Author's reponse

Report #1

Submitted on 25 Feb 2023
Anonymous referee #1

egusphere-2022-1085 revision

They responded well to the comments of the reviewers. I appreciate their efforts to present this material and to clarify my misunderstandings in the previous review.

I still have a question about whether the non-tidal processes at short lags, which apparently contain much more variance than the tidal processes for the Argo and mooring data, can significantly bias the estimated amplitude of the demodulate at 48-hr. Since the paper deals with amplitude and amplitude ratios involving the 48-hr demodulate, it would be important to quantify this effect. (See comments regarding Fig 8e+f, below.)

I would suggest "minor revision", but I trust that the authors and editor can decide whether their reply to my remaining query requires another full round of peer review or not.

l52: I'm not sure what "oceanic noise" refers to here.
Ans: *It refers to signals from any other physical process in the ocean. Deleted "oceanic" as instrumental noise can also be modeled as a white noise (thus with 0 acov except at tau=0).*

l88: Please don't conflate approximation with asymptotics ("order of").
Ans.: *Replaced by the approximately equal sign.*

Fig 1: Can you please adjust the aspect ratio so that the circle looks more circular?
Ans.: *Done.*

l163: I still don't understand why the sine term is used in estimating the amplitude envelope. The sample autocovariance is an even function, and so is the true autocovariance. The reply to comments in the discussion paper online mentions that a short derivation is attached, but I don't see it.
Ans.: *We attached it as a supplement, sorry about the inconvenience if this got lost somehow. Here is the original derivation:*

Consider a tidal variability as a sum of two tidal constituents:

$$h(t) = h_0(t) + h_1(t) = A_0 \cos(\omega_0 t + \phi(t)) + A_1 \cos(\omega_1 t + \phi(t)).$$

Here ω_i and A_i are the angular frequency, and the amplitude of the constituent i, respectively. $\phi(t)$ is an AR1 process representing random phase modulations (for simplicity, taken to be identical for both constituents). One can show that the autocovariance of h is the sum of the autocovariance of h_0 and h_1 (assuming that the crosscovariance of h_0 and h_1 is 0). Denoting $R(\tau)$ the autocovariance, following Geoffroy and Nycander, 2022, we have:

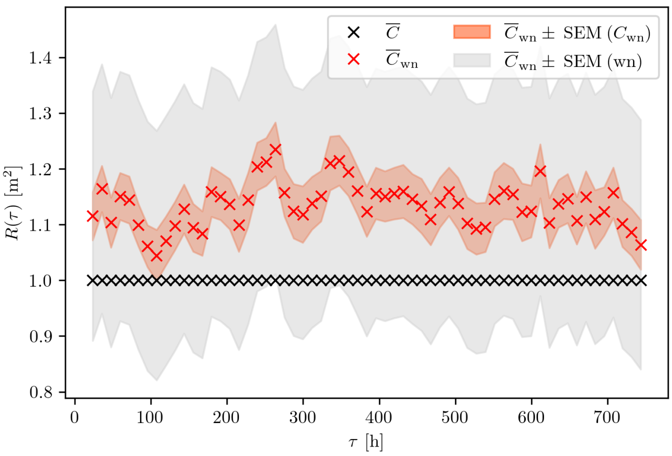
$$R_h(\tau) = R_{h_0}(\tau) + R_{h_1}(\tau) = \exp(-\sigma_\phi^2 + R_\phi(\tau))\big(\frac{A_i^2}{2}\cos(\omega_i\tau) + \frac{A_j^2}{2}\cos(\omega_j\tau)\big).$$

By noting that $A\cos a + B\cos b = A\cos(\frac{a+b+a-b}{2}) + B\cos(\frac{b+a+b-a}{2})$, *expanding and recollecting the terms, one can rewrite the expression for* $R_h(\tau)$ *as:*

$$R_h(\tau) = \exp(-\sigma_\phi^2 + R_\phi(\tau))\Big(\frac{A_i^2 + A_j^2}{2}\cos(\frac{\omega_i+\omega_j}{2}\tau)\cos(\frac{\omega_i-\omega_j}{2}\tau) - \frac{A_i^2 - A_j^2}{2}\sin(\frac{\omega_i+\omega_j}{2}\tau)\sin(\frac{\omega_i-\omega_j}{2}\tau)\Big),$$

here $\frac{\omega_i+\omega_j}{2}$ *is the mean frequency and* $\frac{\omega_i-\omega_j}{2}$ *the beat envelope.*

