errors are smaller than 0.5.

15. Figure 4, caption: Describe the averaging period for (a)–(c).
Response: We added "**averaged over 2012**" and "**averaged over the sea ice-covered regions**" in the Figure 4 caption.

16. Section 3.2.2: There are no description of Figure 5.
Response:
We thank the reviewer for pointing out this mistake, and the description of Figure 5 was lost during our editing processes. We have included the description of Figure 5 in L285-290.

17. Figure 5, caption: Describe the averaging period.

Response:

We thank the reviewer's comment. The comparisons in Figure 5 are against three mooring-based up-looking-sonar observations covering 2012. To clarify, we have added "2012" on the label of Figure 5 and state it explicitly "throughout the year 2012" in L281.

18. Figures 3, 4, 5, and 6: It might be better to use the same colors among these figures for CTL, adjoint-FD, and adjoint-VP.

Response: We thank the reviewer's advice, and we have remade figures 3-6 with the same line colors for the three simulations among these figures.

19. Line 337: Explain why April 10 and September 20 are chosen for this analysis.

20. Figure 9, caption: Explicitly mention that (a)–(e) are for the control run.

21. Figure 9: It seems that the red lines in (a), (f), and (k) are the September SIE from the control run, the black lines in (g)–(j) are from the adjoint-FD, and those in (k)–(o) are from adjointVP.

Response:

We thank the reviewer's comment. The choice of "April 10-September 20" is that the period covers the melting season of the Arctic sea ice. The purpose of Figure 9 is to show differences in the sea ice retreat process in the control run and the two assimilation runs. Since the reviewers suggested simplifying Figure 9 or splitting Figure 9 into more figures, we have replaced Figure 9 with their corresponding temporal variations integrated over the model domain and rephrased the descriptions in L427-453, concentrating on the significant different sea ice melting processes from May 20-June 15.

22. Line 369: It sounds strange that the SIC change through ice–albedo feedback is categorized as F_{oi} rather than F_{ai}.

Response:

Ice-albedo feedbacks are as follows: sea ice retreat leads to more open water-----surface albedo is reduced----more absorption of solar radiation by sea water----then, the warm water further melts sea ice from the bottom;

In the model governing equations and during the melting season, $\boldsymbol{F_{ai}}$ **represents the ice-atmosphere heat flux (radiative and turbulent fluxes) at the ice surface. Over the open fraction of seawater, ocean-atmosphere heat fluxes heat the ocean directly, further melting sea ice from the bottom.** Solar heating through the open water is the primary heat source for warming the sea; therefore, we categorize ice-albedo feedback to F_{oi}.