l175: I don't understand why a "reasonable" estimate of the uncertainty of the envelope is the estimated uncertainty of the oscillating function itself. For example, if we had a constant-amplitude sinusoid plus white noise, the uncertainty of the envelope would be much smaller than the pointwise scatter of the sinusoidal curve, assuming the time series length is much longer than the period of oscillation. (Although -- after finishing the whole manuscript, the error bars don't seem to play a significant role in any conclusions.)
Ans: *Replaced "reasonable" by "conservative". As stated in the text, the demodulate of the sample mean acov is an overestimate of the envelope of the true mean acov, and even more so as time lag increases (see added appendix A for the proof). We illustrate this point below. We computed 100 32-day long synthetic time series of a cosine at the semidiurnal frequency with amplitude 1 plus a white noise (wn) with std* $\sqrt{5}$. *Each time series thus represent a sample acov affected by sampling errors (the white noise). Note that for actual data, the sampling errors would grow with time lag instead of being constant. For each time series we compute the 48-h demodulates. Then, these are averaged over the 100 samples. The result is plotted below (red crosses,* $\overline{C}_{\rm wn}$*) along with the demodulates of the pure cosine (black crosses,* $\overline{C}$*). From there, it is clear that the demodulate of the noisy cosine is an overestimate, in the mean, of the amplitude of the cosine.*



Estimating the uncertainty of a demodulate from its distribution over the 100 samples (red shading), would not reflect this systematic bias. Instead, we chose to use the uncertainty of the underlying stochastic signal as an estimate of the uncertainty of the demodulate. This is a conservative estimate, as shown by the gray shading.

Fig 2: The caption mentions that the zero-lag Argo variance is 121m^2. If 25m^2 of this is due to internal tide, it leaves nearly 100m^2 as "noise" variance -- due to instrument noise or rapidly-decorrelating oceanographic signals. Perhaps this value is plausible for the broadband internal wave variance, but I am not an expert. Could you comment on the origin of the 100m^2 nontidal variance at zero lag? Is it comparable to the value obtained from the mooring in Fig 4?
Ans: *This value is in line with the broadband internal wave variance computed from similar Argo park-phase data by Hennon et al., 2014 (see their Fig. 11, and Fig. 12 for a comparison with GM*

spectrum). The mooring in Fig. 4 shows a variance of 151 m^2 and first demodulate of 29 m^2, yielding a comparable “noise” variance. Note that in the latest revision we systematically subtract an estimate of the acov of the background noise from the sample acov (see appendix C). Thus, the values of the sample acov at 0 time lag we plot now cannot be related to the variance of the noise anymore.

l223: "criteria" --> "criterion"
Ans.: *Corrected.*

l223-224: I thought the latter criterion was already taken care of by the low temperature gradient test. I don't understand why these two tests (low temperature gradient and excessive eta_1000 variance) would need to be applied.
Ans.: *This is right. Still, we found a few HYCOM particles (41 out of 41,644) with an excessive Lagrangian eta_1000 variance having a significant influence on the global mean acov plotted in Fig. 11a. We chose to discard these from our analysis. Including it instead only marginally affects the demodulates of the global mean acov, with no qualitative change to our results. We did not investigate more the cause of these extreme values.*

Fig 8e+f: It is a little suspicious to me that the Argo envelope for lags from about 50 to 150hr extrapolates to zero lag rather close to the zero lag value of HYCOM. Is your procedure for estimating the envelope validated for the case of a large "nugget" at zero lag? I wonder if there is a bias issue connected with how the huge variance at zero lag (in the Argo data) could "leak" into the demodulate amplitude for small lags, somewhat analogous to spectral leakage? It might be useful to look at these autocovariances in the spectral domain, instead.

To make this question concrete: Suppose the Argo floats sample an oceanographic process that decorrelates over 3 hours, but the variance of this process is about 5 times larger than the variance of the IT. How much will your estimate of IT variance, based on the demodulate amplitude at 48hr, be affected by the rapidly-decorrelating process? What if the rapid process decorrelates over 12hr, instead?
Ans.: *This is a very pertinent remark. We carefully assessed how the non-tidal variability (noise) affects the demodulates. The results are presented in the added appendix B. There are three main conclusions: (i) The noise can affect the first demodulates. (ii) the contribution of the noise to the demodulates can be either positive or negative, depending on its characteristic timescale (an effect of filtering the time series) (iii) the Argo demodulates are only marginally biased high.*

However, we expect the stochastic background noise affecting the simulated data to be different from the one recorded by the in situ instruments. Therefore, to limit any systematic bias in the comparisons presented in this work, we chose to consistently correct the sample acov by subtracting an estimate of the acov of the non-tidal variability before performing the demodulation. The way we estimate the acov of the background noise is presented in the added appendix C. This accounts for slight quantitative changes throughout the manuscript. Our results remain unchanged.

l300: The spring-neap period is about 15 days, not twice this.
Ans.: *Corrected.*

l301: "Note that ... process." omit? I don't understand this. Why "strong/short" and "weak/long"? Isn't this exactly the distinction you can make, i.e., you can't distinguish strong/long from weak/short?
Ans.: *Deleted as it was not needed and mainly confusing. This sentence referred to possibly similar stationary limits for (i) strong and short, and (ii) weak and long decorrelations. The decorrelation time scale is unknown here, so we do not know how close to the stationary limit we get with our 32-*

days time series. In other words, we can distinguish strong from weak, but we cannot say anything about the duration of the decorrelation.

Table 2 + Fig 10: Since these involve ratios of the IT estimated at 48hr lag relative to the estimate at 15-d, it would be important to understand the answer to my above question about how the large-variance rapidly-decorrelating processes in the data might bias the estimate at 48-hr lag.
Ans: *See answer above.*

l391-421: It seems like this could be considerably shortened, emphasizing the conclusions of the last paragraph.
Ans.: *There are two main points there: (i) the possible bias due to higher modes not resolved in HYCOM (roughly limited to 5 modes and 3 in the Southern Ocean), and (ii) the possible bias due to the magnification effect of modes 2 and 3 in the Southern Ocean. It is not straightforward to us how to considerably shorten this without losing clarity.*

l440: add "in the model." ?
Ans.: *Added.*

l454: "coarser" --> "finer" ? Also -- shouldn't you use the HYCOM bathymetry, rather than GEBCO bathymetry, to compute HYCOM modes? Although, I'd be surprised if this made much difference.
Ans.: *"coarser" is correct : the 3D data from HYCOM we used are on a 1 deg resolution grid, while the climatology is 1/4th of a deg. Added precision in the text.*

We linearly interpolated the GEBCO bathymetry to the 1 deg resolution grid. At this downgraded resolution and for our purpose (looking at spatial patterns), we expect no sensible difference using the specific HYCOM bathymetry (we did not check). More particularly, our understanding is that both bathymetry sources are based on Smith and Sandwell's work and its updates.

Table 4: It might be interesting to compare the decorrelation timescale T with the estimate in Zaron 2022, "Baroclinic tidal cusps from satellite altimetry." J. Phys. Oceanogr., 52(12):3123--3137, or mention why you don't think the results should be compared.
Ans.: Changed the method of the comparison to include the results of Zaron, 2022. Also mentioned the outcome in the conclusion.

Report #2

Submitted on 09 Mar 2023
Referee #2: Shane Elipot, selipot@rsmas.miami.edu

This is a very valuable and comprehensive study that compares the semidiurnal tidal variance and autocovariance in a global numerical model and in in situ observations from Argo floats. I am pleased with the revision of this manuscript and pending that the authors address a few specific additional comments listed in my report, I can only recommend publication.

Page 1: I read that you are examining the variance and decay rate of the autocovariance function. But you are examining 1) the variance of the semidiurnal IT, and 2) the decay rate of the autocovariance of the semidiurnal IT. Could you rephrase to make this clearer?
Ans: *Rephrased.*

Page 3: Fill values ?
Ans.: *The fact that we use NaN as a fill value here is an important information. The sample autocovariance can be directly computed from time series with NaNs, with no distortion. This is not the case if we were to use spectral methods such as in Hennon et al. 2014.*

Page 5: If that's the case, should this be in section 4? Section 3 is labeled "Methods".
Ans.: *We use a local example to introduce the methods used in the global comparison to come. Made this clearer in the text.*

So the average of the gradient between 950 and 1050 m? How many grid points in the vertical does this include?
Ans.: *The average of the gradient between 900 and 1100 m, this represents 5 vertical levels in z-coordinates. Made clearer in the text.*

Page 7: How? Reference?
Ans.: *Added appendix with the proof. See report #1 for an illustration of this.*

Page 8: I am not quite sure how to read this example: is this to illustrate the method or are these results general? I think it would be good to clarify your intentions here.
Ans.: *This illustrates the method. Added clarification in the introduction of the section.*

What do you mean? The red crosses are what you call the complex demodulate ... what does it mirror?
Ans.: *"The decay [of the demodulates] mirrors the decorrelation observed by the floats". Here the subject of "mirrors" can only be "the decay".*

Page 9: So you compute the autocovariance at all Eulerian grid points that have been occupied by a HYCOM particle?
Ans.: *No. For a given HYCOM particle, we compute the acov at the successive points occupied by the particle sampled every 12 h.*

Page 10: Are these statistics calculated with the actual data points or with the data density points using some weight?
Ans.: *It is calculated form the data points, there are no weights applied.*

Page 11: This is not scatter plot. I would just say that it is a 2d histogram. Warmer colors indicate larger densities.
Ans.: Corrected.

It would be more useful to state that these are your estimates of the variance.
Ans.: *Corrected.*

Page 12: Not sure what "one" refers to? The simluated Argo floats in HYCOM?
Ans.: *Here, “one” refers to the “semidiurnal IT variance”. Rephrased.*

Page 13: Not sure I understand: Figure 7b seems to show decreasing values of the ratio statistic.
Ans.: *Right, the text was wrong. Clarified the text.*

Page 15: You repeat this many times. I think by now we have understood how you get the variance estimates.
Ans.: *Trimmed the sentence.*

Page 19: eq 5 indicates T_{int} so which one is it?
Ans.: *This is a display issue with the track-change pdf file, T_{int} has been correctly replaced by T everywhere.*